CHAPTER ONE

BASIC CONCEPTS AND UNITS

1.1 Introduction

The study of an electrical engineering involves the analysis of the energy transfer from one form to another or from one point to another. So before beginning the actual study of an electrical engineering, it is necessary to discuss the fundamental ideas about the basic elements of an electrical engineering like electromotive force, current, resistance etc. The electricity is related with number of other types of systems like mechanical, thermal etc. To analyze such transfer, it is necessary to revise the **S.I.** units of measurement of different quantities like work, power, energy etc. in various systems.

1.2 The Structure of Matter

The structure of matter plays an important role in the understanding of fundamentals of electricity. The matter which occupies the space may be solid, liquid or gaseous. The atom is composed of three fundamental particles: neutron, proton and the electron.

Fundamental particles of matter	Symbol	Nature of charge possessed	Mass in Kg.
Neutron	n	0	$1.675x10^{-27}$
proton	p+	+	$1.675x10^{-27}$
electron	e^-	-	$9.107x10^{-31}$

1.3 Concept of Charge

Key Point: *Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C).*

The following table shows the different particles and charge possessed by them.

Particle	Charge possessed in Coulomb	Nature
Neutron	0	Neutral
Proton	1.602×10^{-19}	Positive
Electron	1.602×10^{-19}	Negative

1.3.1 Unit of Charge

As seen from the **Table 1.2** that the charge possessed by the electron is very very small hence it is not convenient to take it as the unit of charge. The unit of the measurement of the charge is Coulomb, so one coulomb charge is defined as

1 coulomb= charge on 6.24×10^{18} electrons

The charge associated with one electron can then be determined from

$$\text{Charge/electron} = e = \frac{1C}{6.24\times10^{18}} = 1.602\times10^{-19}C$$

1.4 Concept of Electromotive Force and Current

- The free electrons are responsible for the flow of electric current.
- A conductor is one which has abundant free electrons. The free electrons in such a conductor are always moving in random directions.

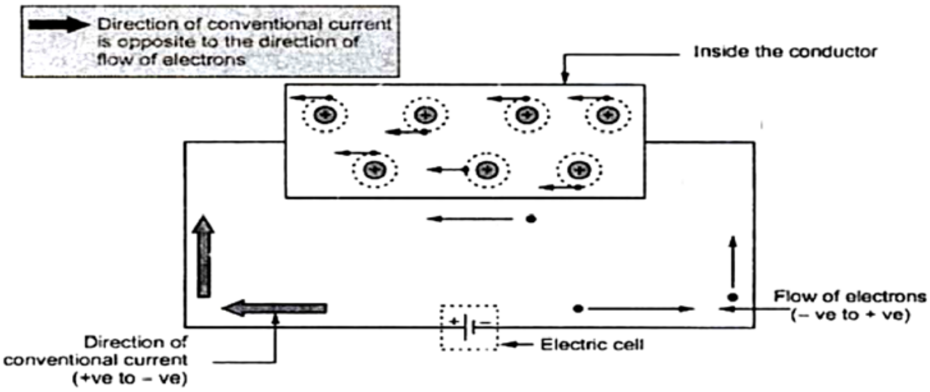


Figure 1.2 the flow of current.

The small electrical effort, externally applied to such conductor makes all such free electrons to drift along the metal in a definite particular direction. This direction depends on how the external electrical effort is applied to the conductor. Such physical phenomenon is represented in the **Fig.1.2**.

- The free electrons as are negatively charged get attracted by positive of the cell connected.
- The flow of electrons from negative to positive of the cell.
- This movement of electron is called an Electric Current. The movement of electrons is always from negative to positive while movement of current is always assumed as from positive to negative. This called direction of conventional current.

1.5 Relation between Charge and Current

The current is flow of electrons. Thus current can be measured by measuring how many electrons are passing through material per second. This can be expressed in terms of the charge carried by those electrons in the material per second.

Key Point: *Electric current is the time rate of change of charge, measured in amperes (A).*

Mathematically we can write the relation between the charge (**Q**) and the electric current (**I**) as,

$$i = \frac{dq}{dt} \tag{1.1}$$

Where current is measured in amperes (A), and

1 ampere = 1 coulomb/second

The charge transferred between time $\mathbf{t_0}$ and **t** is obtained by integrating both sides of **Eq. (1.1)**. We obtain

$$q = \int_{t_0}^{t} idt \tag{1.2}$$

$$I = \frac{Q}{t} \quad \text{Ampere} \tag{1.3}$$

Where **I** = average current flowing, **Q** = total charge transferred

t = time required for transfer of charge.

Definition of 1 Ampere: A current of 1 Ampere is said to be flowing in the conductor when a charge of one coulomb is passing any given point on it in one second.

1 Ampere current = Flow of 6.24×10^{18} electrons per second

Example 1.1: Determine the time required for 4×10^{16} electrons to pass through the imaginary surface of **Fig. 1.5** if the current is 5 mA.

Solution: Determine **Q**

$$Q = 4 \times 10^{16} \frac{1C}{6.24 \times 10^{18} electron} = 0.641 \times 10^{-2}\, C = 6.41 mC$$

Calculate **t** $\quad t = \frac{Q}{I} = \frac{6.41\times10^{-3}C}{5\times10^{-3}A} = 1.282\ s$

1.6 Concept of Electric Potential and Potential Difference

> **Key Point:** ***potential is the energy required to move a unit charge through an element, measured in volts (V).***

The electric potential at point due to a charge is one volt if one joule of work is done in moving a unit positive charge i.e. positive charge of one coulomb from Infinity to that point. Mathematically it is expressed as

Eledrica1 Potential = work done/charge = dw/dq= W/Q (1.4)

1 volt = 1 joule/coulomb = 1 newton.meter/coulomb

In electric circuit flow of current is always from higher electric potential to lower electric potential. So we can define potential difference as below:

> Key Point: ***the difference between the electric potential at any two given points in a circuit is known as potential difference (p.d.) and measured in volts (V).***

Thus, when two points have different potential, the electric current flows from higher potential to lower potential i.e. the electrons start flowing from lower potential to higher potential. No current can flow if the potential difference between the two points is zero.

1.7 Resistance

When the electrons begins flow in the metal. The ions get formed which are charged particles as discussed earlier. Now free electrons are moving in specific direction when connected to external source of **e.m.f.** So such ions always become obstruction for the flowing electrons. So there is collision between ions and free flowing electrons. This not only reduces the speed of electrons but also produced the heat. The effect of this is nothing but the reduction of flow of current. Thus the material opposes the flow of current.

> **Key Point**: ***This property of an electric current circuit tending to prevent the flow of current and at the same time causes electrical energy to be converted to heat is called resistance.***

The resistance is denoted by the symbol '**R**' and is measured in ohm symbolically represented as **Ω**. We can define unit ohm as below.

> Key Point: ***1 Ohm: Is the resistance of a circuit, when a current of 1 Ampere generates the heat at the rate of one joules per second.***

1.7.1 Factors Affecting the Resistance

1. Length of the material: The Length is denoted by '***l***'.

2. Cross-section area: The cross sectional area is denoted by ' **a**'.

3. The type and nature of the material:

4. Temperature: The temperature of the material affects the value of the resistance.

So for a certain material at a certain temperature we can write a mathematical expression as,

$$\boldsymbol{R = \frac{\rho l}{a}} \qquad (1.5)$$

Where ***l***= length in meters, **a**= cross-sectional area in square meters

ρ= resistivity in ohms-meters, **R**= resistance in ohms

1.8 Resistivity and Conductivity

The resistivity or specific resistance of a material depends on nature of material and denoted by **ρ** (rho). From the **eq. (1.6)** of resistance it can be expressed as,

$$\boldsymbol{\rho = \frac{Ra}{l} \quad i.e. \quad \frac{\Omega - m^2}{m} = \Omega - m} \qquad (1.6)$$

Definition: The resistance of the material having unit length and unit cross-sectional area is known as its specific resistance or resistivity.

The Table 1.3 gives the values of resistivity of few common materials.

Material	Resistivity in Ohm-metre at 20° (×10^{-8})	Temperature coefficient at 20° (×10^{-4})
Aluminum, commercial	2.8	40.3
Brass	6-8	20
Carbon	3000-7000	-5
Lead	22	
Copper (annealed)	1.73	39.3
German silver (84% Cu; 12% Ni; 4% Zn)	20.2	2.7
Gold	2.44	36.5
Iron	9.8	65
Manganin (84% Cu; 12% Mn; 4% Ni)	44-48	0.15
Mercury	95.8	8.9
Nichrome (60% Cu; 25% Fe; 15% Cr)	108.5	1.5
Nickel	7.8	54
Platinum	9-15.5	36.7
Silver	1.64	38
Tungsten	5.5	47
Amber	5×10^{14}	
Bakelite	10^{10}	
Glass	10^{10}-10^{12}	
Mica	10^{15}	
Rubber	10^{16}	

1.8.1 Conductance (G)

The conductance of any material is reciprocal of its resistance and ill denoted as **G**. It is the indication of ease with which current can flow through the material. It is measured in Siemens.

So $$G = \frac{1}{R} = \frac{a}{\rho l} = \frac{1}{\rho}\left(\frac{a}{l}\right) = \sigma\left(\frac{a}{l}\right) \tag{1.7}$$

1.8.2 Conductivity

The quantity (**1/ρ**) is called conductivity denoted as **σ** (**sigma**). Thus the conductivity is the reciprocal of resistivity. It is measured in **Siemens/m**.

Key Point: ***A material having highest value of conductivity is the best conductor while with poorest value of conductivity is the best insulator.***

Example 1.2: A coil consists of **2000 turns** of copper wire having a cross-sectional **area of 0.8 mm^2**. The mean **length per turn is 80 cm** and the resistivity of copper is **0.02μΩ-m**. Find the resistance of the coil?

Solution:

Length of the coil, $l = 0.8 * 2000 = 1600$ m

$A = 0.8\ mm^2 = 0.8 * 10^{-6}\ m^2$.

$R = \rho\ (l / A) = 0.02 * 10^{-6} * 1600 / 0.8 * 10^{-6} = 40\ \Omega$.

Example 1.3: The resistance of copper wire 25 m long is found to be 50 **Ω**. If its diameter is 1**mm**, calculate the resistivity of copper

Solution: l= 25 m, **d** = 1 mm, **R** =50Ω

$$\mathbf{a} = \pi/4(d^2) = \pi/4(1^2) = 0.7853\ mm^2$$

$$\rho = \frac{Ra}{l} = \frac{50\times0.7853\times10^{-6}}{25} = 1.57\times10^{-6}\ \Omega\text{-m} = 1.57\ \mu\Omega\text{-m}$$

Example 1.4: A silver wire has resistance of 2.5 **Ω**. What will be the resistance of a manganin wire having a diameter, half of the silver wire and length one third? The specific resistance of manganin is 30 times that of silver.

Solution: R_s = silver resistance= 2.5 Ω, d_m= manganin diameter = $d_s/2$

l_m= manganin length=$l_s/3$, ρ_m= manganin specific resistance = 30 ρ_s

Now $a_s = \pi/4(d_s^2)$ =. area of cross section for silver

$$R_s = \frac{\rho_s l_s}{a_s} = \frac{\rho_s l_s}{\frac{\pi}{4}(d_s)^2} = 2.5\ \Omega$$

$$R_m = \frac{\rho_m l_m}{a_m} = \frac{30\rho_s\times\left(\frac{l_s}{3}\right)}{\frac{\pi}{4}(d_m)^2} = \frac{10\rho_s l_s}{\frac{\pi}{4}\left(\frac{d_s}{2}\right)^2}$$

$$= 40\ \frac{\rho_s l_s}{\frac{\pi}{4}(d_s)^2} = 40R_s = 100\ \Omega$$ Resistance of manganin

1.9 Effect of Temperature on Resistance

The resistance of the material affected as temperature of a material change. As example, Atomic structure theory says that under normal temperature when the metal is subjected to potential difference, ions i.e. unmovable charged particles get formed inside the metal. The electrons which are moving randomly get aligned in a particular direction as shown in the **fig. 1.3**. If temperature increases, the ions gain energy and start oscillating about their mean position and cause collision and obstruction to the flowing electrons. Due to collision and obstruction due to higher amplitude of oscillations of ions, the resistance of material increases as temperature increases. But this is not true for all materials. In some cases the resistance decreases as temperature increase.

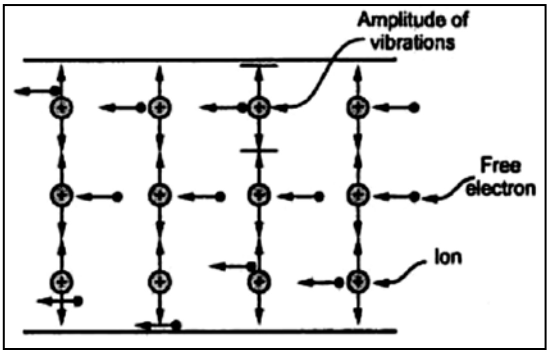


Figure 1.3 Vibrating ions in conductor.

1.9.3 Effect of Temperature on Alloys

The resistance of alloys increase as the temperature increase but rate of increase is not significant. In fact, some of alloys show almost no change in resistance for considerable change in the temperature like Manganin (alloy of copper, manganese and nickel), Eureka (alloy of copper and nickel) etc. Due to this property alloys are used to manufacture the resistance boxes. **Fig.1.5** shows the effect of temperature on metals, insulating materials and alloys.

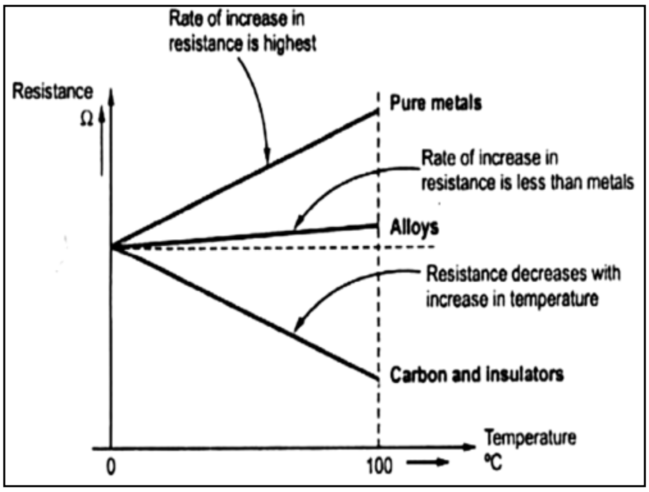


Figure 1.5 effect of temperature on resistance.

1.9.4 Effect of Temperature on Semiconductors

The materials having conductivity between that of metals and insulators are called semiconductors such as silicon, germanium etc. At absolute zero temperature, the semiconductors behave as perfect insulators.

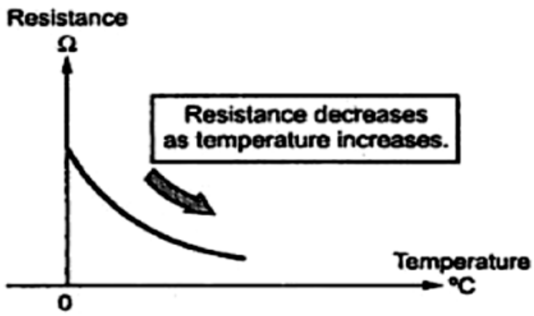


Figure 1.6 Effect of temperature on semiconductor.

For semiconductor materials, an increase in temperature will result in a decrease in the resistance level. Consequently, semiconductors have negative temperature coefficients.

The thermistor and photoconductive cell are excellent examples of semiconductor devices with negative temperature coefficients.

1.10 Resistance Temperature Coefficient (R.T.C.)

From the discussion up till now we can conclude that the change in resistance is,

1) Directly proportional to the initial resistance.

2) Directly proportional to the change in temperature.

3) Depends on the nature of the material whether it is a conductor, alloy or insulator.

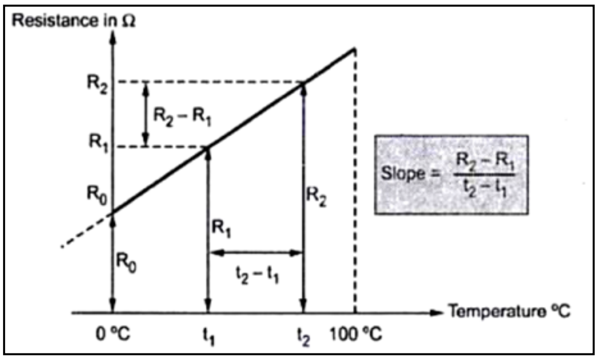


Figure 1.7 resistance vs. temperature.

Let us consider a conductor, the resistance of which increases with temperature linearly.

Let **R_0**= Initial resistance at 0 C^o, **R_1**= Resistance at t_1 C^o, **R_2**=Resistance at t_2 C^o

As shown in the **Fig. 1.7**. $\mathbf{R_2 > R_1 > R_0}$

Definition of R.T.C.: The resistance temperature coefficient at **t** C^o is the ratio of change in resistance per degree Celsius to the resistance at **t** C^o. the unit of **R. T.C.** is 1/C^o.

From the **Fig. 1.7**, change in resistance = $\mathbf{R_2 - R_1}$, change in temperature = $\mathbf{t_2 - t_1}$

$$\text{change in resistance per C}^o = \frac{\Delta R}{\Delta t} = \frac{R_2 - R_1}{t_2 - t_1} = \text{the slope of graph}$$

Hence according to the definition of **R.T.C.** we can write $\boldsymbol{\alpha_1}$ at $\mathbf{t_1}$ C^o as,

$$\alpha_1 = \frac{\text{change in resistance per } C^o}{\text{resistance at } t_1\ C^o} = \frac{\left(R_2 - R_1 / t_2 - t_1\right)}{R_1}$$

1.10.1 Use of R.T.C. In Calculating Resistance at t C^o

Let $\boldsymbol{\alpha_0}$= **R.T.C.** at 0 C^o, $\mathbf{R_0}$ =Resistance at 0 C^o, $\mathbf{R_t}$ = Resistance at **t** C^o

Then
$$\alpha_0 = \frac{\left(R_t - R_0 / t - 0\right)}{R_0} = \frac{R_t - R_0}{tR_0} \Longrightarrow R_t = R_0\,(1 + \alpha_0\, t) \tag{1.8}$$

In general, above result can be expressed as

$$R_{final} = R_{initial}\,[1 + \alpha_{initial}\,\Delta t] \tag{1.9}$$

1.10.2 Effect of Temperature on R.T.C.

From the above discussion, it is clear that the value of **R.T.C.** also changes with the temperature. As the temperature increases, its value decreases. For any metal its value is maximize at 0 C^o.

If starting temperature is $\mathbf{t_1}$ = 0 C^o and a at **t** C^o i.e. $\boldsymbol{\alpha_t}$ is required then we can write,

$$\boxed{\alpha_t = \frac{\alpha_0}{1 + \alpha_0(t - 0)} = \frac{\alpha_0}{1 + \alpha_0 t}} \tag{1.10}$$

1.10.3 Effect of Temperature on Resistivity

Similar to the resistance, the specific resistance or resistivity is a function of temperature. So similar to resistance temperature coefficient we can define temperature coefficient of resistivity as fractional change in resistivity per degree centigrade change in temperature from the given reference temperature.

if $\boldsymbol{\rho_1}$= resistivity at $\mathbf{t_1}$ C^o, $\boldsymbol{\rho_2}$= resistivity at $\mathbf{t_2}$ C^o

Then temperature coefficient of resistivity $\boldsymbol{\alpha}$ at $\mathbf{t_1}$ C^o can be defined as,

$$\alpha_{t1} = \frac{(\rho_2 - \rho_1)/(t_2 - t_1)}{\rho_1} \tag{1.11}$$

Similarly we can write the expression for resistivity at time t C^o as,

$$\boxed{\begin{array}{l}\rho_t = \rho_0\,(1+\alpha_0 t)\\ \rho_{t2} = \rho_{t1}\,[1+\alpha_{t1}\,(t_2-t_1)]\end{array}} \qquad (1.12)$$

Example 1.5: A certain winding made up of copper has a resistance of 100 Ω at room temperature. If resistance temperature coefficient of copper at 0 C° is 0.00428/C°, calculate the winding resistance if temperature is increased to 50 C°. Assume room temperature as 25 C°.

Solution: t_1=25C°, R_1=100 Ω, t_2= 50 C°, α_0= 0.00428/C°

Now

$$\alpha_t = \frac{\alpha_0}{1+\alpha_0 t}$$

$$\alpha_1 = \frac{\alpha_0}{1+\alpha_0 t_1} = \frac{0.00428}{1+0.00428\times 25} = 0.003866/C^o$$

Use $R_2 = R_1\,[1+\alpha_1\,(t_2-t_1)]$ = 100[1+0.003866(50-25)]

=109.6657 Ω resistance at 50 C°

Example 1.6: A specimen of copper has a resistivity (ρ) and a temperature coefficient of 1.6×10^{-6} ohm-cm at 0 C° and 1/254.5 at 20 C° respectively. Find both of them at 60 C°.

Solution: $\rho_0=1.6\times10^{-6}$ ohm-cm = 1.6×10^{-8} ohm-m, $\alpha_1 = \frac{1}{254.5}\ /C^o$ at 20 C°

Now

$$\alpha_t = \frac{\alpha_0}{1+\alpha_0 t} \qquad \alpha_1 = \frac{\alpha_0}{1+\alpha_0\times 20}$$

$$\frac{1}{254.5} = \frac{\alpha_0}{1+20\alpha_0} \quad \Rightarrow \quad 1+20\alpha_0 = 254.5\,\alpha_0$$

$$\alpha_0 = \frac{1}{234.5}\ /C^o \ \text{ at 0 C}^o$$

$$\alpha_{60} = \frac{\alpha_0}{1+\alpha_0\times 60} = \frac{1/234.5}{1+60/234.5} = \frac{1}{294.5}\ /C^o$$

$\rho_t = \rho_0\,(1+\alpha_0 t)$

$$\rho_{60} = 1.6\times10^{-8}\left(1+\frac{1}{234.5}\times 60\right) = 2\times10^{-8}\ \Omega - m$$

1.11 Fundamental Quantities and Units

Scientists and engineers know that the terms they use, the quantities they measure must all be defined precisely. Such precise and standard measurements can be specified only if there is common system of indication of such measurements. This common system of unit is called **'SI'** system i.e. **International System** of Units. The **SI** system is divided into six base units and two supplementary units. The six fundamental or base units are length, mass, time, electric current, temperature, amount of substance and luminous intensity, see **table 1.4**. The two supplementary units are plane angle and solid angle. All other units are derived which are obtained from the above two classes of units. The derived units are classified into three main groups.

1. Mechanical units, 2. Electrical units, 3. Heat units

TABLE 1.4 the six basic SI units.

Quantity	Basic unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd

1.11.1 Multiples and sub-multiples

One great advantage of the SI unit is that it uses prefixes based on the power of 10 to relate larger and smaller units to the basic unit. **Table 1.5** shows the **SI** prefixes and their symbols. For example, the following are expressions of the same distance in meters (m): 600,000,000 mm = 600,000 m = 600 km.

TABLE 1.5 the SI prefixes.

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

- ✓ For American automobiles, engine power is rated in a unit called "**horsepower**," then

1 Horsepower = 745.7 Watt

1.11.3 Electrical Units

The various electrical units are,

1. Electrical work: In an electric circuit, movement of electrons i.e. transfers of charge is an electric current. The electrical work is done when there is a transfer of charge. The unit of such work is Joule.

So if V is potential difference in volts and Q is charge in coulombs then we can write,

Electrical work = $\mathbf{W = V \times Q}$ **J** But $\mathbf{I = Q/t}$,

$$\mathbf{W = V.I.t} \quad \mathbf{J} \qquad \text{where} \quad \mathbf{t = \text{time in second}} \tag{1.21}$$

2. Electrical power: The rate at which electrical work is done in an electric circuit is called an electrical power.

Electrical power = **P = electrical work / time = W / t = V.I.t / t**

$$\mathbf{P = V.I} \qquad \mathbf{J/sec\ i.e.\ watts} \tag{1.22}$$

Thus power consumed in an electric circuit is 1 wall if the potential difference of 1 volt applied across the circuit causes 1 ampere current to flow through it.

3. Electrical energy: An electrical energy is the total amount of electrical work done in an electric circuit.

$$\text{Electrical energy} = \mathbf{E = Power \times Time = V.I.t \ \ joules} \tag{1.23}$$

The unit of energy is joule or watt-sec.

As watt-sec unit is very small, the electrical energy is measured in bigger units as watt-hour (**Wh**) and kilo watt-hour (**kWh**). When a power of 1 kW is utilized for 1 hour, the energy consumed is said to be **1 kWh**. This unit is called a **Unit**.

1.11.5 Efficiency

The efficiency can be defined the ratio of energy output to energy input. It can be also expressed as ratio of power output to power input. Its value is always less than 1. Higher its value, more efficient is the system of equipment. Generally it is expressed in percentage, its symbol **η**.

$$\begin{aligned} \%\,\boldsymbol{\eta} &= \text{Energy output/ Energy input} \times 100 \\ &= \text{Power output/ power input} \times 100 \end{aligned} \tag{1.27}$$

Example 1.7: An electric pump lifts 60 m3 of water per hour to a height of 25 m. The pump efficiency is 82 % and the motor efficiency is 77 %. The pump is used for 3 hours daily. Find the energy consumed per week, if the mass of 1 - m3 of water is 1000 kg.

Solution: 1 m^3 = 1000 kg hence m = 60 m^3 = 60000 kg

h = 25 m, ηm = 77 %, ηp = 82 %, time= 1 hour= 3600 sec

Energy output = mgh = $60000 \times 9.81 \times 25 = 14.715 \times 10^6$ J

$$\text{Pout} = \frac{\text{energy}}{\text{time}} = \frac{14.715 \times 10^6}{3600} = 4087.5 \text{ W}$$

$$\text{Pin} = \frac{P_{out}}{\eta_m \eta_p} = \frac{4087.5}{0.82 \times 0.77} = 6473.7092 \text{ w}$$

Per day 3 hours running hence,

Daily consumption= 6473.7092 × 3 = 19.421 kWh

Weekly power consumption = 7 × 19.421 = 135.947 kWh

Weekly energy consumption = $135.947 \times 10^3 \times 3600 = 489.4124 \times 10^6$ J

CHAPTER TWO

Basic Laws

2.1 Introduction

Chapter 1 introduced basic concepts in an electric circuit. To actually determine the values of these variables in a given circuit requires that we understand some fundamental laws that govern electric circuits. These laws, known as **Ohm's law** and **Kirchhoff's laws**, form the foundation upon which electric circuit analysis is built. In addition to these laws, we shall discuss some techniques commonly applied in circuit design and analysis.

2.2 Network Terminology

In this section, we shall define some of the basic terms which are commonly associated with a network.

1. Network: Any arrangement of the various, electrical energy source along with the different circuit elements is called an electrical network. Such a network is shown in the **Fig. 2.1**.

2. Network Element: Any individual circuit element with two terminals which can be connected to other circuit element is called a network element. Network elements can be either active elements or passive elements.

3. Branch: A part of the network which connects the various points of the network with one another is called a branch. In the **Fig. 2.1**, **AB, BC, CD, DA, DE, CF** and **EF** are the various branches. The branch may consist of more than one element.

4. Junction Point: A point where three or more branches meet is called a junction point. Points D and C are the junction points in the network shown in the **Fig. 2.1**.

5. Node: A point at which two or more elements are joined together is called node. The junction points are also the nodes of the network. In the network shown in the **Fig. 2.1**, **A, B, C, D, E** and **F** are the nodes of the network.

6. Mesh (or Loop): Mesh (or Loops) is a set of branches forming a closed path in a network in such way that if one branch is removed then remaining branches do not form a closed path. In the **Fig. 2.1** paths **A-B-C-D-A, A-B-C-F-E-D-A, D-C-F-E-D** etc are the loops of the network.

In this chapter, the analysis of **d.c.** circuits consisting of pure resistors and **d.c.** sources is included.

2.3 Classification of Electric Networks

The behavior of the entire network depends on the behavior and characteristics of its elements. Based on such characteristics electrical network can be classified as below,

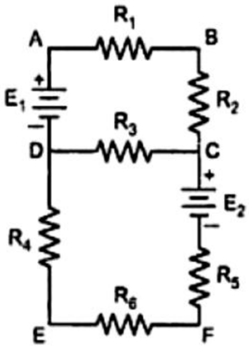


Figure 2.1 an electrical network.

i) Linear Network: A circuit or network whose parameter i.e. elements are always constant irrespective of the change in time, voltage, temperature etc. is known as **linear network**.

ii) Nonlinear Network: A circuit whose parameters change their values with change in time, temperature, voltage etc. is known as **nonlinear network**.

iii) Active Network: A circuit whose contain at least one source of energy is called active. An energy source may be a voltage or current source.

iv) Passive Network: A circuit which contains no energy source is called passive circuit. This is shown in the **Fig 2.2**.

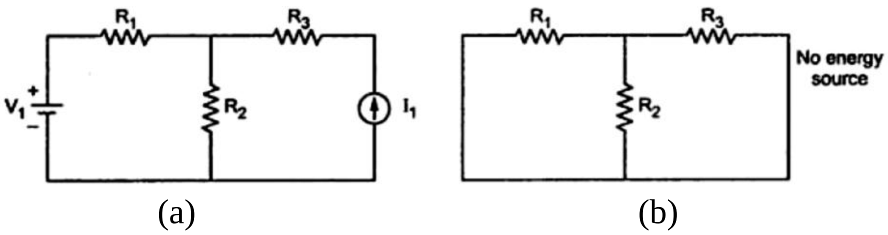


Figure 2.2 (a) active network, (b) passive network

2.4 OHM'S LAW

As shows in chapter one, the materials in general have a characteristic behavior of resisting the flow of electric charge. The resistance ***R*** of any material with a uniform cross-sectional area ***A*** depends on ***A*** and its length ***l***.

The circuit element used to model the current-resisting behavior of a material is the resistor. For the purpose of constructing circuits, resistors are usually made from metallic alloys and carbon compounds. The circuit symbol for the resistor is shown in **Fig. 2.3**, where ***R*** stands for the resistance of the resistor. The resistor is the simplest passive element. Georg Simon Ohm (1787–1854), a German physicist, is credited with finding the relationship between current and voltage for a resistor. This relationship is known as Ohm's law.

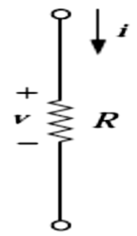


Figure 2.3 Circuit symbol for resistance.

Key Point: *Ohm's law states that the voltage* ***v*** *across a resistor is directly proportional to the current* ***i*** *flowing through the resistor.*

Ohm defined the constant of proportionality for a resistor to be the resistance; **R**. (The resistance is material property which can change if the internal or external conditions of the element are altered, e.g., if there are changes in the temperature.) Thus,

$$v = iR \tag{2.1}$$

The resistance ***R*** of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).

Then $$\mathbf{R = v/i} \tag{2.2}$$

so that $\mathbf{1\ \Omega = 1\ V/A}$

It should be pointed out that not all resistors obey Ohm's law. A resistor that obeys Ohm's law is known as a linear resistor. It has a constant resistance and thus its current-voltage characteristic is as illustrated in **Fig. 2.4(a)**. A nonlinear resistor does not obey Ohm's law. Its resistance varies with current and its i-v characteristic is typically shown in **Fig. 2.4 (b)**. Examples of devices with nonlinear resistance are the light bulb and the diode. A useful quantity in circuit analysis is the reciprocal of resistance **R**, known as conductance and denoted by **G**:

$$\mathbf{G = 1/R = i/v} \tag{2.3}$$

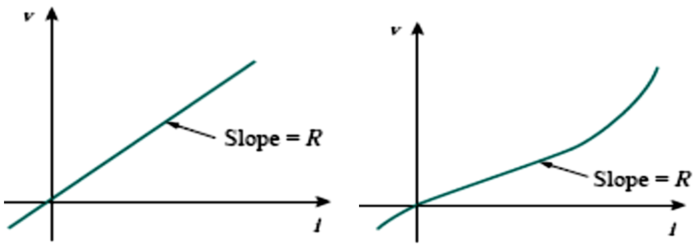


Figure 2.4 The i-v characteristic of: (a) a linear resistor, (b) a nonlinear resistor.

The conductance is a measure of how well an element will conduct electric current. The unit of conductance is the mho (ohm spelled backward) or reciprocal ohm, with symbol ℧, the inverted omega. Although engineers often use the mhos, in this lectures we prefer to use the **Siemens** (**S**), the **SI** unit of conductance:

1 S = 1 ℧ = 1 A/V

Thus,

Conductance is the ability of an element to conduct electric current; it is measured in mhos *(℧)* or Siemens (S).

From **Eq. (2.3)**, we may write

$$\mathbf{i = Gv} \tag{2.4}$$

The power dissipated by a resistor can be expressed in terms of **R**. Using **Eqs. (1.23)** and **(2.1)**,

$$\mathbf{p = vi = i^2R = v^2/R} \tag{2.5}$$

The power dissipated by a resistor may also be expressed in terms of **G** as

$$\mathbf{p = vi = v^2G = i^2/G} \tag{2.6}$$

We should note two things from **Eqs. (2.5)** and **(2.6)**:

1. The power dissipated in a resistor is a nonlinear function of either current or voltage.
2. Since **R** and **G** are positive quantities, the power dissipated in a resistor is always positive. Thus, a resistor always absorbs power from the circuit.

2.4.1 Limitations of Ohm's Law

The Limitations of the Ohm's law are,

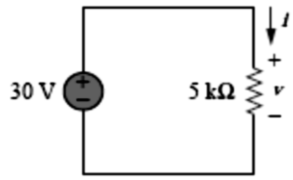
1) It is not applicable to the nonlinear devices such as diode, zener diode, voltage regulators.
2) It does not hold good for non-metallic conductors such as silicon carbide. The law for such conductors is given by, $V = kI^m$ where **k, m** are constants.

EXAMPLE 2.1: An electric iron draws 2 A at 120 V. Find its resistance.

Solution:

From Ohm's law, **R = v/i =120/2 = 60 Ω**

EXAMPLE 2.2: In the circuit shown below, calculate the current i, the conductance G, and the power **P**.



Solution:

The voltage across the resistor is the same as the source voltage (30 V) because the resistor and the voltage source are connected to the same pair of terminals. Hence, the current is

$$\mathbf{i = v / R = 30 \times 5 \times 10^{3} = 6\ mA}$$

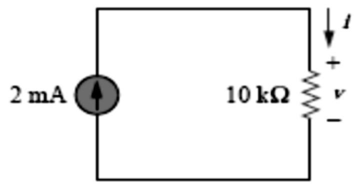
The conductance is $\mathbf{G = 1/ R = 1/ 5 \times 10^{3} = 0.2\ mS}$

We can calculate the power in various ways using either **Eqs. (1.29)**, **(2.5)**, or **(2.6)**.

$$\mathbf{p = vi = 30 \times (6 \times 10^{-3}) = 180\ mW}$$

PRACTICE PROBLEM 2.1: For the circuit shown below, calculate the voltage v, the conductance G, and the power p.

Answer: 20 V, 100 μS, 40 mW.



2.5 SERIES RESISTORS

A series circuit is one in which several resistances are connected one after the other. There is only one path for the flow of current. Consider the resistances shown in the **Fig. 2.5**. The resistance $\mathbf{R_1}$, $\mathbf{R_2}$ and $\mathbf{R_3}$, said to be in series.

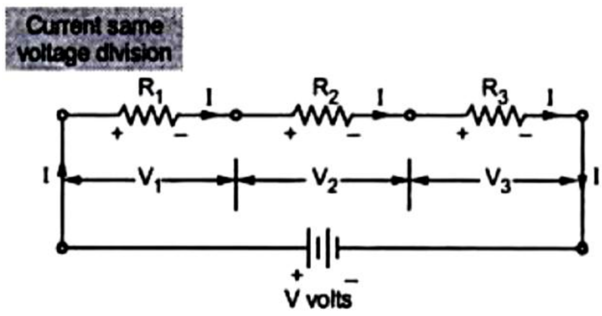


Fig. 2.5 series circuit

Req= Equivalent resistance of the circuit.

$$\mathbf{Req = R_1 + R_2 + R_3}$$

i.e. total or equivalent resistance of the series circuit is arithmetic sum of the resistances connected in series.

For **N** resistances in series, $\mathbf{R = R_1 + R_2 + R_3 + \ldots + R_N}$ (2.7)

If $\mathbf{R_1 = R_2 = \cdots = R_N = R}$, then

$$\mathbf{R_{eq} = N \times R} \quad (2.8)$$

2.5.1 Characteristics of Series Circuits

1) The same current flows through each resistance.

2) The supply voltage **V** is the sum of the individual voltage drops across the resistances.

$$\mathbf{V = V_1 + V_2 + V_3 + \ldots + V_N} \qquad (2.9)$$

3) The equivalent resistance is equal to the sum of the individual resistances.

4) The equivalent resistance is the largest of all the individual resistances.

i.e. $\mathbf{R > R_1, R > R_2, \ldots R > R_N}$

2.6 PARALLEL RESISTORS

The parallel circuit is one in which several resistances are connected across one another in such a way that one terminal of each is connected to form a junction point while the remaining ends are also joined to form another junction point. Consider a parallel circuit shown in the **Fig. 2.6**.

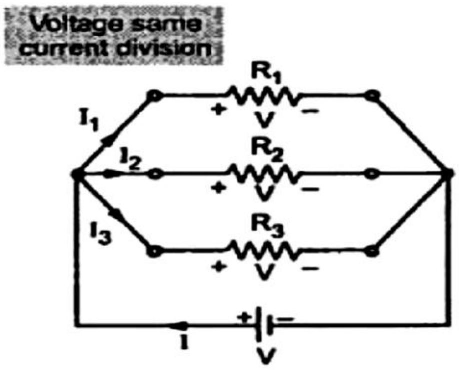


Fig. 2.6 A parallel circuit.

R_{eq}= Total or equivalent resistance of the circuit,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In general if '**N**' resistances are in parallel,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N} \qquad (2.10)$$

Note that **Req** is always smaller than the resistance of the smallest resistor in the parallel combination. If $\mathbf{R_1 = R_2 = \cdots = R_N = R}$, then

$$\mathbf{R_{eq} = R/N} \qquad (2.11)$$

Conductance (G):

It is known that, **1/R = G** (conductance) hence,

$$\mathbf{G = G_1 + G_2 + G_3 + \ldots + G_N} \qquad (2.12)$$

Important result:

Now If **N** = 2, two resistance are in parallel then,.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \ \text{or} \ R = \frac{R_1 R_2}{R_1 + R_2} \qquad (2.13)$$

2.6.1 Characteristics of Parallel Circuits

1) The same potential difference gets across all the resistances in parallel.

2) The total current gets divided into the number of paths equal to the number of resistances in parallel. The total current is always sum of the individual currents.

3) The reciprocal of the equivalent resistance of a parallel circuit is equal to the sum of the reciprocal of the individual resistances.

4) The equivalent resistance is the smallest of all the resistances $\mathbf{R < R_1, R<R_2, R<R_N}$.

5) The equivalent conductance is the arithmetic addition of the individual conductances.

In general, it is often convenient and possible to combine resistors in series and parallel and reduce a resistive network to a single equivalent resistance **Req**.

<u>Example 2.3:</u> Find **Req** for the circuit shown in **Fig. 1**.

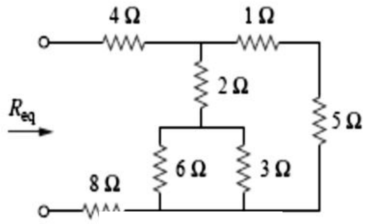


Figure 1

To get $\mathbf{R_{eq}}$, we combine resistors in series and in parallel. The 6-Ω and 3-Ω resistors are in parallel, so their equivalent resistance is

$$\mathbf{6\ \Omega \parallel 3\ \Omega = 6 \times 3/ (6 + 3) = 2\ \Omega}$$

(The symbol || is used to indicate a parallel combination.) Also, the 1-Ω and 5-Ω resistors are in series; hence their equivalent resistance is

$$\mathbf{1\ \Omega + 5\ \Omega = 6\ \Omega}$$

Thus the circuit in **Fig. 1** is reduced to that in **Fig. 2(a)**. **In Fig. 2(a)**, we notice that the two 2-Ω resistors are in series, so the equivalent resistance is

$$\mathbf{2\ \Omega + 2\ \Omega = 4\ \Omega}$$

This 4-Ω resistor is now in parallel with the 6-Ω resistor in **Fig. 2 (a)**; their equivalent resistance is

$$\mathbf{4\ \Omega \parallel 6\ \Omega = 4 \times 6/ (4 + 6) = 2.4\ \Omega}$$

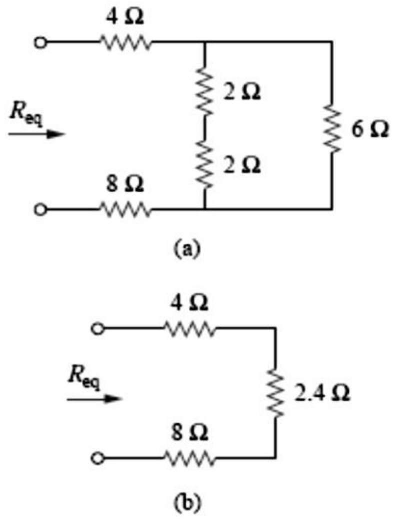


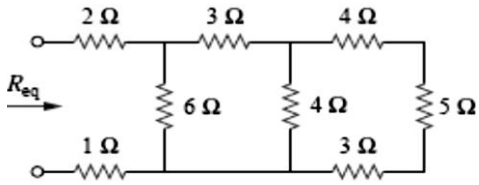
Figure 2

The circuit in **Fig. 2 (a)** is now replaced with that in **Fig. 2 (b)**. In **Fig. 2 (b)**, the three resistors are in series. Hence, the equivalent resistance for the circuit is

$$\mathbf{R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega}$$

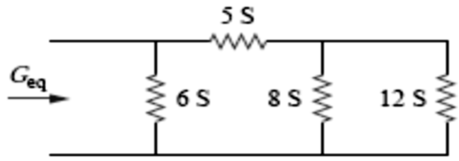
<u>PRACTICE PROBLEM 2.2:</u> By combining the resistors in Figure below, find **Req**.

Answer: 6 Ω.



PRACTICE PROBLEM 2.3: Find the conductance $\mathbf{G_{eq}}$ for the circuit in Figure below.

Answer: 10 S.



2.7 Short and Open Circuits

In the network simplification, short circuit or open circuit existing in the network plays an important role. Since the value of **R** can range from zero to infinity, it is important that we consider the two extreme possible values of **R**.

2.7.1 Short Circuit

When any two points in a network are joined directly to each other with a thick metalic conducting wire the two points are said to be short circuited. The resistance of such short circuit is zero.

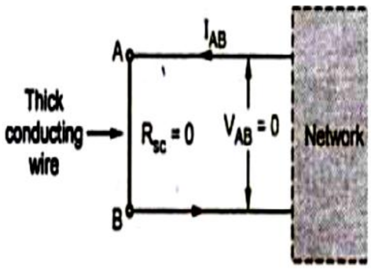


Figure 2.7 Short circuit (R_{sc} = 0)

The part of the network, which is short circuited, is shown in the **Fig. 2.7**. The points A and B are short circuited. The resistance of the branch **AB** is **Rsc**=0. The Current $\mathbf{I_{AB}}$ is flowing through the short circuited path. According to Ohm's law,

$$\mathbf{V_{AB} = R_{sc} \times I_{AB} = 0 \times I_{AB} = 0\ V}$$

> **Key Point:** ***The voltage across short circuit is always zero though current flows through the short circuited path.***

2.7.2 Open Circuit

When there is no connection between the two points of a network, having some voltage across the two points then the two points are said to be open circuited.

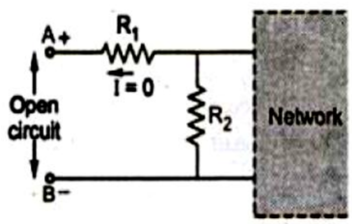


Figure 2.8 Open circuit (R_{OC} =∞).

As there is no direct connection in an open circuit, the resistance of the open circuit is ∞. The part of the network which is open circuited is shown in the **Fig. 2.8**. The points A and B are said to be open circuited. The resistance of the branch AB is $\mathbf{R_{OC}} = \infty\ \Omega$.

According to Ohm's law,

$$\mathbf{I_{OC} = V_{AB} / R_{OC} = V_{AB} / \infty = 0\ A}$$

> **Key Point:** ***The current through open circuit is always zero though there exist voltage across open circuited terminals.***

2.8 The Voltage-divider and Current-divider Circuits

2.8.1 The voltage-divider circuit

Voltage-divider circuit, shown in **Fig.2.9**. We analyze this circuit by directly applying Ohm's law and Kirchhoff's laws. To aid the analysis we introduce the current ***i*** as shown in **Fig.2.9 (b)**. From Kirchhoff's current law $\mathbf{R_1}$ and $\mathbf{R_2}$, carry the same current. Applying **Kirchhoff's voltage law** around the closed loop yields

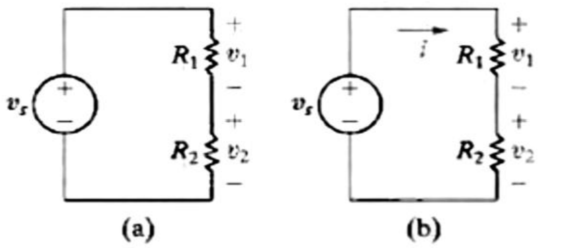


Figure 2.9 (a) A voltage-divider circuit and (b) The voltage-divider circuit with current i indicated

$$\mathbf{v_s = i\,R_1 + i\,R_2}\,,$$

Now we can use Ohm's law to calculate v_1 and, v_2:

$$v_1 = \frac{R_1\, v_s}{R_1 + R_2}, \quad v_2 = \frac{R_2\, v_s}{R_1 + R_2} \qquad (2.14)$$

In general, if a voltage divider has **N** resistors ($\mathbf{R_1, R_2, \ldots, R_N}$) in series with the source voltage $\boldsymbol{v_s}$, the Nth resistor ($\mathbf{R_N}$) will have a voltage drop of

$$v_N = \frac{R_N\, v_s}{R_1 + R_2 + \cdots + R_N} = \frac{R_N\, v_s}{R_{eq}} \qquad (2.15)$$

2.8.2 The current-divider circuit

The **current-divider circuit** shown in **Fig. 2.10.** The current divider is designed to divide the current $\boldsymbol{i_s}$ between $\mathbf{R_1}$ and $\mathbf{R_2}$. We find the relationship between the current $\boldsymbol{i_s}$, and the current in each resistor (that is, $\boldsymbol{i_1}$ and $\boldsymbol{i_2}$) by directly applying **Ohm's law and Kirchhoff's current law**. The voltage across the parallel resistors is

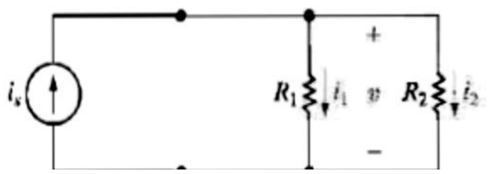


Figure2.10 the current-divider circuit.

$$v = i_1 R_1 = i_2 R_2 = \frac{R_1 R_2}{R_1 + R_2}\, i_s$$

$$i_1 = \frac{R_2\, i_s}{R_1 + R_2}, \quad i_2 = \frac{R_1\, i_s}{R_1 + R_2} \qquad (2.16)$$

If we divide both the numerator and denominator by $\mathbf{R_1R_2}$, **Eq. (2.16)** become

$$i_1 = \frac{G_1\, i_s}{G_1 + G_2}, \quad i_2 = \frac{G_2\, i_s}{G_1 + G_2} \qquad (2.17)$$

Thus, in general, if a current divider has **N** conductors (**G_1, G2, . . . , G_N**) in parallel with the source current i, the nth conductor (**G_N**) will have current

$$i_N = \frac{G_N\, i_s}{G_1 + G_2 + \cdots + G_N} = \frac{R_{eq}\, i_s}{R_N} \tag{2.18}$$

<u>EXAMPLE 2.4:</u> Find i_o and v_o in the circuit shown in **Fig. 1(a)**. Calculate the power dissipated in the 3-Ω resistor.

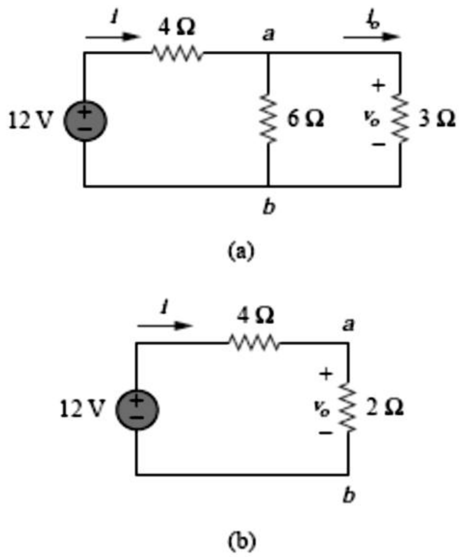


Figure 1(a) Original circuit, (b) Its equivalent circuit.

<u>Solution:</u> The 6-Ω and 3-Ω resistors are in parallel, so their combined resistance is

$$6\ \Omega \parallel 3\ \Omega = 6 \times 3/(6+3) = 2\ \Omega$$

By apply voltage division, since the 12 V in **Fig. 1(b)** is divided between the 4-Ω and 2-Ω resistors. Hence,

$$\boldsymbol{v_o = 2(12\ V)/(2+4) = 4\ V}$$

Apply current division to the circuit in **Fig. 1(a)** now that we know ***i***, by writing

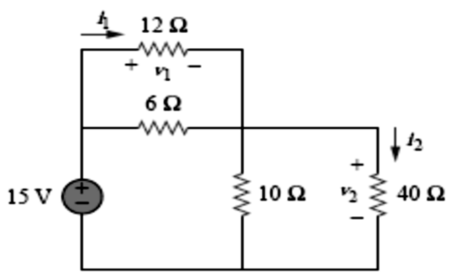
$$\boldsymbol{i = 12/\ 4 + 2 = 2\ A}$$
$$\boldsymbol{i_o = 6\, i\, /(6+3) = 4\,/3\ A}$$

The power dissipated in the 3-Ω resistor is

$$p_o = v_o\, i_o = 4(4/3) = 5.333\ W$$

<u>PRACTICE PROBLEM 2.4:</u> Find v_1 and v_2 in the circuit shown in Figure below. Also calculate i_1 and i_2 and the power dissipated in the 12-Ω and 40-Ω resistors.

<u>Answer:</u> v_1 = 5 V, i_1 = 416.7 mA, p_1 = 2.083 W, v_2 = 10 V, i_2 = 250 mA, p_2 = 2.5 W.



2.9 WYE-DELTA TRANSFORMATIONS

Situations often arise in circuit analysis when the resistors are neither in parallel nor in series. For example, consider the bridge circuit in **Fig. 2.11**. How do we combine resistors $\mathbf{R_1}$ through $\mathbf{R_6}$ when the resistors are neither in series nor in parallel? Many circuits of the type shown in **Fig. 2.11** can be simplified by using three-terminal equivalent networks. These are the wye (**Y**) or tee (**T**) network shown in **Fig. 2.12** and the delta (**Δ**) or pi (**π**) network shown in **Fig. 2.13**.

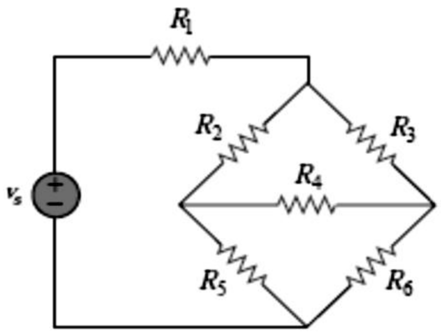


Figure 2.11The bridge network.

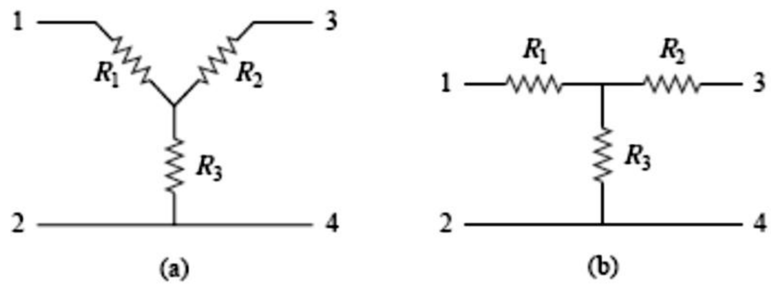


Figure 2.12 Two forms of the same network: (a) Y, (b) T.

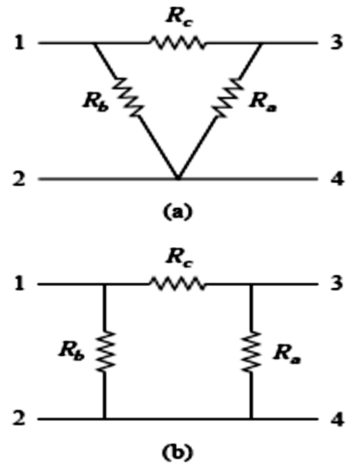


Figure 2.13 Two forms of the same network: (a) Δ, (b) π.

Delta to Wye Conversion

Suppose it is more convenient to work with a **wye** network in a place where the circuit contains a delta configuration. We superimpose a **wye** network on the existing **delta** network and find the equivalent resistances in the **wye** network. For terminals 1 and 2 in **Figs. 2.12** and **2.13**, for example, $\mathbf{R_{12}(Y) = R1 + R3}$, $\mathbf{R_{12}\,(\Delta) = R_b \parallel (R_a + R_c)}$ (2.19)

Setting $\mathbf{R_{12}(Y) = R_{12}\,(\Delta)}$ gives

$$\mathbf{R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c}}$$

$$\mathbf{R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}} \qquad \mathbf{R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c}} \tag{2.20}$$

By solving previous equations, we get

$$\mathbf{R_1 = \frac{R_b\, R_c}{R_a + R_b + R_c}} \tag{2.21}$$

$$\mathbf{R_2 = \frac{R_c\, R_a}{R_a + R_b + R_c}} \tag{2.22}$$

$$\mathbf{R_3 = \frac{R_a\, R_b}{R_a + R_b + R_c}} \tag{2.23}$$

Wye to Delta Conversion

Reversing the **Δ-to-Y** transformation also is possible. That is, we can start with the **Y** structure and replace it with an equivalent **Δ** structure. The expressions for the three **Δ**-connected resistors as functions of the three **Y**-connected resistors are

$$\mathbf{Ra} = \frac{\mathbf{R1R2 + R2R3 + R3R1}}{\mathbf{R1}} \quad (2.24)$$

$$\mathbf{Rb} = \frac{\mathbf{R1R2 + R2R3 + R3R1}}{\mathbf{R2}} \quad (2.25)$$

$$\mathbf{Rc} = \frac{\mathbf{R1R2 + R2R3 + R3R1}}{\mathbf{R3}} \quad (2.26)$$

The Y and Δ networks are said to be balanced when

$$\mathbf{R_1 = R_2 = R_3 = R_Y, R_a = R_b = R_c = R_\Delta} \quad (2.27)$$

Under these conditions, conversion formulas become

$$\mathbf{R_Y = R_\Delta / 3 \text{ or } R_\Delta = 3R_Y} \quad (2.28)$$

EXAMPLE 2.5: Obtain the equivalent resistance $\mathbf{R_{ab}}$ for the circuit in **Fig. 1** and use it to find current **i**.

Solution:

In this circuit, there are two **Y**-networks and one **Δ**-network. Transforming just one of these will simplify the circuit. If we convert the **Y**-network comprising the 5-Ω, 10-Ω, and 20-Ω resistors, we may select

$$\mathbf{R_1 = 10\ \Omega, \quad R_2 = 20\ \Omega, \quad R_3 = 5\ \Omega}$$

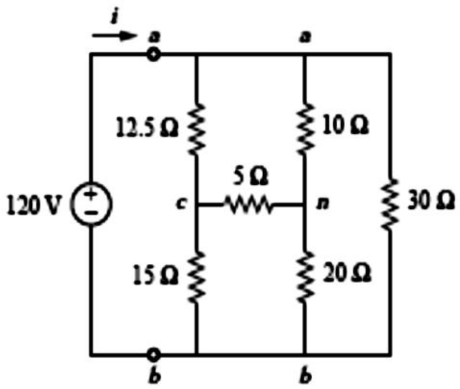


Figure 1.

Thus, from Eqs. (2.24) to (2.26) we have

$$\mathbf{Ra} = \frac{\mathbf{R1R2 + R2R3 + R3R1}}{\mathbf{R1}} = \frac{\mathbf{10{\times}20{+}20{\times}5{+}5{\times}10}}{\mathbf{10}} = \frac{350}{10} = 35\ \Omega$$

$$\mathbf{Rb} = \frac{\mathbf{R1R2 + R2R3 + R3R1}}{\mathbf{R2}} = \frac{350}{20} = 17.5\Omega$$

$$\mathbf{Rc} = \frac{\mathbf{R1R2 + R2R3 + R3R1}}{\mathbf{R3}} = \frac{350}{5} = 70\Omega$$

With the **Y** converted to **Δ**, the equivalent circuit (with the voltage source removed for now) is shown in **Fig. 2 (a)**. Combining the three pairs of resistors in parallel, we obtain

$$\mathbf{70 \parallel 30 = 70 \times 30/ (70 + 30) = 21\ \Omega}$$

$$\mathbf{12.5 \parallel 17.5 = 12.5 \times 17.5/ (12.5 + 17.5) = 7.2917\ \Omega}$$

$$\mathbf{15 \parallel 35 = 15 \times 35/ (15 + 35) = 10.5\ \Omega}$$

so that the equivalent circuit is shown in **Fig. 2 (b)**. Hence, we find

$$R_{ab} = (7.292 + 10.5) \parallel 21 = 17.792 \times 21/ (17.792 + 21) = 9.632\ \Omega$$

Then

$$i = vs/ Rab = 120/ 9.632 = 12.458\ A$$

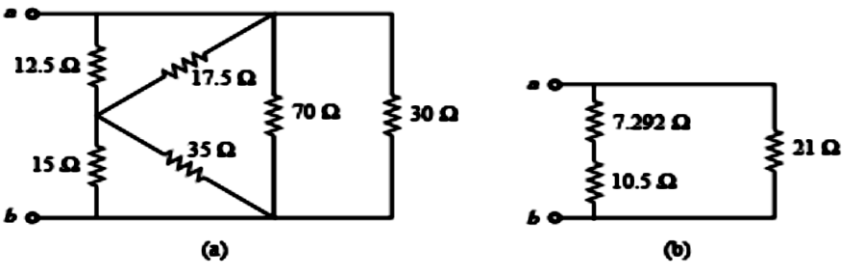
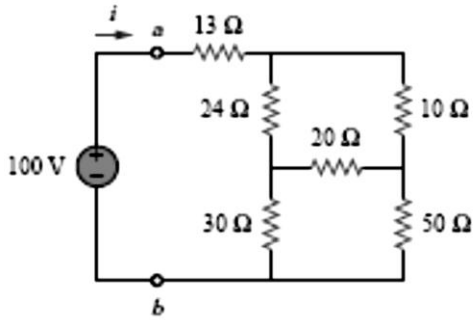


Figure 2 Equivalent circuits to Fig. 1, with the voltage removed.

PRACTICE PROBLEM 2.5: For the bridge network in Figure below, find R_{ab} and **i**.

Answer: 40 Ω, 2.5 A.



2.10 Energy Sources

There are basically two types of energy sources; voltage source and current source. These sources are classified as i) Ideal source and ii) Practical source. Let us see the difference between Ideal and practical sources.

2.10.1 Voltage Source

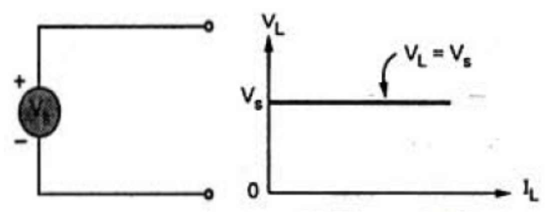
***Ideal voltage source:**

Ideal voltage source is defined as the energy source which gives constant voltage across its terminals irrespective of the current drawn through its terminals. This is indicated by **V- I** characteristics shown in the **Fig. 2.14 (b)**.

***Practical voltage source:**

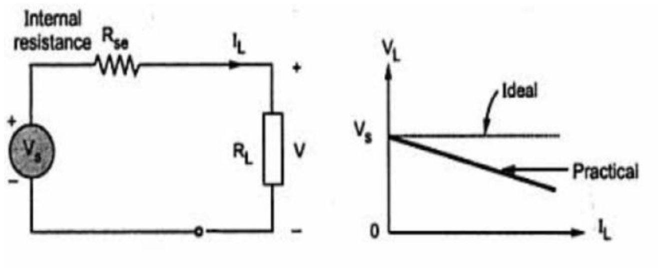
But practically, every voltage source has small internal resistance shown in series with voltage source and is represented by R_{se} as shown in the **Fig. 2.15**. Because of the R_{se}, voltage across terminals decreases slightly with increase in current and it is given by expression,

$$V_L = V_S - I_L R_L$$



(a) symbol **(b) characteristics**

Figure 2.14 Ideal voltage source.



(a) circuit **(b) characteristics**

Figure 2.15 Practical voltage source.

Voltage sources are further classified as follows,

i) Time invariant Sources:

The sources in which voltage is not varying with time are known as time invariant voltage source or **D.C.** sources. These are denoted by capital letters. Such a source is represented in the **Fig. 2.16 (a)**.

ii) **Time Variant Source**:

The sources in which voltage is varying with time are known as time variant voltage sources or **A.C.** sources. These are denoted by small letters. This is shown in the **Fig. 2.16 (b)**.

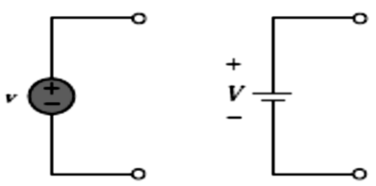


Figure 2.16 (a) D.C. sources.

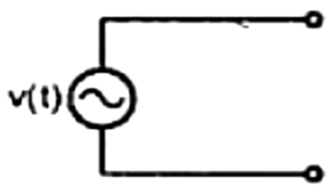


Figure 2.16(b) A.C. source.

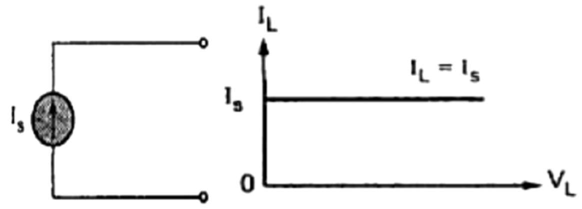
2.10.2 Current Source

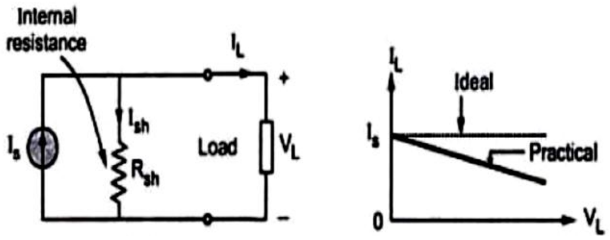
***Ideal current source:**

Ideal current source is the source which gives constant current at its terminals irrespective of the voltage appearing across its terminal. This is explained by **V-I** characteristics shown in the **Fig. 2.17 (b)**.

***Practical current source:**

But practically, every current source has high internal resistance, shown in parallel with current source and It is represented by $\mathbf{R_{sh}}$. This is shown in the **Fig. 2.18**. Because of $\mathbf{R_{sh}}$, current through its terminals decreases slightly with voltage at its terminals.





(a) symbol **(b) characteristics** **(a) circuit** **(b) characteristics**

Figure 2.17 ideal current source. **Figure 2.18 ideal current source.**

Similar to voltage sources, current sources are classified as follows,

i) **<u>Time Invariant Sources:</u>**

The sources in which current is not varying with time are known as time invariant current sources or D.C. sources. These are denoted by capital letters. Such a current source is represented in the **Fig. 2.19 (a)**.

ii) **<u>Time Variant Sources:</u>**

The sources in which current is varying with time are known as time variant current sources or **A.C. sources**. These are denoted by small letters. Such source is represented in the **Fig. 2.19 (b)**.

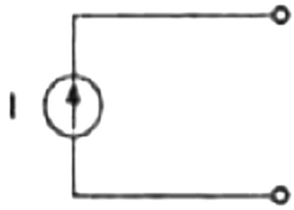


Figure 2.19 (a) D.C. source.

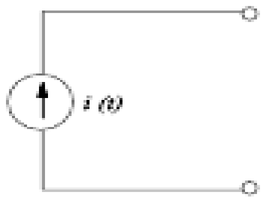


Fig. 2.19 (b) A.C. source.

The sources, which are discussed above are independent sources because these sources does not depend on other voltage or currents in the network for their value. These are represented by a circle with a polarity of voltage or direction of current indicated inside

<u>2.10.3 Dependent Sources</u>

Dependent source are those whose value of source depends on voltage or current in the circuit. Such sources are indicated by diamond as shown in the **Fig. 2.20** and further classified as,

i) **Voltage-Controlled Voltage Source (VCVS)**: It produces a voltage as a function of voltage elsewhere in the given circuit. It is shown in the **Fig. 2.20 (a)**. The controlling voltage is named v_x the equation that determines the supplied voltage v_s is

$v_s = \mu\ v_x$, and the reference polarity for v_s is as indicated. Note that μ is a multiplying constant that is dimensionless.

ii) **Current-Controlled Voltage Source (CCVS)**: It produces voltage as a function of current elsewhere in the given circuit. It is shown In the **Fig. 2.20(b)**. the controlling current is $\boldsymbol{i_x}$ the equation for the supplied voltage $\boldsymbol{v_s}$ is $\boldsymbol{v_s = \rho\, i_x}$,

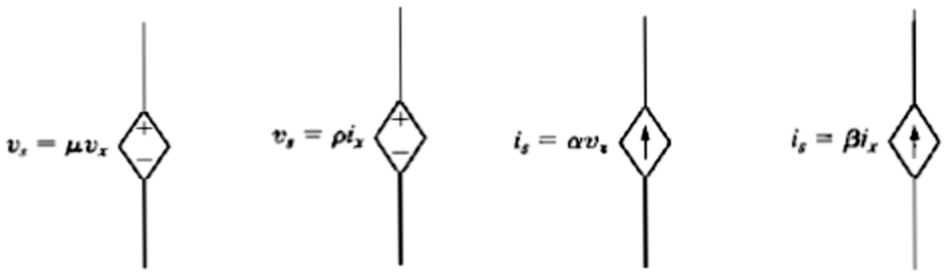
the reference polarity is as shown and the multiplying constant **ρ** has the dimension volts per ampere

iii) **Voltage-Controlled Current Source (VCCS)**: It produces current as a function of voltage elsewhere in the given circuit. It is shown in the **Fig. 2.20(c)**. The controlling voltage is $\boldsymbol{v_x}$, the equation for the supplied current $\boldsymbol{i_s}$ is $\boldsymbol{i_s = \alpha\, v_x}$,

the reference direction is as shown and the multiplying constant **α** has the dimension amperes per volt.

iv) **Current-Controlled Current Source (CCCS)**: It produces current as a function of current elsewhere in the given circuit. It is shown in the **Fig. 2.20 (d)**. the controlling current is $\boldsymbol{i_x}$ the equation for the supplied current $\boldsymbol{i_s}$ is $\boldsymbol{i_s = \beta\, i_x}$,

the reference direction is as shown, and the multiplying constant $\boldsymbol{\beta}$ is dimensionless.



(a) (b) (c) (d)

Figure 2.20 The circuit symbols a) an ideal dependent voltage-controlled voltage source, (b) an ideal dependent current-controlled voltages source, (c) an ideal dependent voltage-controlled current source (d) an ideal dependent current-controlled current source.

Dependent sources are useful in modeling elements such as transistors, operational amplifiers and integrated circuits. An example of a current controlled voltage source is shown on the right-hand side of **Fig. 2.21**, where the voltage 10i of the voltage source depends on the current $\boldsymbol{i}$ through element C.

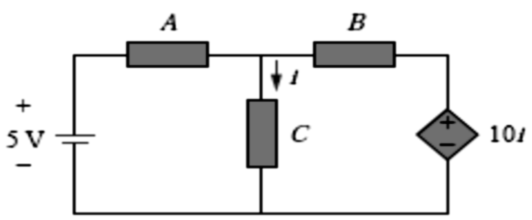


Figure 2.21 the source on the right-hand side is a current-controlled voltage source.

2.11 Combinations of Sources

In a network consisting of many sources, series and parallel combinations of sources exist. If such combinations are replaced by the equivalent source then the network simplification becomes much easier. Let us consider such series and parallel combinations of energy sources.

2.11.1 Voltage Sources in Series

If two voltage sources are in series then the equivalent is dependent on the polarities of the two sources. Consider the two sources as shown in the **Fig. 2.22**.

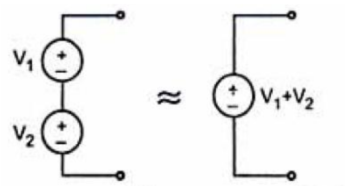


Figure 2.22

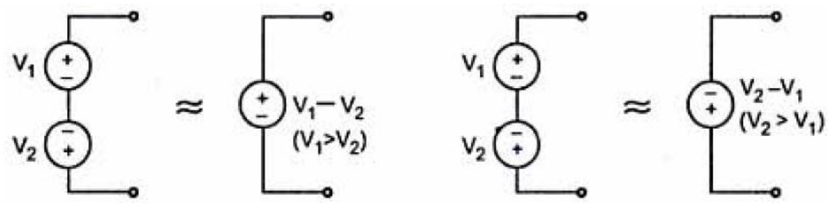


Figure 2.23

If the polarities of the two sources are same then the equivalent single source is the addition of the two sources with polarities same as that of the two sources.

Consider the two sources as shown in the **Fig. 2.23**. If the polarities of the two sources are different then the equivalent single source is the difference between the two voltage sources. The polarity of such source is same as that of the greater of the two sources.

> **Key Point:** *the voltage sources to be connected in series must have same current rating through their voltage ratings may be same or different.*

2.11.2 Voltage Sources in Parallel

Consider the two voltage source in parallel as shown in the **Fig. 2.24**. The equivalent single source has a value same as $\mathbf{V_1}$ and$\mathbf{V_2}$. It must be noted that all the open circuit voltage provided by each source must be equal as the sources are in parallel.

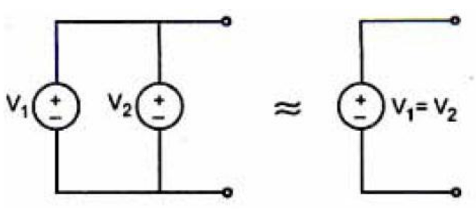


Figure 2.24

> **Key Point:** *the voltage sources to be connected in parallel must have same voltage rating through their current ratings may be same or different.*

2.11.3 Current Sources in Series

Consider the two current sources in series is shown in the **Fig. 2.25**, the equivalent single source has a value same as $\mathbf{I_1}$ and $\mathbf{I_2}$.

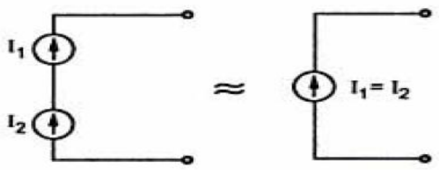
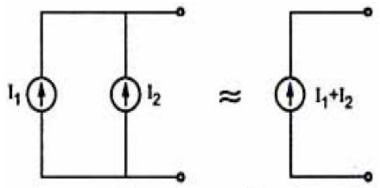


Figure 2.25

> **Key Point:** *the current sources to be connected in series must have same current rating through their voltage ratings may be same or different.*

2.11.4 Current Sources in Parallel

Consider the two current sources in parallel as shown in the **Fig. 2.26**.



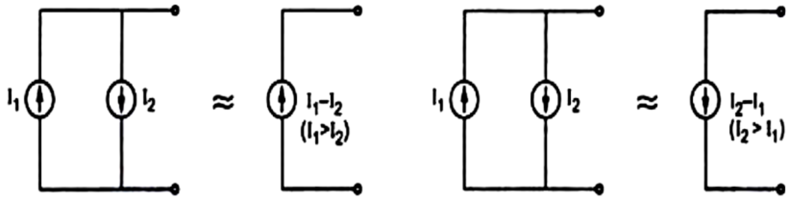


Figure 2.27

Figure 2.26

if the directions of the currents of the sources connected in parallel are same then the equivalent single source is the addition of the two sources with direction same as that of the two sources.

Consider the two current sources with opposite directions connected in parallel as shown in the **Fig. 2.27**. If the directions of the two sources are different then the equivalent single source has a direction same as greater of the two sources with value equal to the difference between the two voltage sources.

> **Key Point:** *the current sources to be connected in parallel must have same voltage rating through their current ratings may be same or different.*

2.12 NOTATION: it will play an increasingly important role in the analysis.

i) Double-Subscript Notation

The fact that voltage is an across variable and exists between two points has resulted in a double-subscript notation that defines the first subscript as the higher potential. In **Fig. 2.28(a)**, the two points that define the voltage across the resistor **R** are denoted by **a** and **b**. Since a is the first subscript for V_{ab}, point a must have a higher potential than point **b** if V_{ab} is to have a positive value. If, in fact, point b is at a higher potential than point a, V_{ab} will have a negative value, as indicated in **Fig. 2.28(b)**.

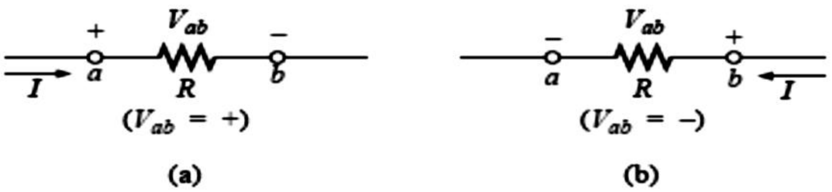


Figure 2.28 defining the sign for double-subscript notation.

In summary:

The voltage V_{ab} is the voltage at point a with respect to (w.r.t.) point b.

ii) <u>Single-Subscript Notation</u>

If point **b** of the notation $\mathbf{V_{ab}}$ is specified as ground potential (zero volts), then a single subscript notation can be employed that provides the voltage at a point with respect to ground.

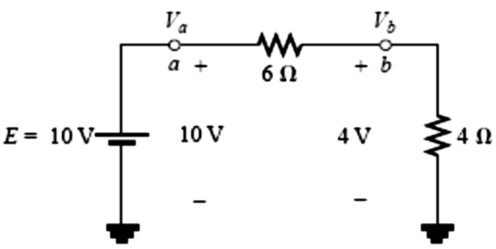


Figure 2.29 defining the use of single-subscript notation for voltage levels.

In **Fig. 2.29**, $\mathbf{V_a}$ is the voltage from point a to ground. In this case it is obviously **10 V** since it is right across the source voltage **E**. The voltage $\mathbf{V_b}$ is the voltage from point **b** to ground. Because it is directly across the **4-Ω** resistor, $\mathbf{V_b}$ = 4 V.

In summary:

The single-subscript notation Va specifies the voltage at point a with respect to ground (zero volts). If the voltage is less than zero volts, a negative sign must be associated with the magnitude of Va .

<u>General Comments</u>

A particularly useful relationship can now be established that will have extensive applications in the analysis of electronic circuits. For the above notational standards, the following relationship exists:

$$\boldsymbol{V_{ab} = V_a - V_b} \tag{2.29}$$

In other words, if the voltage at points **a** and **b** is known with respect to ground, then the voltage $\mathbf{V_{ab}}$ can be determined using **Eq. (2.29)**. In **Fig. 2.29**, for example,

$$\boldsymbol{V_{ab} = V_a - V_b = 10\text{ V} - 4\text{ V} = 6\text{ V}}$$

<u>2.13 KIRCHHOFF’S LAWS</u>

Ohm’s law by itself is not sufficient to analyze circuits. However, when it is coupled with Kirchhoff’s two laws, we have a sufficient, powerful set of tools for analyzing a large variety of electric circuits. Kirchhoff's laws were first introduced in 1847 by the German physicist Gustav Robert Kirchhoff (1824–1887). These laws are formally known as Kirchhoff's current law (**KCL**) and Kirchhoff's voltage law (**KVL**).

2.13.1 Kirchhoff's current law

Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero or the sum of the currents entering a node is equal to the sum of the currents leaving the node.

Mathematically, **KCL** implies that

$$\sum_{n=1}^{N} i_n = 0 \qquad (2.30)$$

where **N** is the number of branches connected to the node and i_n is the nth current entering (or leaving) the node.

Consider the node in **Fig. 2.30**. Applying **KCL** gives

$$\mathbf{i_1 + (-i_2) + i_3 + i_4 + (-i_5) = 0} \qquad (2.31)$$

since currents $\mathbf{i_1}$, $\mathbf{i_3}$, and $\mathbf{i_4}$ are entering the node, while currents $\mathbf{i_2}$ and $\mathbf{i_5}$ are leaving it. By rearranging the terms, we get

$$\mathbf{i_1 + i_3 + i_4 = i_2 + i_5} \qquad (2.32)$$

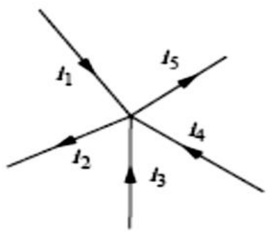


Figure 2.30 Currents at a node illustrating KCL.

A simple application of **KCL** is combining current sources in parallel. The combined current is the algebraic sum of the current supplied by the individual sources. For example, the current sources shown in **Fig. 2.31(a)** can be combined as in **Fig. 2.31(b)**. The combined or equivalent current source can be found by applying **KCL** to node **a**.

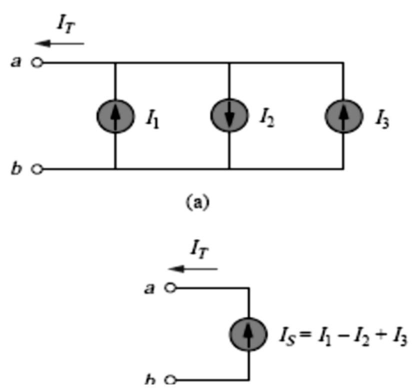


Figure 2.31 Current sources in parallel: (a) original circuit, (b) equivalent circuit.

$$\mathbf{I_T + I_2 = I_1 + I_3}$$

or

$$\mathbf{I_T = I_1 - I_2 + I_3}$$

A circuit cannot contain two different currents, $\mathbf{I_1}$ and $\mathbf{I_2}$, in series, unless $\mathbf{I_1} = \mathbf{I_2}$; otherwise **KCL** will be violated.

2.13.2 Kirchhoff's voltage law

Kirchhoff's second law is based on the principle of conservation of energy:

Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

Expressed mathematically, **KVL** states that

$$\sum_{m=1}^{M} v_m = 0 \tag{2.33}$$

Where **M** is the number of voltages in the loop (or the number of branches in the loop) and $\boldsymbol{v_m}$ is the mth voltage.

To illustrate **KVL**, consider the circuit in **Fig. 2.32**. The sign on each voltage is the polarity of the terminal encountered first as we travel around the loop. We can start with any branch and go around the loop either clockwise or counterclockwise. Suppose we start with the voltage source and go clockwise around the loop as shown; then voltages would be $\mathbf{-v_1}$, $\mathbf{+v_2}$, $\mathbf{+v_3}$, $\mathbf{-v_4}$, and $\mathbf{+v_5}$, in that order. For example, as we reach branch 3, the positive terminal is met first; hence we have $+v_3$. For branch 4, we reach the negative terminal first; hence, $-v_4$. Thus, **KVL** yields

$$-\mathbf{v_1} + \mathbf{v_2} + \mathbf{v_3} - \mathbf{v_4} + \mathbf{v_5} = \mathbf{0} \tag{2.34}$$

Rearranging terms gives

$$\mathbf{v_2} + \mathbf{v_3} + \mathbf{v_5} = \mathbf{v_1} + \mathbf{v_4} \tag{2.35}$$

which may be interpreted as

Sum of voltage drops = Sum of voltage rises (2.36)

This is an alternative form of **KVL**. Notice that if we had traveled counterclockwise, the result would have been $\mathbf{+v_1}$, $\mathbf{-v_5}$, $\mathbf{+v_4}$, $\mathbf{-v_3}$, and $\mathbf{-v_2}$, which is the same as before, except that the signs are reversed. Hence, **Eqs. (2.34) and (2.35)** remain the same.

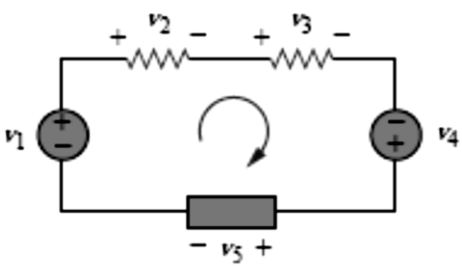


Figure 2.32 A single-loop circuit illustrating KVL.

When voltage sources are connected in series, **KVL** can be applied to obtain the total voltage. The combined voltage is the algebraic sum of the voltages of the individual sources.

2.13.3 Steps to Apply Kirchhoff. Laws to Get Network Equations

The steps are stated based on the branch current method.

Step 1: Draw the circuit diagram from the given information and insert all the value of sources with appropriate polarities and all the resistances.

Step 2: Mark all the branch currents with assumed directions using **KCL** at various nodes and junction points. Kept the number of unknown currents as minimum as far as possible to limit the mathematical calculations required to solve them later on. Assumed directions may be wrong; in such case answer of such current will be mathematically negative which indicates the correct direction of the current.

Step 3: Mark all the polarities of voltage drops and rises as per directions of the assumed branch currents flowing through various branch resistance of the network. This is necessary for application of **KVL** to various closed loops.

Step 4: Apply **KVL** to different closed paths in the network and obtain the corresponding equations. Each equation must contain some element which is not considered in any preview equation.

2.14 Solving Simultaneous Equations and Cramer's Rule

Electric circuit analysis with the help of Kirchhoff's laws usually involves solution of two or three simultaneous equations. These equations can be solved by a systematic elimination of the variables but the procedure is often lengthy and laborious and hence more liable to error. Determinants and Cramer's rule provide a simple and straight method for solving network equations through manipulation of their coefficients. Of course, if the number of simultaneous equations happens to be very large, use of a digital computer can make the task easy. Let us assume that set of simultaneous equations obtained is, as follows,

$$
\begin{aligned}
&a_{11} x_1 + a_{12} x_2 + \ldots\ldots\ldots\ldots + a_{1n} x_n = C_1 \\
&a_{21} x_1 + a_{22} x_2 + \ldots\ldots\ldots\ldots + a_{2n} x_n = C_2 \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad . \\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad . \\
&a_{n1} x_1 + a_{n2} x2 + \ldots\ldots\ldots\ldots + a_{nn} x_n = C_n
\end{aligned}
$$

where $\mathbf{C_1}$, $\mathbf{C_2}$, **………**, $\mathbf{C_n}$ constants. Then Cramer's rule says that form a system determinant **Δ** or **D** as,

$$\Delta = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ & \vdots & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} = D$$

Then obtain the subdeterminant **Dj** by replacing **j**th column of **Δ** by the column of constants existing on right hand side of equations i.e. $\mathbf{C_1, C_2, \ldots. C_n}$;

$$D_1 = \begin{bmatrix} C_1 & a_{12} & \cdots & a_{1n} \\ C_n & a_{22} & \cdots & a_{2n} \\ & \vdots & & \\ C_n & a_{n2} & \cdots & a_{nn} \end{bmatrix}, \qquad D_2 = \begin{bmatrix} a_{11} & C_1 & \cdots & a_{1n} \\ a_{21} & C_2 & \cdots & a_{2n} \\ & \vdots & & \\ a_{n1} & C_n & \cdots & a_{nn} \end{bmatrix}$$

and

$$D_n = \begin{bmatrix} a_{11} & a_{12} & \cdots & C_1 \\ a_{21} & a_{22} & \cdots & C_2 \\ & \vdots & & \\ a_{n1} & a_{n2} & \cdots & C_n \end{bmatrix}$$

The unknowns of the equations are given by Cramer's rule as,

$$X_1 = \frac{D_1}{D}, \quad X_2 = \frac{D_2}{D}, \cdots, X_n = \frac{D_n}{D}$$

Where $\mathbf{D_1}$, $\mathbf{D_2}$, …, $\mathbf{D_n}$ and **D** are values of the respective determents

Example 2.6: Apply Kirchhoff's laws to the circuit shown in figure 1 below Indicate the various branch currents.

Write down the equations relating the various branch currents.

Solve these equations to find the values of these currents.

Is the sign of any of the calculated currents negative?

If yes, explain the significance of the negative sign.

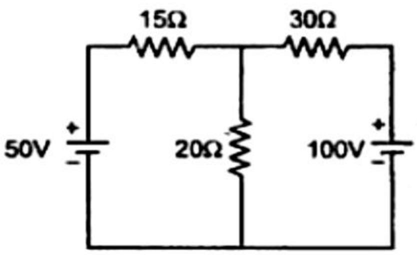


Figure 1

Solution: Application Kirchhoff's laws:

Step 1and 2: Draw the circuit with all the values which are same as the given network.

Mark all the branch currents starting from **+ve** of any of the source, say +ve of 50 V source

Step 3: Mark all the polarities for different voltages across the resistance. This is combined with step 2 shown in the network below in Fig. 1 (a).

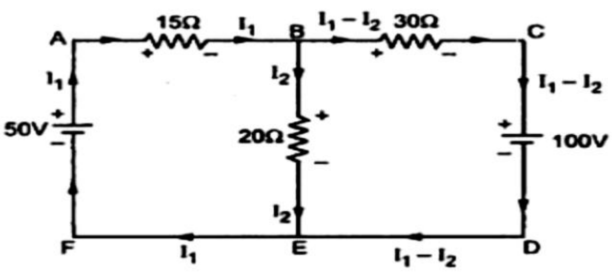


Figure 1 (a)

Step 4: Apply **KVL** to different loops.

Loop 1: **A-B-E-F-A**, $-15\ I_1 - 20\ I_2 + 50 = 0$

Loop 2: **B-C-D-E-D**, $-30\ (I_1 - I_2) - 100 + 20\ I_2 = 0$

Rewriting all the equations, taking constants on one side,

$$15\ I_1 + 20\ I_2 = 50, \qquad -30\ I_1 + 50\ I_2 = 100$$

Apply Cramer's rule, $D = \begin{vmatrix} 15 & 20 \\ -30 & 50 \end{vmatrix} = 1350$

Calculating **D_1**, $D_1 = \begin{vmatrix} 50 & 20 \\ 100 & 50 \end{vmatrix} = 500$

$$I_1 = \frac{D_1}{D} = \frac{500}{1350} = 0.37\ A$$

Calculating **D_2**, $D_2 = \begin{vmatrix} 15 & 50 \\ -30 & 100 \end{vmatrix} = 3000$

$$I_2 = \frac{D_2}{D} = \frac{3000}{1350} = 2.22\ A$$

For I_1 and I_2 as answer is positive, assumed direction is correct.

For I_1 answer is **0.37** A. For I_2 answer is **2.22** A

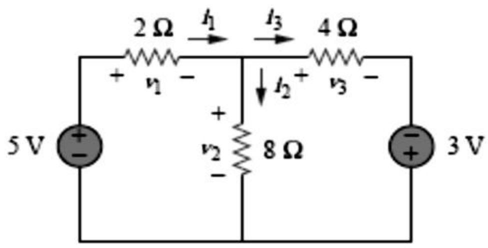
$$I_1 - I_2 = 0.37 - 2.22 = -1.85\ A$$

Negative sign indicates assumed direction is wrong.

i.e. $I_1 - I_2$ = 1.85 A flowing in opposite direction to that of the assumed direction.

Practice problem 2 .6: Find the currents and voltages in the circuit shown below.

Answer: v_1 **= 3 V,** v_2 **= 2 V,** v_3 **= 5 V,** i_1 **= 1.5 A,** i_2 **= ٠.25 A,** i_3 **=1.25 A.**



2.15 SOURCE TRANSFORMATION

We have noticed that series-parallel combination and wye-delta transformation help simplify circuits. Source transformation is another tool for simplifying circuits. We can substitute a voltage source in series with a resistor for a current source in parallel with a resistor, or vice versa, as shown in **Fig. 2.33**. Either substitution is known as a ***source transformation***.

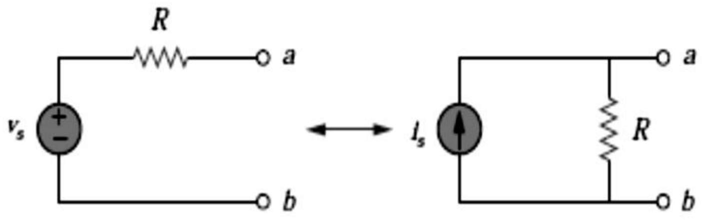


Figure 2.33 Transformation of independent sources.

> **Key Point:** *A source transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source is in parallel with a resistor R, or vice versa*.

We need to find the relationship between v_s and i_s that guarantees the two configurations in **Fig. 2.33** are equivalent with respect to nodes **a, b**.

Suppose $\mathbf{R_L}$, is connected between nodes **a, b** in **Fig.2.33(a)**. Using Ohms law, the current in $\mathbf{R_L}$ is.

$$i_L = \frac{v_s}{(R+R_L)} \qquad \mathbf{R} \text{ and } \mathbf{R_L} \text{ in series} \tag{2.37}$$

If it is to be replaced by a current source then load current must be $\frac{V}{(R+R_L)}$

Now suppose the same resistor $\mathbf{R_L}$, is connected between nodes **a**, **b** in **Fig. 4.4 (b)**. Using current division, the current in $\mathbf{R_L}$, is

$$i_L = i_s \frac{R}{(R+R_L)} \tag{2.38}$$

If the two circuits in **Fig. 4.4** are equivalent, these resistor currents must be the same. Equating the right-hand sides of **Eqs.4.5** and **4.6** and simplifying

$$i_s = \frac{v_s}{R} \quad or \quad v_s = i_s R \tag{2.39}$$

Source transformation also applies to dependent sources, provided we carefully handle the dependent variable.As shown in**Fig.2.34**, a dependent voltage source in series with a resistor can be transformed to a dependent current source in parallel with the resistor or vice versa.

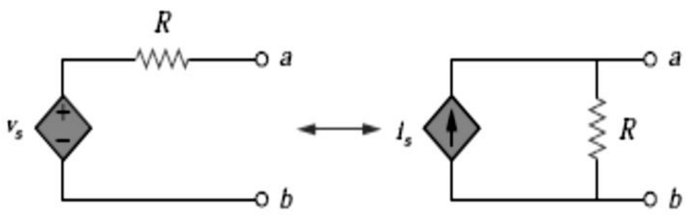


Figure 2.34 Transformation of dependent sources.

However, we should keep the following points in mind when dealing with source transformation.

1. Note from **Fig. 2.33** (or **Fig. 2.34**) that the arrow of the current source is directed toward the positive terminal of the voltage source.

2. Note from **Eq. (2.39)** that source transformation is not possible when **R = 0**, which is the case with an ideal voltage source. However, for a practical, nonideal voltage source, **R ≠ 0**. Similarly, an ideal current source with **R** =∞cannot be replaced by a finite voltage source.

<u>Example 2.7:</u> Use source transformation to find v_o in the circuit in **Fig. 2.35**.

<u>Solution:</u>

We first transform the current and voltage sources to obtain the circuit in **Fig. 2.37(a)**. Combining the 4-Ω and 2-Ω resistors in series and transforming the 12-V voltage source gives us **Fig. 2.37(b)**. We now combine the 3-Ω and 6-Ω resistors in parallel to get 2-Ω. We also combine the 2-A and 4-A current sources to get a 2-A source. Thus, by repeatedly applying source transformations, we obtain the circuit in**Fig.2.37 (c)**.

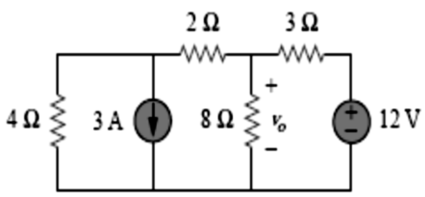


Figure 2.36

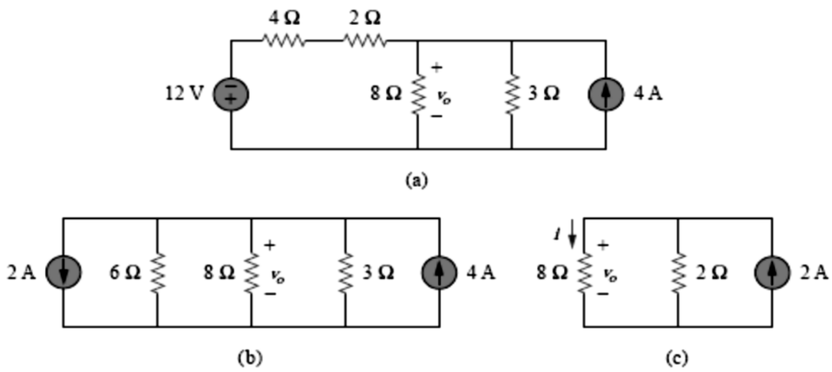


Figure 2.37

Alternatively, since the 8-Ω and 2-Ω resistors in **Fig. 2.37(c)** are in parallel, they have the same voltage ***vo*** across them. Hence,

$$\boldsymbol{v_o = (8||2)(2\ A) = \frac{8\times2}{10}(2) = 3.2\ V}$$

CHAPTER THREE

METHODS OF ANALYSIS

3.1 INTRODUCTION

Having understood the fundamental laws of circuit theory (**Ohm's law** and **Kirchhoff's laws**), we are now prepared to apply these laws to develop two powerful techniques for circuit analysis: nodal analysis, which is based on a systematic application of Kirchhoff's current law (**KCL**), and mesh analysis, which is based on a systematic application of Kirchhoff's voltage law (**KVL**). The two techniques are so important that this chapter should be regarded as the most important in the lectures.

3.2 NODAL ANALYSIS

Nodal analysis provides a general procedure for analyzing circuits using node voltages as the circuit variables. Choosing node voltages instead of element voltages as circuit variables is convenient and reduces the number of equations one must solve simultaneously. To simplify matters, we shall assume in this section that circuits do not contain voltage sources. Circuits that contain voltage sources will be analyzed in the next section.

Steps to Determine Node Voltages:

1. Select a node as the reference node. Assign voltages $\mathbf{v_1}$, $\mathbf{v_2}$,. . $\mathbf{v_{n-1}}$ to the remaining **n-1** nodes. The voltages are referenced with respect to the reference node.
2. Apply **KCL** to each of the **n-1** nonreference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

We shall now explain and apply these three steps.

The first step in nodal analysis is selecting a node as the reference or datum node. The reference node is commonly called the ground since it is assumed to have zero potential. A reference node is indicated by any of the three symbols in **Fig. 3.1**. We shall always use the symbol in **Fig. 3.1(b)**. Once we have selected a reference node, we assign voltage designations to nonreference nodes. Consider, for example, the circuit in **Fig. 3.2(a)**. Node 0 is the reference node ($\mathbf{v = 0}$), while nodes 1 and 2 are assigned voltages $\mathbf{v_1}$ and $\mathbf{v_2}$, respectively. Keep in mind that the node voltages are defined with respect to the reference node. As illustrated in **Fig. 3.2(a)**, each node voltage is the voltage with respect to the reference node.

The number of nonreference nodes is equal to the number of independent equations that we will derive.

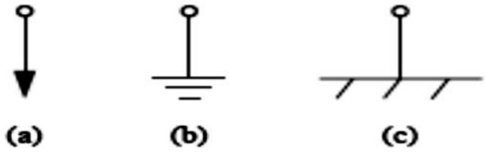


Figure 3.1 Common symbols for indicating a reference node.

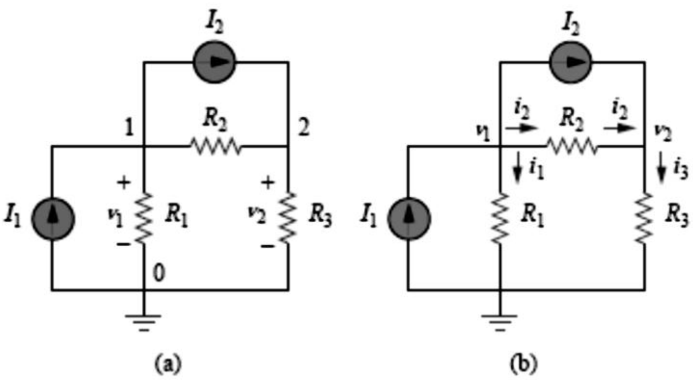


Figure 3.2 Typical circuits for nodal analysis.

As the second step, we apply **KCL** to each nonreference node in the circuit. To avoid putting too much information on the same circuit, the circuit in **Fig. 3.2(a)** is redrawn in **Fig. 3.2(b)**, where we now add $\mathbf{i_1}$, $\mathbf{i_2}$, and $\mathbf{i_3}$ as the currents through resistors $\mathbf{R_1}$, $\mathbf{R_2}$, and $\mathbf{R_3}$, respectively. At node 1, applying **KCL** gives

$$\mathbf{I_1 = I_2 + i_1 + i_2} \tag{3.1}$$

At node 2,

$$\mathbf{I_2 + i_2 = i_3} \tag{3.2}$$

We now apply Ohm's law to express the unknown currents $\mathbf{i_1}$, $\mathbf{i_2}$, and $\mathbf{i_3}$ in terms of node voltages.

Current flows from a higher potential to a lower potential in a resistor.

We can express this principle as

$$\boldsymbol{i = \frac{v_{higher} - v_{lower}}{R}} \tag{3.3}$$

Note that this principle is in agreement with the way we defined resistance in Chapter 2 (see **Fig. 2.3**). With this in mind, we obtain from **Fig. 3.2(b)**,

$$i_1 = \frac{v_1 - 0}{R_1},$$

$$i_2 = \frac{v_1 - v_2}{R_2},$$

$$i_3 = \frac{v_2 - 0}{R_3}, \tag{3.4}$$

Substituting **Eq. (3.4) in Eqs. (3.1)** and **(3.2)** results, respectively, in

$$I_1 = I_2 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} \tag{3.5}$$

$$I_2 + v_1 - \frac{v_2}{R_2} = \frac{v_2}{R_3} \tag{3.6}$$

In terms of the conductances, Eqs. (3.5) and (3.6) become

$$I_1 = I_2 + G_1 v_1 + G_2 (v_1 - v_2) \tag{3.7}$$

$$I_2 + G_2 (v_1 - v_2) = G_3 v_2 \tag{3.8}$$

The third step in nodal analysis is to solve for the node voltages. If we apply **KCL** to **n−1** nonreference nodes, we obtain **n−1** simultaneous equations such as **Eqs. (3.5)** and **(3.6)** or **(3.7)** and **(3.8)**. For the circuit of **Fig. 3.2**, we solve **Eqs. (3.5)** and **(3.6)** or **(3.7)** and **(3.8)** to obtain the node voltages $\mathbf{v_1}$ and $\mathbf{v_2}$ using any standard method, such as the substitution method, the elimination method, Cramer's rule, or matrix inversion. To use either of the last two methods, one must cast the simultaneous equations in matrix form. For example, **Eqs. (3.7)** and **(3.8)** can be cast in matrix form as

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix} \tag{3.9}$$

which can be solved to get $\mathbf{v_1}$ and $\mathbf{v_2}$.

Example 3.1: Calculate the node voltages in the circuit shown in **Fig. 3.3(a)**.

Solution:

Consider **Fig. 3.3(b)**, where the circuit in **Fig. 3.3(a)** has been prepared for nodal analysis. Notice how the currents are selected for the application of **KCL**. Except for the branches with current sources, the labeling of the currents is arbitrary but consistent. (By consistent, we mean that if, for example, we assume that i2 enters the 4_resistor from the left-hand side, i2 must leave the resistor from the right-hand side.) The reference node is selected, and the node voltages v1 and v2 are now to be determined.

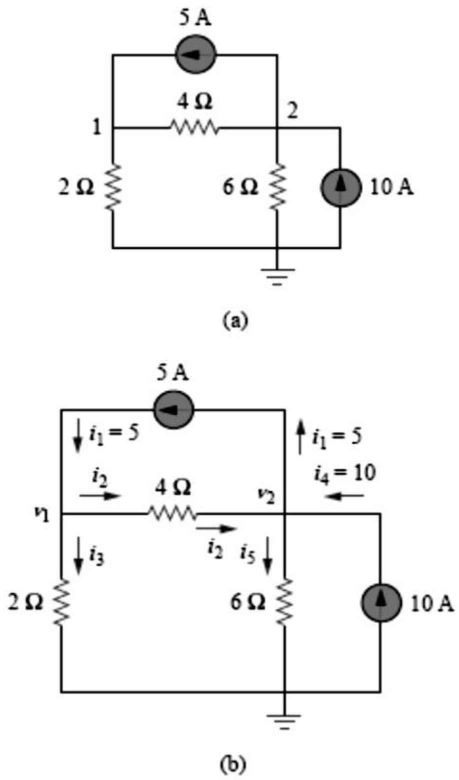


Figure 3.3 For Example 3.1: (a) original circuit, (b) circuit for analysis

At node 1, applying **KCL** and **Ohm's law** gives

$$\mathbf{i_1 = i_2 + i_3 \Rightarrow 5 = \frac{v1 - v2}{4} + \frac{v1 - 0}{2}}$$

Multiplying each term in the last equation by 4, we obtain

$$\mathbf{20 = v_1 - v_2 + 2v_1}$$

or

$$\mathbf{3v_1 - v_2 = 20} \qquad (3.1.1)$$

At node 2, we do the same thing and get

$$\mathbf{i_2 + i_4 = i_1 + i_5 \Rightarrow \frac{v1 - v2}{4} + 10 = 5 + \frac{v2 - 0}{6}}$$

Multiplying each term by 12 results in

$$3v_1 - 3v= + 120 = 60 + 2v_2$$

or

$$-3v_1 + 5v_2 = 60 \qquad (3.1.2)$$

Now we have two simultaneous **Eqs. (3.1.1)** and **(3.1.2)**. We can solve the equations using any method and obtain the values of v1 and v2.

<u>METHOD 1:</u> Using the elimination technique, we add **Eqs. (3.1.1)** and **(3.1.2)**.

$$4v_2 = 80 \Rightarrow v_2 = 20 \text{ V}$$

Substituting $v_2 = 20$ in Eq. (3.1.1) gives

$$3v_1 - 20 = 20 \Rightarrow v_1 = 40/3 = 13.33 \text{ V}$$

<u>METHOD 2:</u> To use **Cramer's rule**, we need to put **Eqs. (3.1.1)** and **(3.1.2)** in matrix form as

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix} \qquad (3.1.3)$$

The determinant of the matrix is

$$\boldsymbol{\Delta = D = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - 3 = 12}$$

We now obtain $\mathbf{v_1}$ and $\mathbf{v_2}$ as

$$\boldsymbol{v_1 = \frac{D_1}{D} = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{D} = \frac{100+60}{12} = 13.33V}$$

$$\boldsymbol{v_2 = \frac{D_2}{D} = \frac{\begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}}{D} = \frac{180+60}{12} = 20V}$$

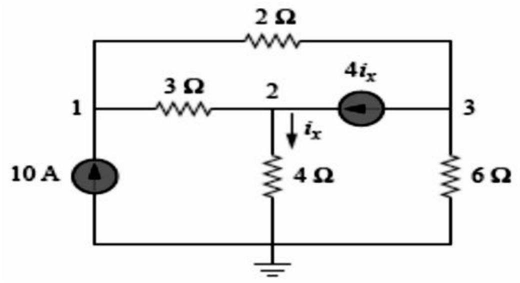
If we need the currents, we can easily calculate them from the values of the nodal voltages.

$\mathbf{i_1 = 5\ A},\ i_2 = \frac{v_1 - v_2}{4} = -1.6667\ A,\ i_3 = \frac{v_1}{2} = 6.666A,\ \mathbf{i_4 = 10\ A},\ i_5 = \frac{v_2}{6} = 3.333A$

The fact that $\mathbf{i_2}$ is negative shows that the current flows in the direction opposite to the one assumed.

Practice problem 3.1: Find the voltages at the three nonreference nodes in the circuit of Figure below.

Answer: $v_1 = 80$ V, $v_2 = -64$ V, $v_3 = 156$ V.



3.2.1 NODAL ANALYSIS WITH VOLTAGE SOURCES

We now consider how voltage sources affect nodal analysis. We use the circuit in **Fig. 3.4** for illustration. Consider the following two possibilities.

CASE 1: If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source. In **Fig. 3.4**, for example,

$$\mathbf{v_1 = 10\ V} \tag{3.10}$$

Thus our analysis is somewhat simplified by this knowledge of the voltage at this node.

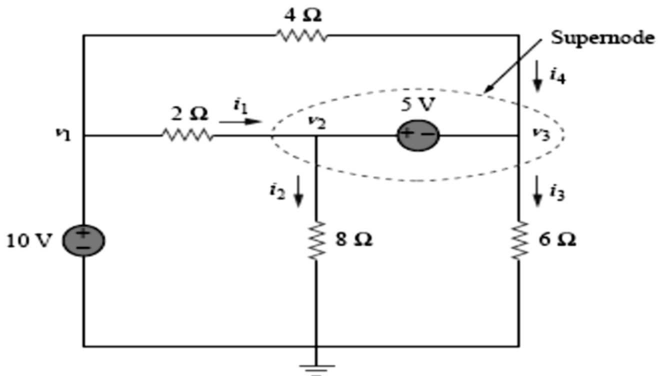


Figure 3.4 A circuit with a supernode.

CASE 2: If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a generalized node or supernode; we apply both **KCL** and **KVL** to determine the node voltages.

A supernode is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.

In **Fig. 3.4**, nodes 2 and 3 form a supernode. (We could have more than two nodes forming a single supernode. For example, see the circuit in the **Practice problem 3.4**). We analyze a circuit with supernodes using the same three steps mentioned in the previous section except that the supernodes are treated differently. Why? Because an essential component of nodal analysis is applying **KCL**, which requires knowing the current through each element. There is no way of knowing the current through a voltage source in advance. However, **KCL** must be satisfied at a supernode like any other node. Hence, at the supernode in **Fig. 3.5**,

$$\mathbf{i_1 + i_4 = i_2 + i_3} \qquad (3.11a)$$

or

$$\frac{v1 - v2}{2} + \frac{v1 - v3}{4} = \frac{v2 - 0}{8} + \frac{v3 - 0}{6} \qquad (3.11b)$$

To apply Kirchhoff's voltage law to the supernode in **Fig. 3.4**, we redraw the circuit as shown in **Fig. 3.5**. Going around the loop in the clockwise direction gives

$$\mathbf{-v_2 + 5 + v_3 = 0 \Rightarrow v_2 - v_3 = 5} \qquad (3.12)$$

From **Eqs. (3.10), (3.11b),** and **(3.12)**, we obtain the node voltages.

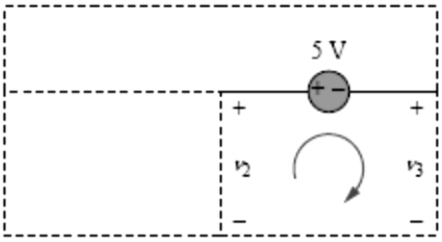


Figure 3.5 Applying KVL to a supernode.

<u>Example 3.2:</u> For the circuit shown in **Fig. 3.6**, find the node voltages.

<u>Solution:</u>

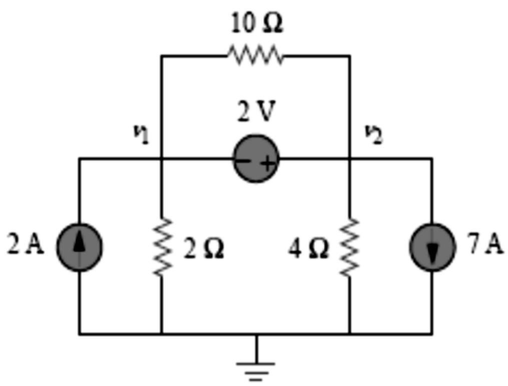


Figure 3.6 For Example 3.2.

The supernode contains the 2-V source, nodes 1 and 2, and the 10-Ω resistor. Applying **KCL** to the supernode as shown in **Fig. 3.7(a)** gives

$$\mathbf{2 = i_1 + i_2 + 7}$$

Expressing $\mathbf{i_1}$ and $\mathbf{i_2}$ in terms of the node voltages

$$\mathbf{2} = \frac{v1 - 0}{2} + \frac{v2 - 0}{4} + \mathbf{7}$$

or

$$\mathbf{v_2 = -20 - 2v_1} \qquad (3.2.1)$$

To get the relationship between $\mathbf{v_1}$ and $\mathbf{v_2}$, we apply **KVL** to the circuit in **Fig. 3.7(b)**. Going around the loop, we obtain

$$-v_1 - 2 + v_2 = 0 \Rightarrow v_2 = v_1 + 2 \qquad (3.3.2)$$

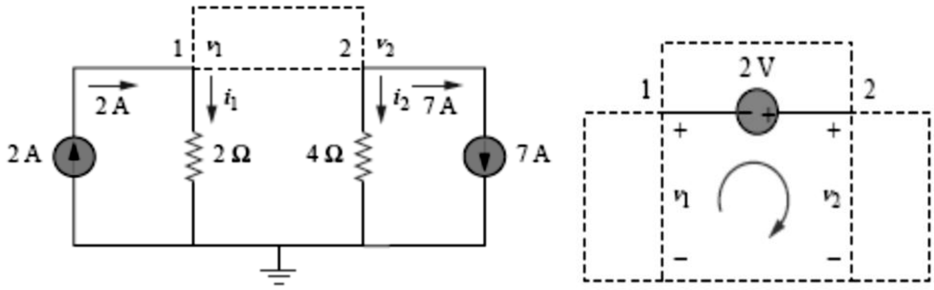
From **Eqs. (3.2.1)** and **(3.2.2)**, we write

$$v_2 = v_1 + 2 = -20 - 2v_1$$

or

$$3v_1 = -22 \Rightarrow v_1 = -7.333 \text{ V}$$

and $\mathbf{v_2 = v_1 + 2 = -5.333}$ **V**. Note that the 10-Ω resistor does not make any difference because it is connected across the supernode.

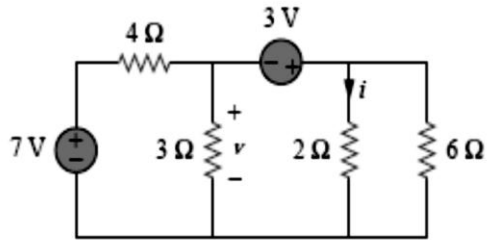


(a) **(b)**

Figure 3.7 Applying: (a) KCL to the supernode, (b) KVL to the loop.

<u>Practice problem 3.2:</u> Find **v** and **i** in the circuit in Figure below.

<u>Answer:</u> −0.2 V, 1.4 A.



<u>3.3 MESH ANALYSIS</u>

Mesh analysis provides another general procedure for analyzing circuits, using mesh currents as the circuit variables. Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously. Recall that a loop is a closed path with no node passed more than once. A mesh is a loop that does not contain any other loop within it.

Nodal analysis applies **KCL** to find unknown voltages in a given circuit, while mesh analysis applies **KVL** to find unknown currents. Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is planar. A **planar** circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is **nonplanar**. A circuit may have crossing branches and still be **planar** if it can be redrawn such that it has no crossing branches. For example, the circuit in **Fig. 3.8 (a)** has two crossing branches, but it can be redrawn as in **Fig. 3.8 (b)**. Hence, the circuit in **Fig. 3.8 (a)** is planar. However, the circuit in **Fig. 3.9** is **nonplanar**, because there is no way to redraw it and avoid the branches crossing. **Nonplanar** circuits can be handled using nodal analysis, but they will not be considered in this text.

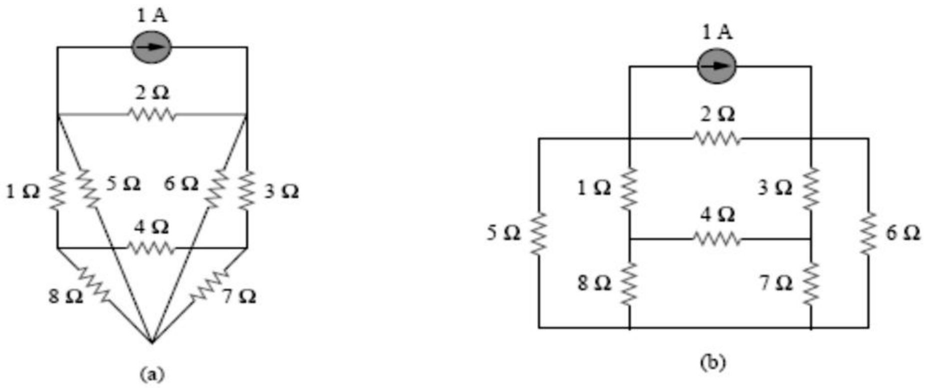


Figure 3.8 (a) A planar circuit with crossing branches, (b) the same circuit redrawn with no crossing branches.

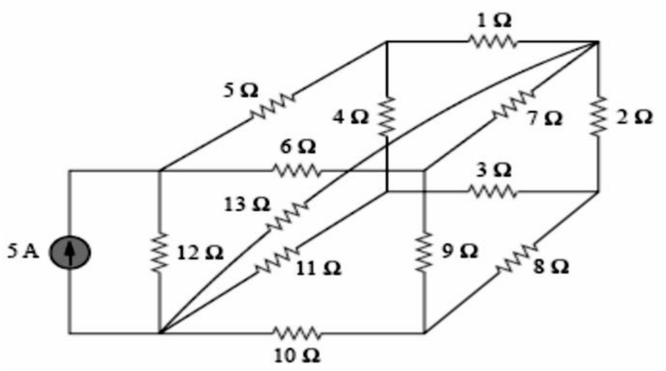


Figure 3.9 A nonplanar circuit.

To understand mesh analysis, we should first explain more about what we mean by a mesh. A mesh is a loop which does not contain any other loops within it.

In **Fig. 3.10**, for example, paths **abefa** and **bcdeb** are meshes, but path **abcdefa** is not a mesh. The current through a mesh is known as mesh current. In mesh analysis, we are interested in applying **KVL** to find the mesh currents in a given circuit.

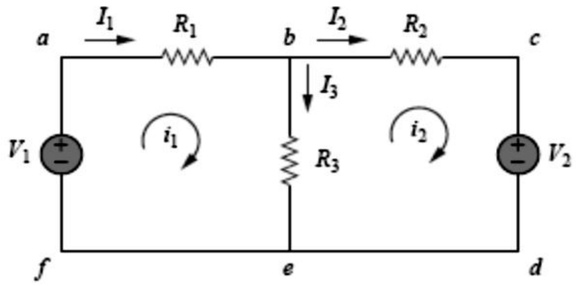


Figure 3.10 circuit with two meshes.

In this section, we will apply mesh analysis to planar circuits that do not contain current sources. In the next sections, we will consider circuits with current sources. In the mesh analysis of a circuit with **n** meshes, we take the following three steps.

Steps to Determine mesh currents:

1. Assign mesh currents $\mathbf{i_1}$, $\mathbf{i_2}$, . . . , $\mathbf{i_n}$ to the n meshes.
2. Apply **KVL** to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

To illustrate the steps, consider the circuit in **Fig. 3.10**. The first step requires that mesh currents $\mathbf{i_1}$ and $\mathbf{i_2}$ are assigned to meshes 1 and 2. Although a mesh current may be assigned to each mesh in an arbitrary direction, it is conventional to assume that each mesh current flows clockwise.

As the second step, we apply **KVL** to each mesh. Applying **KVL** to mesh 1, we obtain

$$\mathbf{-V_1 + R_1 i_1 + R_3 (i_1 - i_2) = 0}$$

or

$$\mathbf{(R_1 + R_3)\, i1 - R_3 i_2 = V_1} \qquad (3.13)$$

For mesh 2, applying KVL gives

$$\mathbf{R_2 i_2 + V_2 + R_3 (i_2 - i_1) = 0}$$

or

$$\mathbf{-R_3 i_1 + (R_2 + R_3)\, i_2 = -V_2} \qquad (3.14)$$

Note in **Eq. (3.13)** that the coefficient of $\mathbf{i_1}$ is the sum of the resistances in the first mesh, while the coefficient of $\mathbf{i_2}$ is the negative of the resistance common to meshes 1 and 2. Now observe that the same is true in **Eq. (3.14)**. This can serve as a shortcut way of writing the mesh equations.

The third step is to solve for the mesh currents. Putting **Eqs. (3.13)**. and **(3.14)** in matrix form yields

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ -V_2 \end{bmatrix} \qquad (3.15)$$

which can be solved to obtain the mesh currents $\mathbf{i_1}$ and $\mathbf{i_2}$. We are at liberty to use any technique for solving the simultaneous equations. If a circuit has **n** nodes, **b** branches, and ***l*** independent loops or meshes, then $l = b - n + 1$. Hence, ***l*** independent simultaneous equations are required to solve the circuit using mesh analysis.

Notice that the branch currents are different from the mesh currents unless the mesh is isolated. To distinguish between the two types of currents, we use **i** for a mesh current and **I** for a branch current. The current elements $\mathbf{I_1}$, $\mathbf{I_2}$, and $\mathbf{I_3}$ are algebraic sums of the mesh currents. It is evident from **Fig. 3.13** that

$$\mathbf{I_1 = i_1, \quad I_2 = i_2, \quad I_3 = i_1 - i_2} \qquad (3.16)$$

<u>Example 3.3:</u> For the circuit in **Fig. 3.11**, find the branch currents $\mathbf{I_1}$, $\mathbf{I_2}$, and $\mathbf{I_3}$ using mesh analysis.

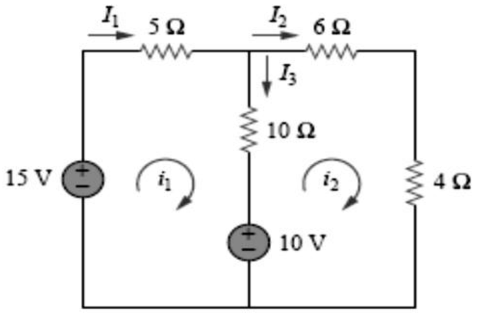


Figure 3.11 For Example 3.3.

<u>Solution:</u>

We first obtain the mesh currents using **KVL**. For mesh 1,

$$\mathbf{-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0}$$

or

$$\mathbf{3i_1 - 2i_2 = 1} \qquad (3.5.1)$$

For mesh 2,

$$\mathbf{6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0}$$

or

$$\mathbf{i_1 = 2i_2 - 1} \qquad (3.5.2)$$

Using the substitution method, we substitute **Eq. (3.3.2)** into **Eq. (3.3.1)**, and write

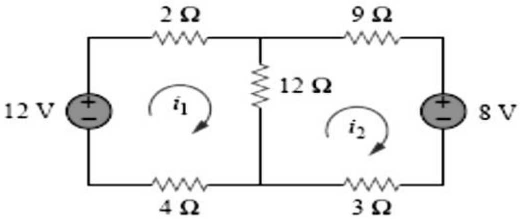
$$\mathbf{6i_2 - 3 - 2i_2 = 1 \Rightarrow \quad i_2 = 1\ A}$$

From **Eq. (3.5.2)**, $\mathbf{i_1 = 2i_2 - 1 = 2 - 1 = 1\ A}$. Thus,

$$\mathbf{I_1 = i_1 = 1\ A,\ I_2 = i_2 = 1\ A,\ I_3 = i_1 - i_2 = 0}$$

Practice problem 3.3: Calculate the mesh currents $\mathbf{i_1}$ and $\mathbf{i_2}$ in the circuit of Figure below.

Answer: $\mathbf{i_1 = 2/3\ A,\ i_2 = 0\ A.}$



3.3.1 MESH ANALYSIS WITH CURRENT SOURCES

Applying mesh analysis to circuits containing current sources (dependent or independent) may appear complicated. But it is actually much easier than what we encountered in the previous section, because the presence of the current sources reduces the number of equations. Consider the following two possible cases.

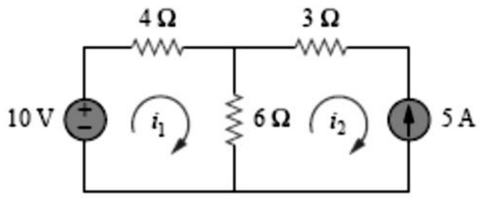


Figure 3.12 A circuit with a current source.

CASE 1: When a current source exists only in one mesh: Consider the circuit in **Fig. 3.12**, for example. We set $\mathbf{i_2}$ = −5 A and write a mesh equation for the other mesh in the usual way, that is,

$$\mathbf{-10 + 4i_1 + 6(i_1 - i_2) = 0 \Rightarrow i_1 = -2\ A} \tag{3.17}$$

CASE 2: When a current source exists between two meshes: Consider the circuit in **Fig. 3.13(a)**, for example. We create a supermesh by excluding the current source and any elements connected in series with it, as shown in **Fig. 3.13(b)**. Thus,

A supermesh results when two meshes have a (dependent or independent) current source in common.

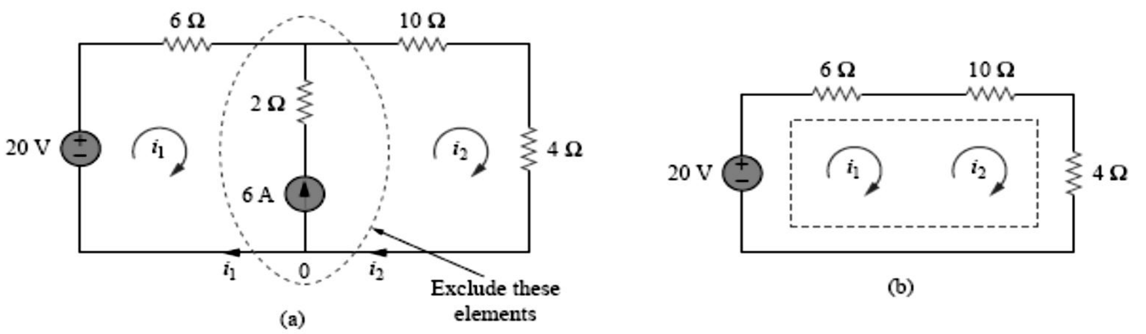


Figure 3.13 (a) Two meshes having a current source in common, (b) a supermesh, created by excluding the current source.

As shown in **Fig. 3.13(b)**, we create a supermesh as the periphery of the two meshes and treat it differently. (If a circuit has two or more supermeshes that intersect, they should be combined to form a larger supermesh.) Why treat the supermesh differently? Because mesh analysis applies **KVL**—which requires that we know the voltage across each branch—and we do not know the voltage across a current source in advance. However, a supermesh must satisfy **KVL** like any other mesh.

Therefore, applying **KVL** to the supermesh in **Fig. 3.13(b)** gives

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

or

$$6i_1 + 14i_2 = 20 \qquad (3.18)$$

We apply **KCL** to a node in the branch where the two meshes intersect.

Applying **KCL** to node 0 in **Fig. 3.13(a)** gives

$$i_2 = i_1 + 6 \qquad (3.19)$$

Solving **Eqs. (3.18)** and **(3.19)**, we get

$$i_1 = -3.2\ \text{A},\ i_2 = 2.8\ \text{A} \qquad (3.20)$$

Example 3.4: For the circuit in **Fig. 3.14**, find $\mathbf{i_1}$ to $\mathbf{i_4}$ using mesh analysis.

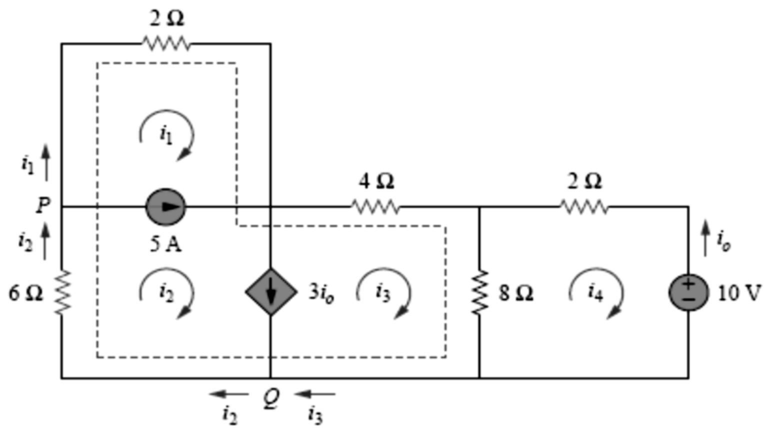


Figure 3.14 For Example 3.4.

Solution:

Note that meshes 1 and 2 form a supermesh since they have an independent current source in common. Also, meshes 2 and 3 form another supermesh because they have a dependent current source in common. The two supermeshes intersect and form a larger supermesh as shown. Applying **KVL** to the larger supermesh,

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

or

$$\mathbf{i_1 + 3i_2 + 6i_3 - 4i_4 = 0} \tag{3.4.1}$$

For the independent current source, we apply **KCL** to node P:

$$\mathbf{i_2 = i_1 + 5} \tag{3.4.2}$$

For the dependent current source, we apply KCL to node Q:

$$\mathbf{i_2 = i_3 + 3i_o}$$

But $\mathbf{i_o = -i_4}$, hence,

$$\mathbf{i_2 = i_3 - 3i_4} \tag{3.4.3}$$

Applying **KVL** in mesh 4,

$$\mathbf{2i_4 + 8(i_4 - i_3) + 10 = 0}$$

or

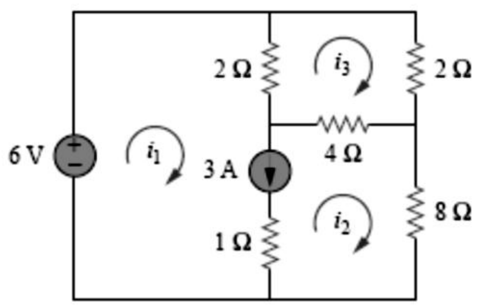
$$\mathbf{5i_4 - 4i_3 = -5} \tag{3.4.4}$$

From **Eqs. (3.4.1)** to **(3.4.4)**,

$$\mathbf{i_1 = -7.5\ A,\ i_2 = -2.5\ A,\ i_3 = 3.93\ A,\ i_4 = 2.143\ A}$$

<u>Practice problem 3.4:</u> Use mesh analysis to determine $\mathbf{i_1}$, $\mathbf{i_2}$, and $\mathbf{i_3}$ in Figure shown below.

<u>Answer:</u> $\mathbf{i_1 = 3.474\ A,\ i_2 = 0.474\ A,\ i_3 = 1.105\ A.}$



CHAPTER FOUR

CIRCUIT THEOREMS

4.1 INTRODUCTION

The growth in areas of application of electric circuits has led to an evolution from simple to complex circuits. To handle the complexity, engineers over the years have developed some theorems to simplify circuit analysis. Such theorems include **Thevenin's and Norton's theorems**. Since these theorems are applicable to *linear* circuits, we first discuss the concept of circuit linearity. In addition to circuit theorems, we discuss the concepts of **superposition and maximum power transfer** in this chapter.

4.2 SUPERPOSITION

The idea of superposition rests on the linearity property.

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

However, to apply the superposition principle, we must keep two things in mind:

1. We consider one independent source at a time while all other independent sources are turned off. This implies that we replace every voltage source by 0 V (or a short circuit), and every current source by 0 A (or an open circuit).

2. Dependent sources are left intact because they are controlled by circuit variables. With these in mind, we apply the superposition principle in three steps:

Steps to Apply Super position Principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Analyzing a circuit using superposition has one major disadvantage: it may very likely involve more work. Keep in mind that superposition is based on linearity.

Example 4.2: Use the superposition theorem to find **v** in the circuit in **Fig. 4.2**.

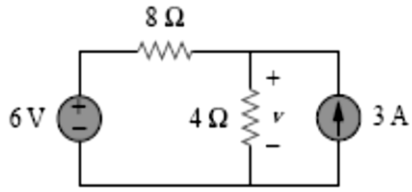


Figure 4.2 for Example 4.2.

Solution:

Since there are two sources, let

$$\mathbf{v = v_1 + v_2}$$

Where $\mathbf{v_1}$ and $\mathbf{v_2}$ are the contributions due to the 6-V voltage source and the 3-A current source, respectively. To obtain $\mathbf{v_1}$, we set the current source to zero, as shown in **Fig. 4.3(a)**. Applying **KVL** to the loop in **Fig. 4.3(a)** gives

$$\mathbf{12i_1 - 6 = 0 \Rightarrow i_1 = 0.5\ A}$$

Thus,

$$\mathbf{v_1 = 4i_1 = 2\ V}$$

We may also use voltage division to get $\mathbf{v_1}$ by writing

$$\boldsymbol{v_1 = \frac{4}{4+8}(6) = 2\ V}$$

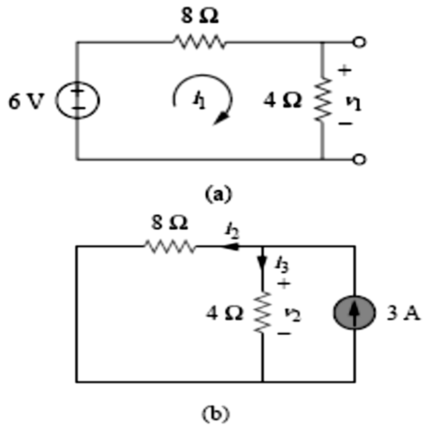


Figure 4.3 for Example 4.2:
(a) Calculating v_1, (b) calculating v_2.

To get $\mathbf{v_2}$, we set the voltage source to zero, as in **Fig. 4.3(b)**. Using current division,

$$\boldsymbol{i_3 = \frac{8}{4+8}(3) = 2\ A}$$

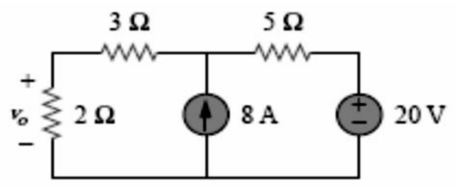
Hence, $\mathbf{v_2 = 4i_3 = 8\ V}$

And we find $\mathbf{v = v_1 + v_2 = 2 + 8 = 10\ V}$

Practice problems:

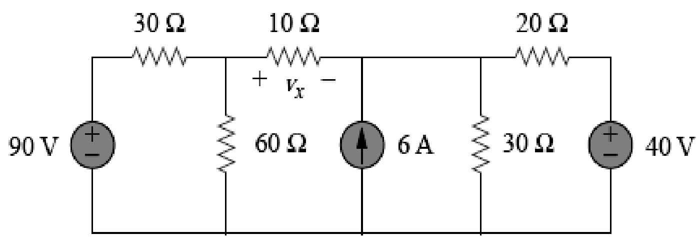
1-Using the superposition theorem, find $\mathbf{v_o}$ in the circuit in Figure below.

Answer: 12 V.



2- Use superposition to obtain v_x in the circuit of Figure below.

Answer: v_x = -8.572V.



4.3 THEVENIN'S THEOREM

It often occurs in practice that a particular element in a circuit is variable (usually called the load) while other elements are fixed. As a typical example, a household outlet terminal may be connected to different appliances constituting a variable load. Each time the variable element is changed, the entire circuit has to be analyzed all over again. To avoid this problem, Thevenin's theorem provides a technique by which the fixed part of the circuit is replaced by an equivalent circuit.

According to Thevenin's theorem, the linear circuit in **Fig. 4.8(a)** can be replaced by that in **Fig. 4.8(b)** is known as the Thevenin equivalent circuit; it was developed in 1883 by M. Leon Thevenin (1857–1926), a French telegraph engineer.

Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source $\boldsymbol{V_{Th}}$ in series with a resistor $\boldsymbol{R_{Th}}$, where $\boldsymbol{V_{Th}}$ is the open-circuit voltage at the terminals and $\boldsymbol{R_{Th}}$ is the input or equivalent resistance at the terminals when the independent sources are turned off.

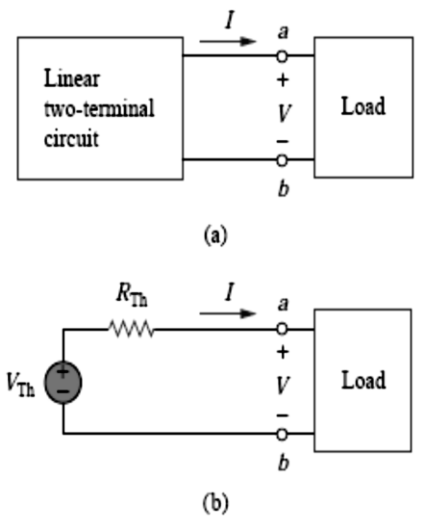


Figure 4.8 Replacing a linear two-terminal circuit by its Thevenin equivalent: (a) original circuit, (b) the Thevenin equivalent circuit.

To find the Thevenin equivalent voltage $\mathbf{V_{Th}}$ and resistance $\mathbf{R_{Th}}$, suppose the two circuits in **Fig. 4.8** are equivalent. The open-circuit voltage across the terminals a-b in **Fig. 4.8(a)** must be equal to the voltage source $\mathbf{V_{Th}}$ in **Fig. 4.8(b)**, since the two circuits are equivalent. Thus $\mathbf{V_{Th}}$ is the open-circuit voltage across the terminals as shown in **Fig. 4.9(a)**; that is,

$$\mathbf{V_{Th} = voc} \qquad (4.8)$$

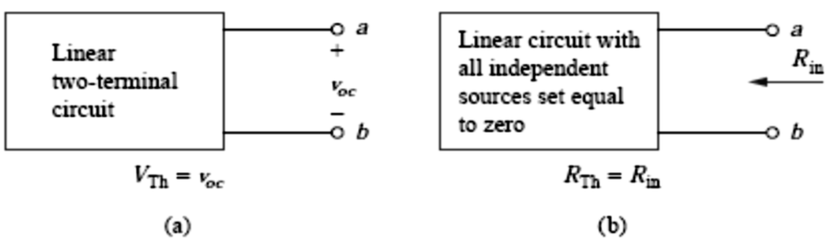


Figure 4.9 Finding V_{Th} and R_{Th}.

$\mathbf{R_{Th}}$ is the input resistance at the terminals when the independent sources are turned off, as shown in **Fig. 4.9(b)**; that is,

$$\mathbf{R_{Th} = R_{in}} \qquad (4.9)$$

To apply this idea in finding the Thevenin resistance $\mathbf{R_{Th}}$, we need to consider two cases.

CASE 1: If the network has no dependent sources, we turn off all independent sources. $\mathbf{R_{Th}}$ is the input resistance of the network looking between terminals **a** and **b**, as shown in **Fig. 4.9(b)**.

CASE 2: If the network has dependent sources, we turn off all independent sources. As with superposition, dependent sources are not to be turned off because they are controlled by circuit variables. We apply a voltage source $\mathbf{v_o}$ at terminals **a** and **b** and determine the resulting current $\mathbf{i_o}$. Then $\mathbf{R_{Th}} = \mathbf{v_o/i_o}$, as shown in **Fig. 4.10(a)**. Alternatively, we may insert a current source $\mathbf{i_o}$ at terminals **a-b** as shown in **Fig. 4.10(b)** and find the terminal voltage $\mathbf{v_o}$. Again $\mathbf{R_{Th}} = \mathbf{v_o/i_o}$. Either of the two approaches will give the same result. In either approach we may assume any value of $\mathbf{v_o}$ and $\mathbf{i_o}$. For example, we may use $\mathbf{v_o}$ = 1 V or $\mathbf{i_o}$ = 1 A, or even use unspecified values of $\mathbf{v_o}$ or $\mathbf{i_o}$.

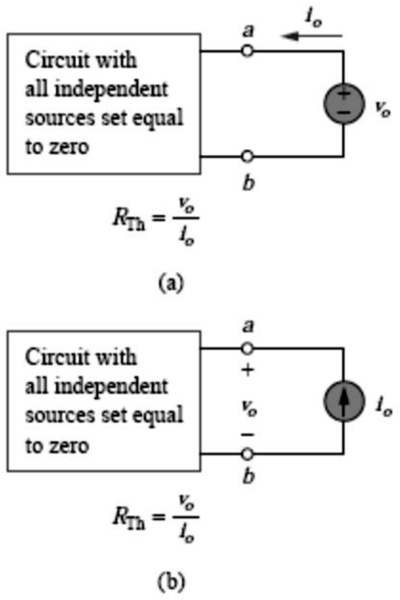


Figure 4.10 Finding RTh when circuit has dependent sources.

It often occurs that $\mathbf{R_{Th}}$ takes a negative value. In this case, the negative resistance (**v = −iR**) implies that the circuit is supplying power. This is possible in a circuit with dependent sources.

The current $\mathbf{I_L}$ through the load and the voltage $\mathbf{V_L}$ across the load are easily determined once the Thevenin equivalent of the circuit at the load's terminals is obtained, as shown in **Fig. 4.11(b)**. From **Fig. 4.11(b)**, we obtain

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} \tag{4.10a}$$

$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th} \tag{4.10b}$$

Note from **Fig. 4.11(b)** that the Thevenin equivalent is a simple voltage divider, yielding $\mathbf{V_L}$ by mere inspection.

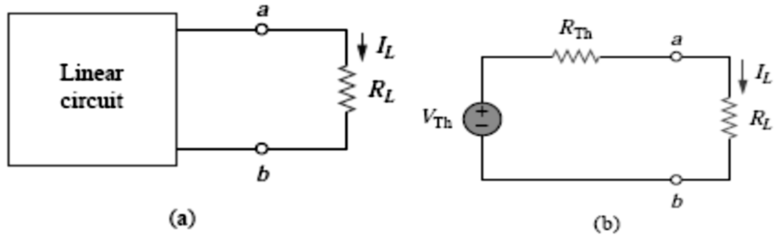


Figure 4.11 A circuit with a load :(a) original circuit, (b) Thevenin equivalent.

Example 4.4: Find the Thevenin equivalent circuit of the circuit shown in **Fig. 4.12**, to the left of the terminals a-b. Then find the current through R_L = 6, 16, and 36 Ω.

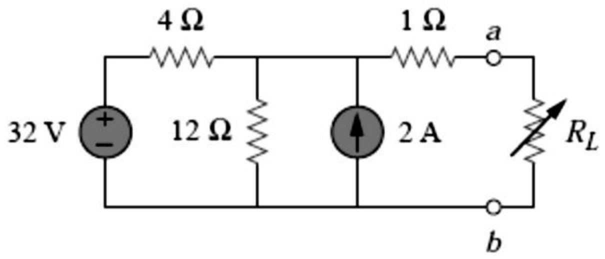


Figure 4.12 For Example 4.4.

Solution:

We find R_{Th} by turning off the 32-V voltage source (replacing it with a short circuit) and the 2-A current source (replacing it with an open circuit). The circuit becomes what is shown in **Fig. 4.13(a)**. Thus,

$$R_{Th} = 4||12 + 1 = \frac{4 \times 12}{16} + 1 = 4\,\Omega$$

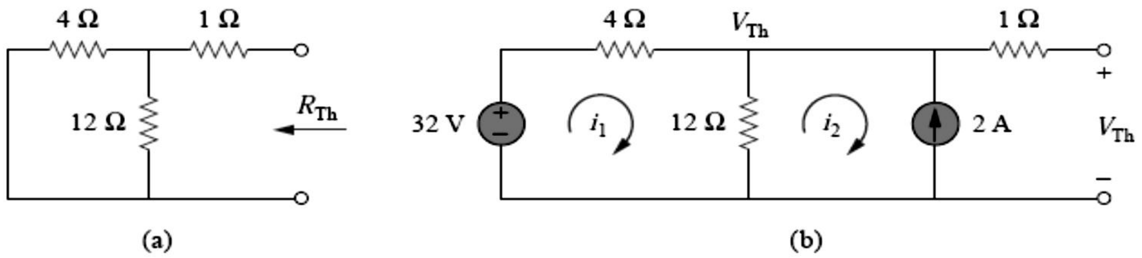


Figure 4.13 For Example 4.4: (a) finding R_{Th}, (b) finding V_{Th}.

To find $\mathbf{V_{Th}}$, consider the circuit in **Fig. 4.13(b)**. Applying mesh analysis to the two loops, we obtain

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2\text{ A}$$

Solving for i_1, we get $i_1 = 0.5$ **A**. Thus,

$$V_{Th} = 12(i1 - i2) = 12(0.5 + 2.0) = 30\text{ V}$$

The Thevenin equivalent circuit is shown in **Fig. 4.14**. The current through $\mathbf{R_L}$ is

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$

When R_L = 6, $\quad I_L = \frac{30}{10} = 3\,A$

When R_L = 16, $\quad I_L = \frac{30}{20} = 1.5\,A$

When RL = 36, $\quad I_L = \frac{30}{40} = 0.75\,A$

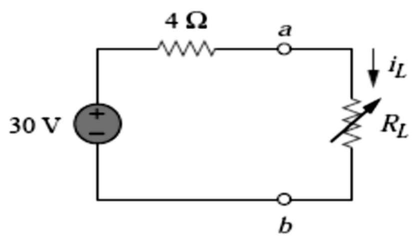
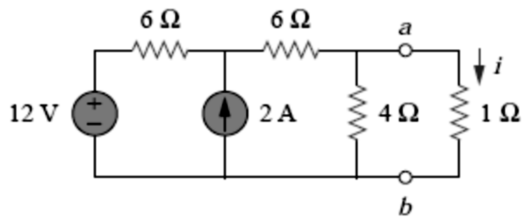


Figure 4.14 The Thevenin equivalent circuit

Practice problem: Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit in Figure below. Then find **i**.

Answer: $V_{Th} = 6$ V, $R_{Th} = 3\ \Omega$, i = 1.5 A.



4.4 NORTON'S THEOREM

In 1926, about 43 years after Thevenin published his theorem, E. L. Norton, an American engineer at Bell Telephone Laboratories, proposed a similar theorem.

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

Thus, the circuit in **Fig. 4.15(a)** can be replaced by the one in **Fig. 4.15(b)**.

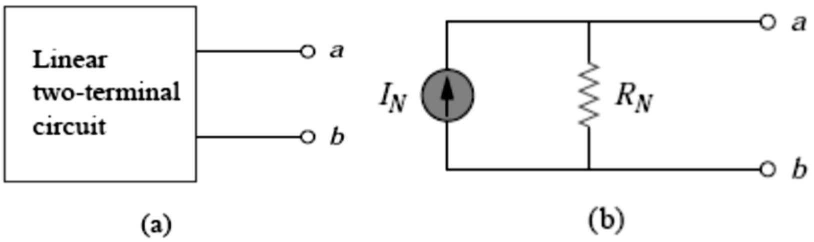


Figure 4.15 (a) Original circuit, (b) Norton equivalent circuit.

We are mainly concerned with how to get $\mathbf{R_N}$ and $\mathbf{I_N}$. We find $\mathbf{R_N}$ in the same way we find $\mathbf{R_{Th}}$. In fact, the Thevenin and Norton resistances are equal; that is,

$$\mathbf{R_N = R_{Th}} \tag{4.11}$$

To find the Norton current $\mathbf{I_N}$, we determine the short-circuit current flowing from terminal a to b in both circuits in **Fig. 4.15**. It is evident that the short-circuit current in **Fig. 4.15(b)** is $\mathbf{I_N}$. This must be the same short-circuit current from terminal a to b in **Fig. 4.15(a)**, since the two circuits are equivalent. Thus,

$$\mathbf{I_N = i_{sc}} \tag{4.12}$$

Dependent and independent sources are treated the same way as in Thevenin's theorem. Observe the close relationship between Norton's and Thevenin's theorems: $\mathbf{R_N = R_{Th}}$ as in **Eq. (4.11)**, and

$$\mathbf{I_N} = \frac{V_{Th}}{R_{Th}} \tag{4.13}$$

This is essentially source transformation. For this reason, source transformation is often called Thevenin-Norton transformation.

We can calculate any two of the three using the method that takes the least effort and use them to get the third using Ohm's law. **Example 4.10** will illustrate this. Also, since

$$\mathbf{V_{Th} = v_{oc}} \tag{4.14a}$$

$$\mathbf{I_N = i_{sc}} \tag{4.14b}$$

$$R_{Th} = \frac{voc}{isc} = R_N \tag{4.14c}$$

the open-circuit and short-circuit tests are sufficient to find any Thevenin or Norton equivalent.

Example 4.5 Find the Norton equivalent circuit of the circuit in **Fig. 4.16**.

Solution:

We find $\mathbf{R_N}$ in the same way we find $\mathbf{R_{Th}}$ in the Thevenin equivalent circuit. Set the independent sources equal to zero. This leads to the circuit in **Fig. 4.17(a)**, from which we find $\mathbf{R_N}$. Thus,

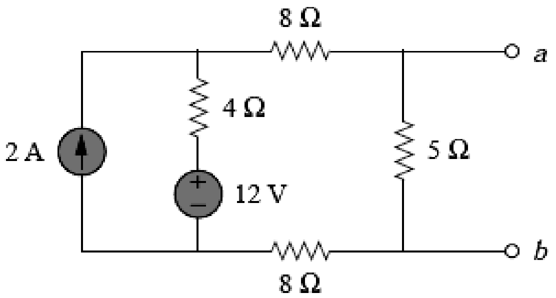


Figure 4.16 For Example 4.5.

$$R_N = 5\|(8+4+8) = 5\|20 = \frac{20 \times 5}{25} = 4\,\Omega$$

To find $\mathbf{I_N}$, we short-circuit terminals a and b, as shown in **Fig. 4.17(b)**. We ignore the 5-Ω resistor because it has been short-circuited. Applying mesh analysis, we obtain

$$\mathbf{i_1 = 2\ A, \qquad 20i_2 - 4i_1 - 12 = 0}$$

From these equations, we obtain

$$\mathbf{i_2 = 1\ A = i_{sc} = I_N}$$

Alternatively, we may determine $\mathbf{I_N}$ from $\mathbf{V_{Th}/R_{Th}}$. We obtain $\mathbf{V_{Th}}$ as the open-circuit voltage across terminals a and b in **Fig. 4.17(c)**. Using mesh analysis, we obtain

$$\mathbf{i_3 = 2\ A}$$

$$\mathbf{25i_4 - 4i_3 - 12 = 0 \quad \Rightarrow i_4 = 0.8\ A}$$

and

$$\mathbf{v_{oc} = V_{Th} = 5i_4 = 4\ V}$$

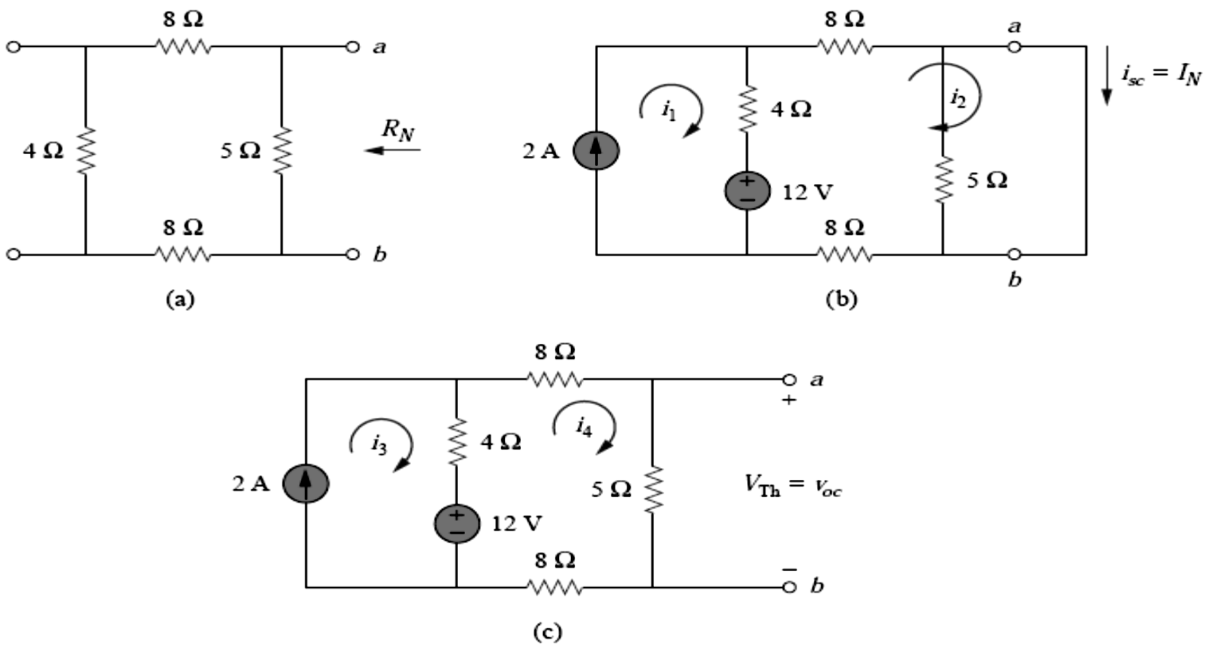


Figure 4.17 For Example 4.5; finding: (a) RN, (b) IN = isc, (c) V_{Th} = voc.

Hence,

$$I_N = \frac{VTh}{RTh} = \frac{4}{4} = 1\ A$$

as obtained previously. This also serves to confirm Eq. that $\mathbf{R_{Th} = v_{oc}/i_{sc} = 4/1 = 4\ \Omega}$. Thus, the Norton equivalent circuit is as shown in **Fig. 4.18**.

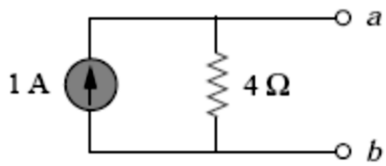
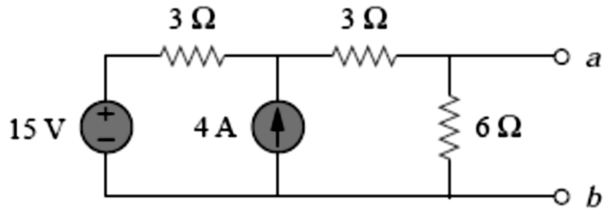


Figure 4.18 Norton equivalent of the circuit in Fig. 4.16.

Practice problem: Find the Norton equivalent circuit for the circuit in Figure below.

Answer: $R_N = 3\ \Omega$, $I_N = 4.5$ A.



4.5 MAXIMUM POWER TRANSFER

In many practical situations, a circuit is designed to provide power to a load. While for electric utilities, minimizing power losses in the process of transmission and distribution is critical for efficiency and economic reasons, there are other applications in areas such as communications where it is desirable to maximize the power delivered to a load. We now address the problem of delivering the maximum power to a load when given a system with known internal losses.

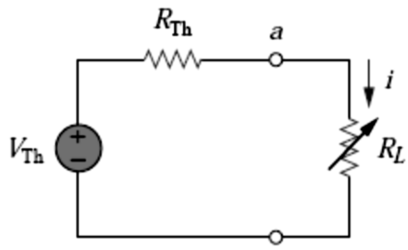


Figure 4.19 The circuit used for maximum power transfer.

The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. We assume that we can adjust the load resistance $\mathbf{R_L}$. If the entire circuit is replaced by its Thevenin equivalent except for the load, as shown in **Fig. 4.19**, the power delivered to the load is

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L \qquad (4.15)$$

For a given circuit, $\mathbf{V_{Th}}$ and $\mathbf{R_{Th}}$ are fixed. By varying the load resistance $\mathbf{R_L}$, the power delivered to the load varies as sketched in **Fig. 4.20**. We notice from **Fig. 4.20** that the power is small for small or large values of $\mathbf{R_L}$ but maximum for some value of $\mathbf{R_L}$ between **0** and ∞. We now want to show that this maximum power occurs when $\mathbf{R_L}$ is equal to $\mathbf{R_{Th}}$. This is known as the maximum power theorem.

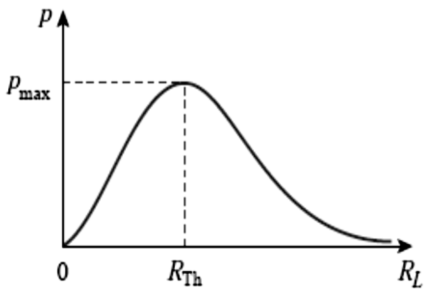


Figure 4.20 Power delivered to the load as a function of RL

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).

To prove the maximum power transfer theorem, we differentiate p in **Eq. (4.15)** with respect to $\mathbf{R_L}$ and set the result equal to zero. We obtain

$$\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(RTh + RL)^2 - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4}\right] = V_{Th}^2 \left[\frac{(R_{Th} + R_L - 2R_L)}{(R_{Th} + R_L)^3}\right] = 0$$

This implies that

$$\mathbf{0 = (R_{Th} + R_L - 2R_L) = (R_{Th} - R_L)} \qquad (4.16)$$

which yields

$$\mathbf{R_L = R_{Th}} \qquad (4.17)$$

showing that the maximum power transfer takes place when the load resistance $\mathbf{R_L}$ equals the Thevenin resistance $\mathbf{R_{Th}}$. We can readily confirm that **Eq. (4.17)** gives the maximum power by showing that $\mathbf{d^2p/dR_L^2 < 0}$.

The maximum power transferred is obtained by substituting **Eq. (4.17)** into **Eq. (4.15)**, for

$$p_{max} = \frac{V_{Th}^2}{4R_{Th}} \qquad (4.18)$$

Equation (4.18) applies only when $\mathbf{R_L = R_{Th}}$. When $\mathbf{R_L \neq R_{Th}}$, we compute the power delivered to the load using **Eq. (4.15)**.

<u>Example 4.6:</u> Find the value of $\mathbf{R_L}$ for maximum power transfer in the circuit of **Fig. 4.21**. Find the maximum power.

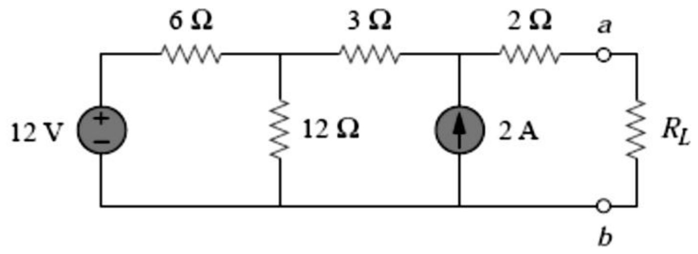


Figure 4.21 For Example 4.6.

Solution:

We need to find the Thevenin resistance $\mathbf{R_{Th}}$ and the Thevenin voltage $\mathbf{V_{Th}}$ across the terminals a-b. To get $\mathbf{R_{Th}}$, we use the circuit in **Fig. 4.22(a)** and obtain

$$R_{Th} = 2 + 3 + 6\|12 = 5 + \frac{6 \times 12}{18} = 9\,\Omega$$

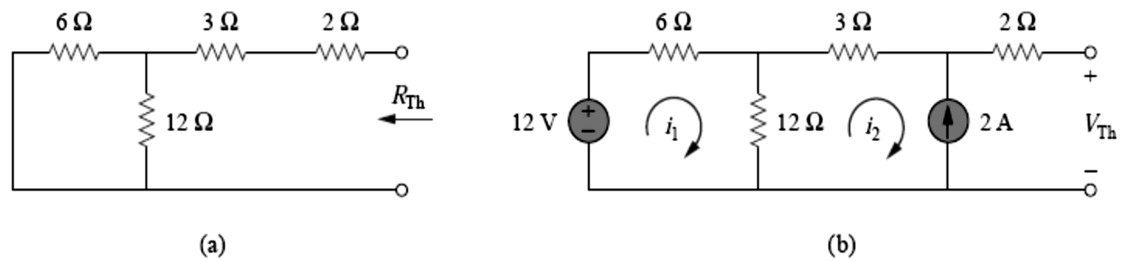


Figure 4.22 For Example 4.6: (a) finding R_{Th}, (b) finding V_{Th}.

To get $\mathbf{V_{Th}}$, we consider the circuit in **Fig. 4.22(b)**. Applying mesh analysis,

$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2\text{ A}$$

Solving for $\mathbf{i_1}$, we get $\mathbf{i_1 = -2/3}$. Applying **KVL** around the outer loop to get $\mathbf{V_{Th}}$ across terminals a-b, we obtain

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0 \Rightarrow V_{Th} = 22\text{ V}$$

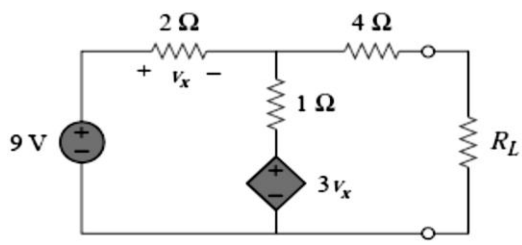
For maximum power transfer,

$$R_L = R_{Th} = 9\,\Omega$$

and the maximum power is

$$p_{max} = \frac{V_{Th}^2}{4RL} = \frac{22^2}{4 \times 9} = 13.44\,W$$

Practice problem: Determine the value of $\mathbf{R_L}$ that will draw the maximum power from the rest of the circuit in Figure below. Calculate the maximum power.



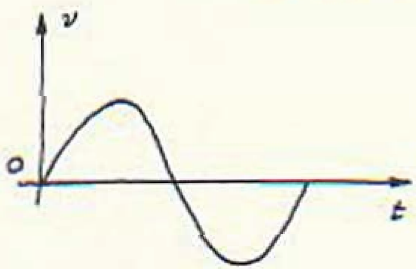
Answer: **4.22 Ω, 2.901 W.**

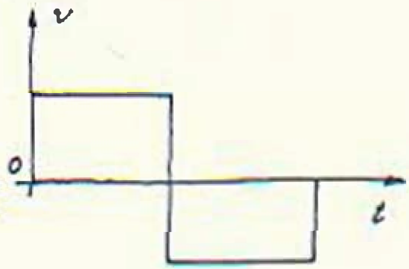
5. Sinusoidal Alternating Waveforms

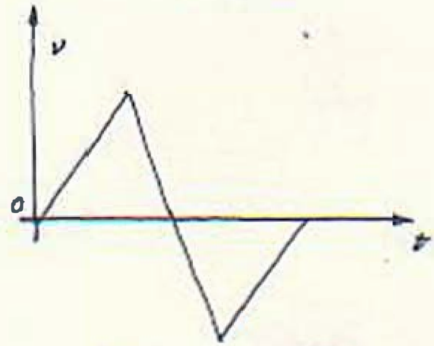
EE6

6.1 Some Representative Types of AC Waveforms

: The terminolog ac voltage or ac current refers to alternating voltage or current. The term alternating indicates only that waveforms alternate between two prescribed levels in a set time sequence. To be absolutely correct the term sinusoidal, square, traingular must be also applied.







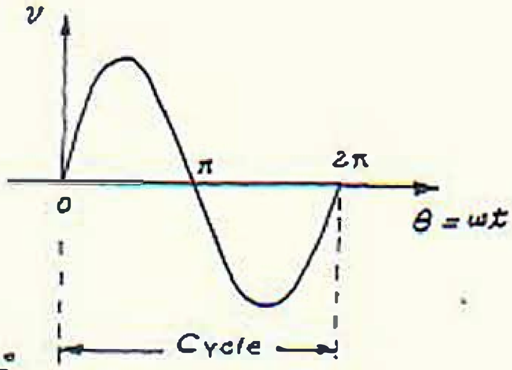
Sinusoidal Waveform   Squar Wave   Triangular Wave.

The above types of waveforms (or signals), are often encountered in electrical engineering applications, but they are not the only one. There are too many different types of waveforms that have different shapes matching certain applications.

:

5.3 Definitions Related to AC Waveforms :

★ The Cycle : One complete set of positive and negative values of an alternating quantity is called a (cycle). Complete cycle is said to spread over 360° or 2π radians



$$\therefore \quad 360^\circ = 2\pi \text{ radians}$$

$$\Rightarrow \quad 1 \text{ rad.} = \frac{360^\circ}{2\pi} = \frac{180^\circ}{\pi}$$

$$\therefore \quad 1 \text{ rad.} = 57.3^\circ$$

- To convert from degrees to radians

$$\boxed{\text{Radians} = \left(\frac{\pi}{180}\right) \times \text{degrees}}$$

- To convert from radians to degrees

$$\boxed{\text{Degrees} = \left(\frac{180}{\pi}\right) \times \text{radians}}$$

For examples

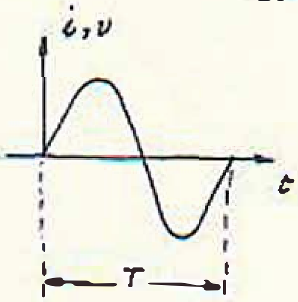
$90^\circ \rightarrow \text{rad.} \quad = \frac{\pi}{180} \times 90^\circ = \frac{\pi}{2}$

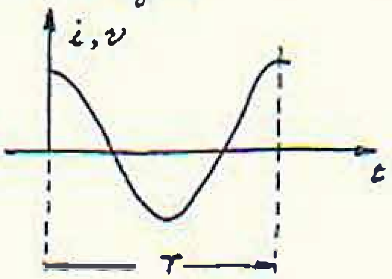
$30^\circ \rightarrow \text{rad.} \quad = \frac{\pi}{180} \times 30 = \frac{\pi}{6}$

$\frac{\pi}{3} \rightarrow \text{deg.} \quad = \frac{180}{\pi} \times \frac{\pi}{3} = 60^\circ$

$\frac{3\pi}{2} \rightarrow \text{deg.} \quad = \frac{180}{\pi} \times \frac{3\pi}{2} = 270^\circ$

★ Time Period (T) : It is the time taken by an alternating quantity to complete one cycle.





$T = \frac{1}{f}$ $\quad \Rightarrow f$ is the frequency.

EEE

* Instantaneous Value

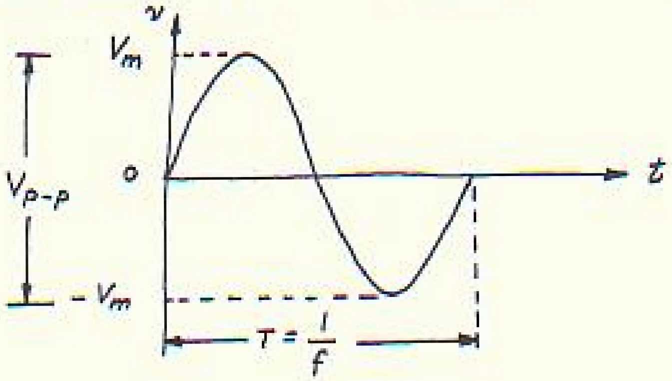
: The magnitude of a waveform (alternating voltage or current) at any instant of time. It is denoted by small letters such as v, e_1, e_2, i_1, i_2, p ... etc.

* Amplitude (or Peak Value or Maximum Value)

: The maximum value (positive or negative) of an alternating quantity is called amplitude. It is denoted by capital letters, such as E_m, V_m, I_m, ... etc.

* Frequency

: It is the number of cycles that occur in one second



* Peak to Peak Value

: It is denoted by E_{p-p} or V_{p-p} and represents the full voltage between positive and negative peaks of the waveform (see the figure above.).

In general form for the sinusoidal voltage or current

$$\boxed{e = E_m \sin\theta} \qquad \theta \text{ in deg.}$$

$$\boxed{e = E_m \sin\omega t} \qquad \theta = \omega t$$

$$\boxed{e = E_m \sin 2\pi f t} \qquad \omega = 2\pi f$$

$$\boxed{e = E_m \sin \frac{2\pi}{T} t} \qquad f = \frac{1}{T}$$

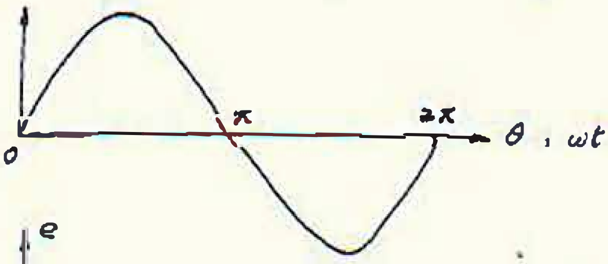
6.4 Phase Relations

EE6

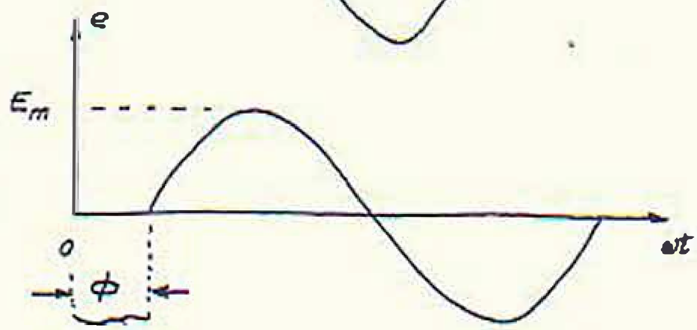
A sinusoidal waveform may start from zero as had been shown earlier, or it may be shifted to the left or to the right as shown.

$$\boxed{e = E_m \sin \omega t}$$

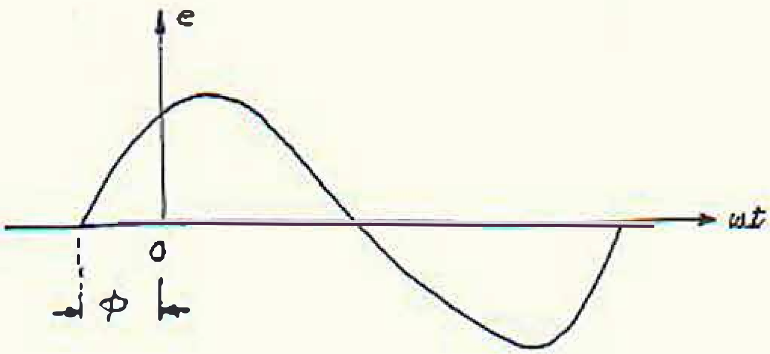
* it has zero phase shift.



$$\boxed{e = E_m \sin(\omega t - \Phi)}$$

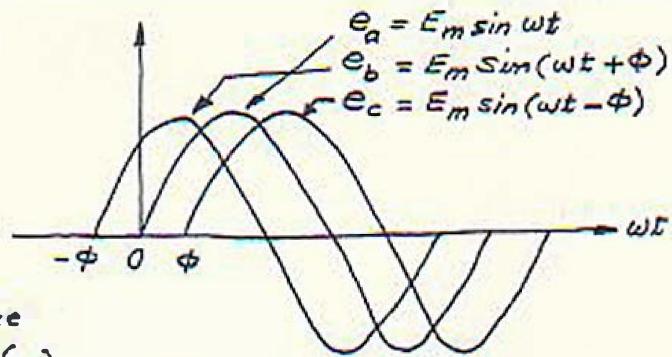


$$\boxed{e = E_m \sin(\omega t + \Phi)}$$



In the figure shown, three sinusoidal waveform are plotted with different phases:

$$e_a = E_m \sin \omega t$$
$$e_b = E_m \sin(\omega t + \Phi)$$
$$e_c = E_m \sin(\omega t - \Phi)$$



(*) A plus (+) sign when used in connection with phase difference denotes (lead) whereas minus (−) sign denotes (lag).

Some Useful Relations

$$\sin(\omega t + 90^\circ) = \sin(\omega t + \tfrac{\pi}{2}) = \cos(\omega t)$$
$$\sin(\omega t - 90^\circ) = \cos(\omega t - \tfrac{\pi}{2}) = \sin \omega t$$
$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$-\sin(\theta) = \sin(\theta \pm 180^\circ)$$
$$-\cos(\theta) = \cos(\theta \pm 180^\circ)$$

Examples

: What is the phase difference between the following sets of voltages and currents?

(a). $v = 10 \sin(\omega t + 30^\circ)$
$i = 5 \sin(\omega t + 70^\circ)$

(b). $v = 10 \sin(\omega t - 20^\circ)$
$i = 15 \sin(\omega t + 60^\circ)$

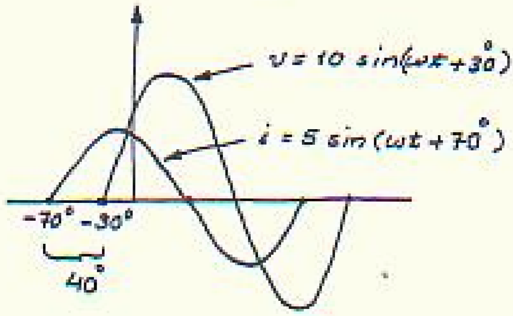
(c). $i = 2 \cos(\omega t + 10^\circ)$
$v = 3 \sin(\omega t - 10^\circ)$

(d). $i = -2\cos(\omega t - 60^\circ)$
$v = 3 \sin(\omega t - 150^\circ)$

(e). $i = -\sin(\omega t + 30^\circ)$
$v = 2 \sin(\omega t + 10^\circ)$

Solutions:

(a).



The phase difference $= 40^\circ$

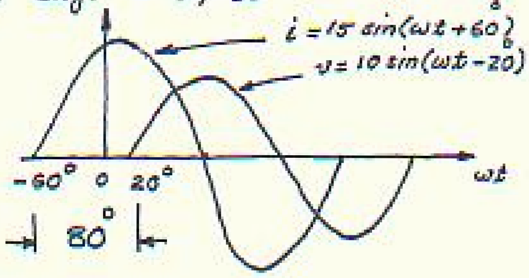
i **leads** v by 40°

$\boxed{\text{or}}$ v **lags** i by 40°

(b). The phase difference $60^\circ + 20^\circ = 80^\circ$

$\therefore$ i **leads** v by 80°

or v lags i by 80°



Similarly you can find the results of c, d and e, and the results are:

(c).

$$i = 2\cos(\omega t + 10^\circ) = 2\sin(\omega t + 10^\circ + 90^\circ) = 2\sin(\omega t + 100^\circ)$$

$$\therefore \left.\begin{aligned} i &= 2\sin(\omega t + 100^\circ) \\ v &= 3\sin(\omega t - 10^\circ) \end{aligned}\right\} \Rightarrow \text{The phase difference} = (100 + 10) = 110^\circ$$

$\therefore$ i lead v by 110°

(d).

$$i = -\sin(\omega t + 30^\circ) = \sin(\omega t + 30^\circ \pm 180^\circ) = \sin(\omega t - 150^\circ)$$

$$\therefore \left.\begin{aligned} i &= \sin(\omega t - 150^\circ) \\ v &= 2\sin(\omega t + 10^\circ) \end{aligned}\right\} \Rightarrow \text{The phase difference} = (150^\circ + 10^\circ) = 160^\circ$$

$\therefore$ v leads i by 160°

or $i = -\sin(\omega t + 30^\circ) = \sin(\omega t + 30^\circ + 180^\circ) = \sin(\omega t + 210^\circ)$

$$\therefore \left.\begin{aligned} i &= \sin(\omega t + 210^\circ) \\ v &= 2\sin(\omega t + 10^\circ) \end{aligned}\right\} \Rightarrow \text{The phase difference is } 200^\circ$$

$\therefore$ i lead v by 200°

(e).

$$i = -2\cos(\omega t - 60^\circ) = 2\cos(\omega t - 60^\circ - 180^\circ) = 2\cos(\omega t - 240^\circ)$$

$$\left.\begin{aligned} &= 2\sin(\omega t - 240^\circ + 90^\circ) = 2\sin(\omega t - 150^\circ) \\ v &= 3\sin(\omega t - 150^\circ) \Rightarrow \text{The phase diff.} = 0^\circ \end{aligned}\right\}$$

V and i are in phase

EEG

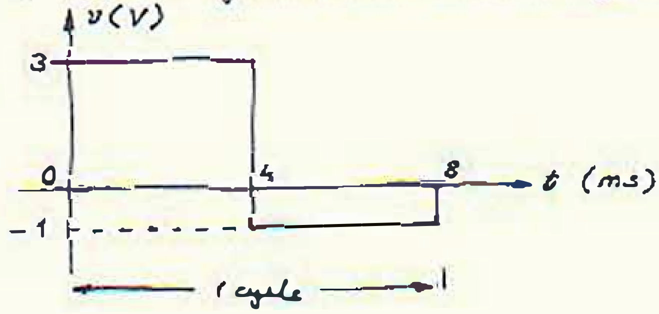
5.5 Average Value

The average value of an alternating waveform is the equivalent (dc) value over a complete cycle. In general the average value of a waveform is given as

$$\boxed{\text{Average value} = \frac{\text{Area under the curve}}{\text{Length of the curve}}}$$

For complex waveforms the area under the curve is difficult to obtain directly, so it can be evaluated by integration over the specified period of time.

Example: Find the average value of the waveform shown over one full cycle.

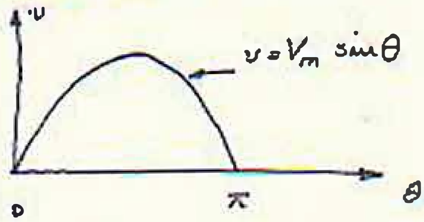


Solution:

$$V_{av.} = \frac{\text{Aera under the curve}}{\text{Length of the curve}}$$

$$= \frac{(3)(4) - (1)(4)}{8} = 1 \text{ Volt}$$

Example: Calculate the average value of the waveform shown.



Solution: - given $v = V_m \sin\theta$

$$\therefore \quad V_{av} = \frac{\int_0^{\pi} V_m \sin\theta \, d\theta}{\pi} \quad \Rightarrow \quad \int_0^{\pi} V_m \sin\theta \, d\theta = \Big| -\cos\theta \Big|_0^{\pi}$$

$$= -V_m(\cos\pi - \cos 0)$$

$$\therefore V_{av} = \frac{2V_m}{\pi} \quad \Leftarrow \quad = 2V_m$$

$$V_{av} = 0.637 \, V_m$$

Note; This value of V_{av} is for one half cycle only!

$\Rightarrow$ For complete cycle (one cycle) $\Rightarrow$ $V_{av} = 0$

5.6 Effective Value (or Root Mean Square Value rms)

EE6

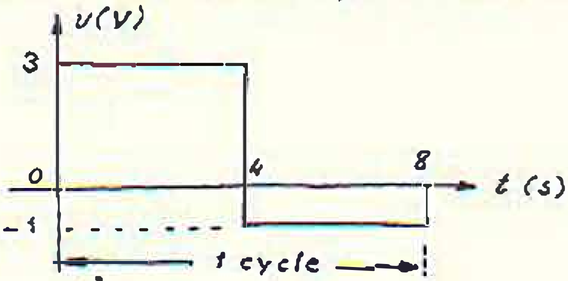
The effective value (or the rms value) of an alternating waveform is given by the steady (dc) current which when flowing through a given circuit, for a given time produces the same heat produced by the alternating current when flowing through the same circuit for the same time.

$$\boxed{I_{rms} = I_{eff} = \frac{\int_0^T i(t)^2\, dt}{T}}$$

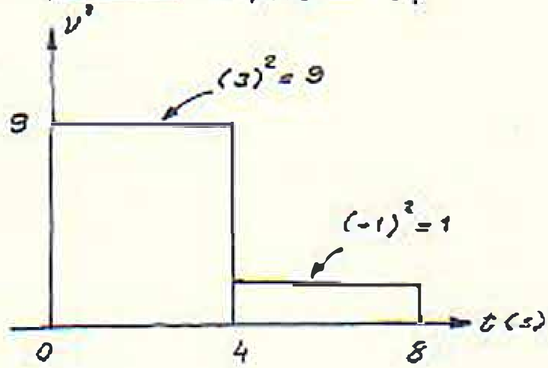
For simple shaped waveforms

$$I_{rms} = I_{eff} = \sqrt{\frac{\text{Area}\left[i(t)^2\right]}{T}}$$

Example : Find the effective or rms value of the waveform shown.



Solution : - finding first $v^2(t)$ as show in the fig.



$$\therefore V_{eff} = \sqrt{\frac{\text{Area}\left[v^2(t)\right]}{T}}$$

$$= \sqrt{\frac{(9)(4) + (1)(4)}{8}}$$

$$= 2.236\ V$$

Example : For the waveform given by ($i = I_m \sin \omega t$), calculate the rms value.

Solution :

$$I_{eff} = I_{rms} = \sqrt{\int_0^{2\pi} \frac{i^2}{2\pi}\, d\theta}$$

we have ; $\quad i = I_m \sin\omega t = I_m \sin\theta$

$$\therefore I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(I_m \sin\theta)^2 d\theta} = \sqrt{\frac{I_m^2}{2\pi}\int_0^{2\pi}\sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{2\pi}\frac{(1-\cos 2\theta)}{2}d\theta} = \sqrt{\frac{I_m^2}{4\pi}\left|\theta - \frac{\sin 2\theta}{2}\right|_0^{2\pi}}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\times 2\pi} = \sqrt{\frac{I_m^2}{2}} = \frac{1}{\sqrt{2}} I_m$$

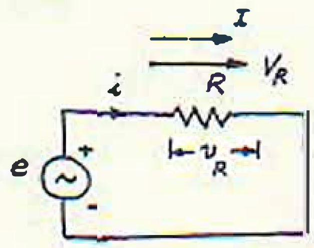
$$\therefore \boxed{I_{rms} = \frac{1}{\sqrt{2}} I_m = 0.707\, I_m}$$

$\Rightarrow$ The rms value of sinusoidal waveforms (voltage or current)

5.7 Circuit Elements in the Phasor Domain :

⊕ AC Through Pure Ohmic Resistor Alone

In the cct. shown, let the applied voltage be given by :



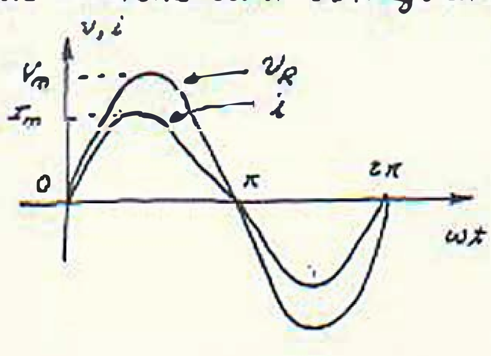
$$e = E_m \sin\omega t = E_m \sin\theta$$

$$\therefore i = \frac{e}{R} = \frac{E_m}{R}\sin\omega t$$

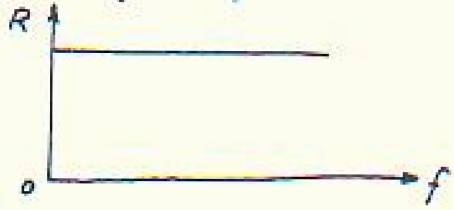
$$\therefore \boxed{i = I_m \sin\omega t}$$

and $\boxed{v_R = iR = I_m R \sin\omega t = V_m \sin\omega t}$

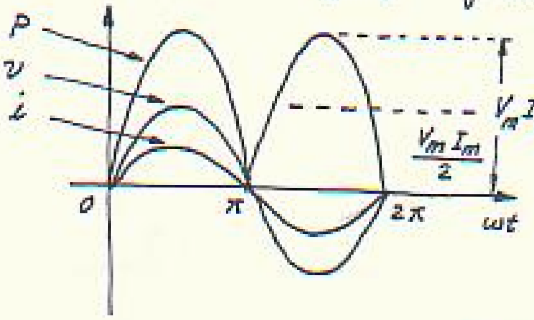
⊕ In resistors, the current and voltage are $\boxed{\text{inphase}}$



* The frequency response of the resistive load is uniform, ie, the value of the resistance doesn't change as the frequency changes.



* The Average Power (Real Power): The instantaneous power to the resistive load is given by:



$$p = vi$$
$$= V_m I_m \sin^2 \omega t = V_m I_m \frac{(1-\cos 2\omega t)}{2}$$

$$\therefore\ p = \underbrace{\frac{V_m I_m}{2}}_{\text{constant term}} - \underbrace{\frac{V_m I_m}{2}\cos 2\omega t}_{\text{Time varying term with average value = 0}}$$

$$\boxed{\therefore\ P_{av} = P = \frac{V_m I_m}{2}} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$
$$= V_{rms} I_{rms} = V_{eff} I_{eff}$$

$$\boxed{\text{or}\ \ P_{av} = P = V_{rms} I_{rms}}$$

* The Power factor is defined as the cosine of the phase angle between the voltage and current, ie,

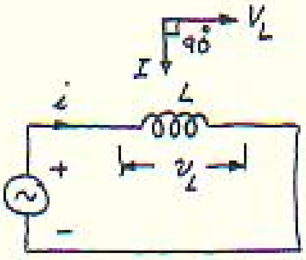
$$\text{Power factor} = P.f = F_p = \cos\phi$$

where ϕ is the phase difference angle between i and v. Since the voltage and current are in phase (ie, the phase difference = 0), then; $\phi = 0$

$$\therefore\ \cos\phi = \cos 0^\circ = 1$$
$$\Rightarrow \text{power factor} = 1$$

$\Rightarrow$ For resistive load only.

⊕ AC Through Pure Inductance Alone



$$v_L = L\frac{di}{dt}$$

If the current i is given by:

$$i = I_m \sin\omega t$$

$$\Rightarrow v_L = L.\frac{d}{dt}(I_m \sin\omega t) = \omega L I_m \cos\omega t$$

$$\text{or} \quad v_L = V_m \cos\omega t = V_m \sin(\omega t + 90°) \Leftarrow V_m = \omega L I_m$$

From the equations of i and v_L, it is clear that v_L lead i by an angle of $(90°)$, or the current i lags v_L by $(90°)$.

- Let us define the reactance of an inductor, in a way similar to Ohm's law:

$$\text{Reactance} = \frac{\text{Cause}}{\text{Effect}} = \frac{\text{Voltage}}{\text{Current}}$$

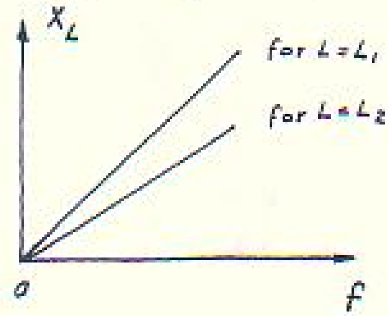
$$\therefore \underset{\text{of an inductor}}{\text{Reactance}} = X_L = \frac{V_m}{I_m} = \frac{\omega L I_m}{I_m} = \omega L$$

$$\boxed{\therefore X_L = \omega L}$$

* The frequency response of the pure inductor is derived from the relation:

$$\boxed{X_L = \omega L = 2\pi f L}$$

and is shown in the figure. X_L increases as the frequency is increased in a linear relationship.



Power factor : The power factor $p.f = \cos\phi$
since $\phi = 90°$

$$\Rightarrow \boxed{\therefore p.f = \cos 90° = 0}$$

The average power : The instantaneous power for the pure inductive circuit is:

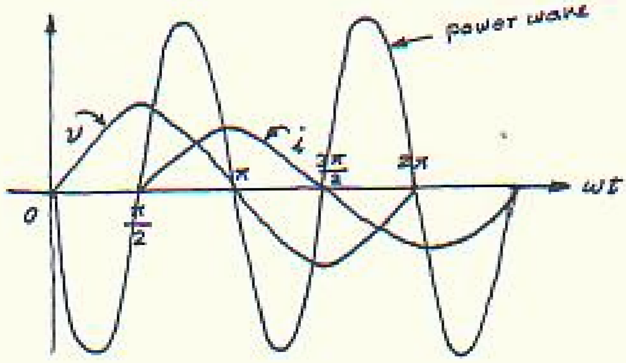
$$p = v_L i$$

EE6

$$\Rightarrow \quad p = v_L \cdot i = V_m I_m \sin \omega t \cdot \sin(\omega t + 90^\circ)$$

$$\therefore \quad p = \frac{V_m I_m}{2} \sin 2\omega t$$

The average value of p is zero $\Rightarrow$ $\boxed{P_{av} = 0}$

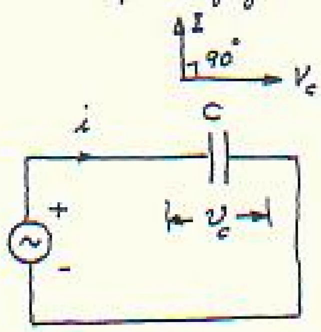


⊛ AC Through a Pure Capacitor

: For the capacitor of the figure shown:

$$i_c = C \frac{dV_c}{dt}$$

let $v_c = V_m \sin \omega t$



$$\therefore \quad i_c = C \frac{d}{dt}(V_m \sin \omega t)$$

$$\Rightarrow \quad i_c = \omega C V_m \cos \omega t$$
$$= \omega C V_m \sin(\omega t + 90^\circ) = I_m \sin(\omega t + 90^\circ)$$

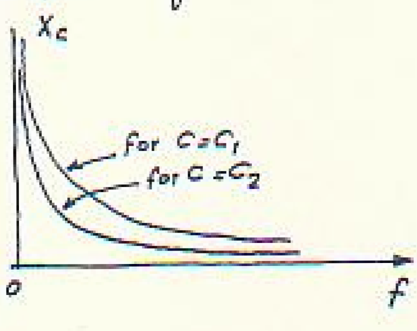
It is clear, from the equations of i_c and v_c, that i_c leads v_c by an angle of 90° or v_c lags i_c by 90°

* The capacitive reactance (X_c) is :

$$X_c = \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

$$\boxed{\therefore \quad X_c = \frac{1}{\omega C}}$$

* The frequency response of a pure capacitor is derived from the relation above and is shown in the figure.
  X_c is decreasing as the frequency is increased in a non-linear behaviour.



EE6

* Power factor

Since the phase difference between i_c and v_c is 90°, this means that $\phi = 90^\circ$, then:

$$\boxed{p.f = \cos\phi = \cos 90^\circ = 0}$$

* The average power

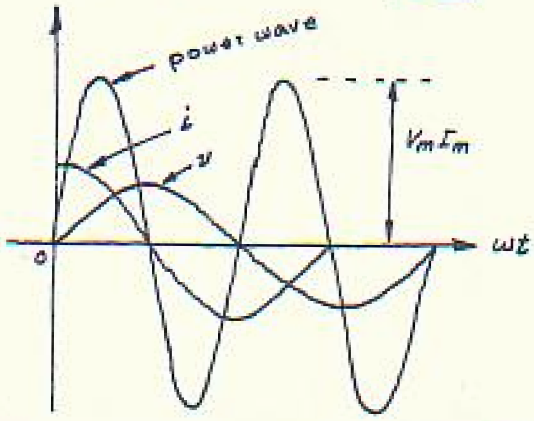
The instantaneous power p is given as:

$$p = v_c i_c = V_m I_m \sin\omega t . \sin(\omega t + 90^\circ)$$

$$\Rightarrow p = \underbrace{\frac{V_m I_m}{2} \sin 2\omega t}$$

This quantity has an average value of zero

$\therefore$ The average power of a pure capacitive load is zero.

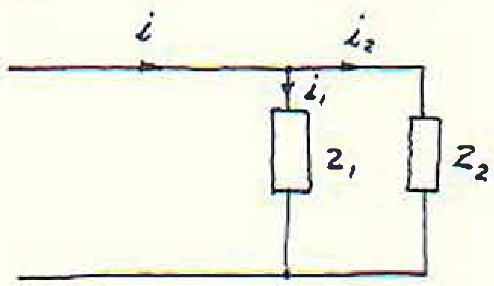


Summary of AC paramters for R, L & C

Element / Paramter	R	L	C
Power factor $\cos\phi$	1	0	0
Average Power $P_{av} = P$	$\frac{V_m I_m}{2} = V_{rms} I_{rms}$	0	0
Impedance $\overline{Z}$	R	$X_L = \omega L$	$X_c = \frac{1}{\omega C}$
Phase difference ϕ between v & i	0	90° v_L leads i or i lags v_L	90° v_c lags i_c or i_c leads v_c
Frequency Response	Uniform (constant)	Linear (Increasing)	Non-linear (Decreasing)

5.8 Calculating Alternating Quantities

Example: For the circuit shown, calculate i, if i_1 and i_2 are:



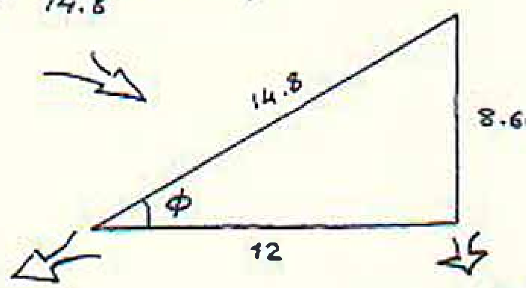
$$i_1 = 7 \sin \omega t$$
$$i_2 = 10 \sin(\omega t + 60^\circ)$$

Solution (time domain calculation)

$$i = i_1 + i_2$$
$$= 7 \sin \omega t + 10 \sin(\omega t + 60^\circ)$$
$$= 7 \sin \omega t + 10 \sin \omega t \cos 60^\circ + 10 \cos \omega t \sin 60^\circ$$
$$\Rightarrow i = 12 \sin \omega t + 8.66 \cos \omega t$$

Divide and multiply be $\sqrt{12^2 + 8.66^2} = 14.8$, we get

$$i = 14.8\left(\frac{12}{14.8} \sin \omega t + \frac{8.66}{14.8} \cos \omega t\right)$$



$$\cos\phi = \frac{12}{14.8} \qquad \sin\phi = \frac{8.66}{14.8}$$

$$\therefore i = 14.8(\cos\phi \sin \omega t + \sin\phi \cos \omega t)$$

$$\Rightarrow i = 14.8 \sin(\omega t + \phi)$$

$$\therefore i = 14.8 \sin(\omega t + 35.8^\circ)$$

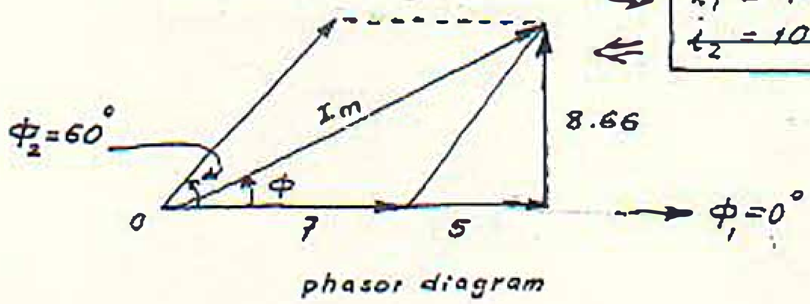
$$\tan\phi = \frac{8.66}{12}$$

$$\text{or } \phi = \tan^{-1}\left(\frac{8.66}{12}\right)$$
$$= 35.8^\circ$$

Solution (phasor domain calculation):

$$i_1 = 7 \sin \omega t \Rightarrow I_{m_1} = 7 \quad \& \ \phi_1 = 0$$
$$i_2 = 10 \sin(\omega t + 60) \Rightarrow I_{m_1} = 10 \quad \& \ \phi_2 = 60^\circ \text{ Leading}$$

In phasor representation $\Rightarrow$ $\boxed{i_1 = 7 \angle 0^\circ \quad i_2 = 10 \angle +60^\circ}$



phasor diagram

* Horizontal components
$$= 7\cos 0^\circ + 10 \cos 60^\circ$$
$$= 12$$

* Vertical components
$$= 7 \sin 0^\circ + 10 \sin 60^\circ$$
$$= 8.66$$

$$\therefore \text{Resultant} = \sqrt{12^2 + 8.66^2} = 14.8 \text{ A}$$

The phase angle $\phi = \tan^{-1} \dfrac{8.66}{12} = 35.8^\circ \quad \Rightarrow \quad i = 14.8 \angle 35.8^\circ$

$\therefore \quad i = 14.8 \sin(\omega t + 35.8^\circ)$ as before.

Phasors

: The phasor is a radius vector that has a constant amplitude at a fixed angle from the positive real axis and that represents a sinusoidal voltage or current in the vector domain, at $t = 0$.

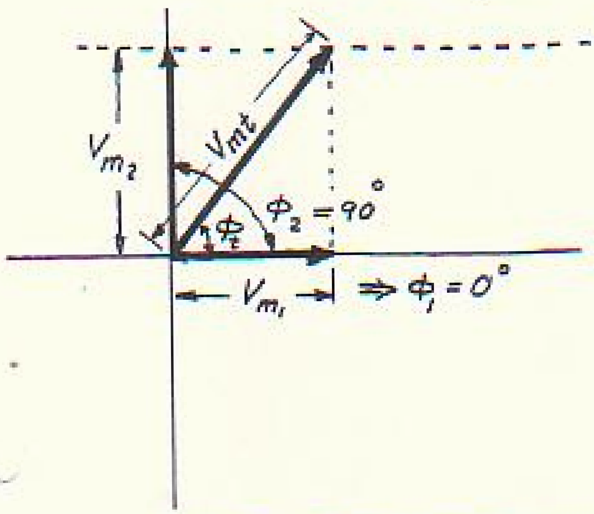
For example, if we have to voltages v_1 and v_2 given by:

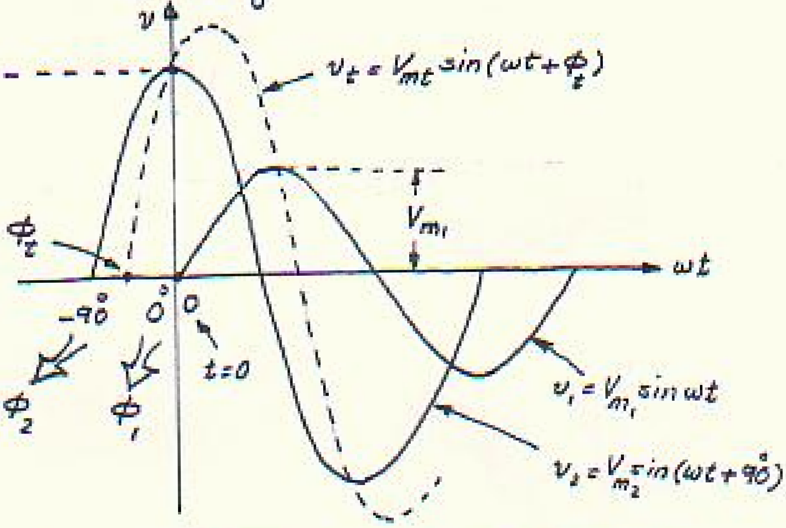
$$v_1 = V_{m_1} \sin \omega t$$

and

$$v_2 = V_{m_2} \sin(\omega t + 90^\circ)$$

We will show the time domain and phasor domain representation of the two voltages and the resultant voltage.





Phasor domain representation		Time domain representation
$v_1 = V_{m_1} \angle 0^\circ$	$\Longleftrightarrow$	$v_1 = V_{m_1} \sin \omega t$
$v_2 = V_{m_2} \angle 90^\circ$		$v_2 = V_{m_2} \sin(\omega t + 90^\circ)$
$v_t = V_{m_1} \angle 0^\circ + V_{m_2} \angle 90^\circ$		$v_t = V_{mt} \sin(\omega t + \theta_T)$

NOTE * In general, it is customary to use the <u>rms</u> (or effective) values (instead of max. values) in the phasor representation, as will be shown in the following examples.

Example: Convert the following quantities from the time domain to the phasor domain.

a. $\sqrt{2}\ 50 \sin\omega t$

b. $69.6 \sin(\omega t + 72^\circ)$

c. $45 \sin \omega t$

d. $80 \cos \omega t$

Solution:

	Time domain	Phasor domain
a.	$\sqrt{2}\,(50) \sin\omega t$	$50 \angle 0^\circ$
b.	$69.6 \sin(\omega t + 72^\circ)$	$49.21 \angle 72^\circ$
c.	$45 \sin \omega t$	$31.82 \angle 0^\circ$
d.	$80 \cos \omega t$	$31.82 \angle 90^\circ$

Example: Write the sinusoidal expression for the following phasors if the frequency is 60 Hz.

a. $\bar{I} = 10 \angle 30^\circ$

b. $\bar{V} = 115 \angle -70^\circ$

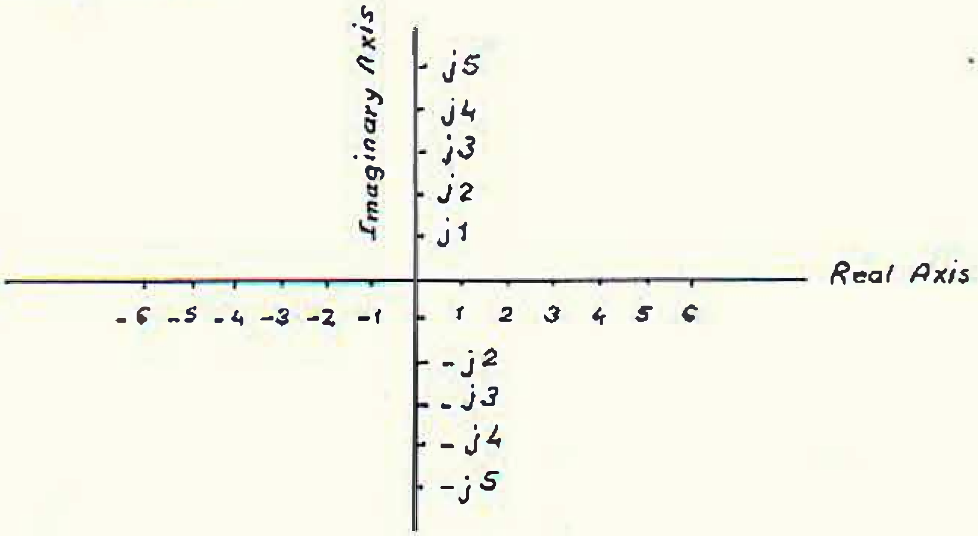
Solution:

Phasor domain	Time domain
a. $\bar{I} = 10 \angle 30^\circ$	$i = \sqrt{2}\,(10) \sin(\omega t + 30^\circ)$ $= 14.14 \sin(2\pi f t + 30^\circ)$ $\therefore i = 14.14 \sin(377t + 30^\circ)$
b. $\bar{V} = 115 \angle -70^\circ$	$v = \sqrt{2}\,(115) \sin(2\pi f t - 70^\circ)$ $\therefore v = 162.6 \sin(377t - 70^\circ)$

5.9 Complex Numbers

EE6

______________ : A complex number is a number that represents a point in a two dimensional plane located with reference to two distinct axes. It defines a vector drawn from the origin to that point. The plane used to represent complex numbers is called the complex plane ; the two axes are called the real and imaginary. It is important that the scale on the axis of imaginaries must be the same as that on the axis of reals.



The complex plane

Complex numbers can be represented in the following forms :

1. Rectangular form	$\Rightarrow$	$\bar{E} = a + jb$
2. Polar form	$\Rightarrow$	$\bar{E} = E_m \angle \Phi$
3. Trigonometric form	$\Rightarrow$	$\bar{E} = E_m (\cos\Phi + j\sin\Phi)$
4. Exponential form	$\Rightarrow$	$\bar{E} = E_m e^{j\Phi}$

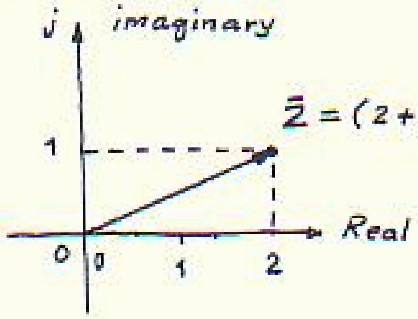
* **Rectangular form** : It is customary in this form to denoted the complex numbers as :

$$\boxed{\bar{Z} = R \mp jX}$$

where ; $\bar{Z}$ is the complex number.
R is the real part.
X is the imaginary part.
j is an operator $= \sqrt{-1}$ and is equivalent to $90°$ phase angle.

EE6

For example, the complex number $\bar{Z} = 2 + j1$ is represented in the complex plane as shown:



Mathematical Operations in the Rectangular Form:

* Equality: If we have two complex numbers $Z = x + jy$ and $\bar{W} = u + jv$, then if $\bar{Z} = \bar{W}$, then it follows that

$$x = u \quad \text{and} \quad v = y$$

Example: Given that $\bar{Z}_1 = 5 + j10$, and $Z_2 = 5 + jX$. If $\bar{Z}_1 = \bar{Z}_2$, find the value of X

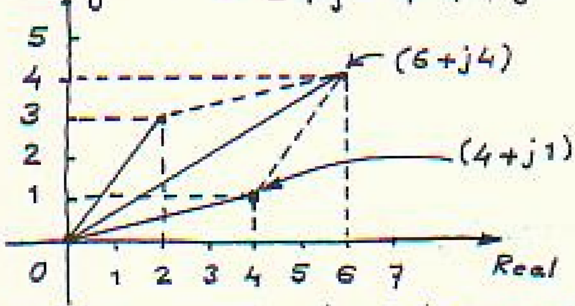
Solution: $X = 10$

* Addition and Subtraction: The sum of two complex numbers has a real number equal to the sum of the real components and an imaginary number equal to the sum of the imaginary components.

Example: Given $\bar{Z}_1 = (2 + j3)$, and $\bar{Z}_2 = (4 + j1)$. Find $\bar{Z}_T$, if $\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2$.

Solution:

$$\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2$$
$$= 2 + j3 + 4 + j1 = 6 + j4$$



The parallelogram method is used for adding and subtraction of quantities in the complex plane.

EE6

* Multiplication

- The product of a real and imaginary number is imaginary ; Thus:

$$2(j3) = j6$$

- The product of two positive imaginary numbers is real and negative, thus:

$$(j2)(j3) = -6$$

and

$$(j3)(-j4) = +12$$

since $(j)(j) = -1$

Complex numbers are multiplied by the ordinary rules of algebra. As an example ;

$$(2+j3)(4+j1) = (2)(4) + (2)(j1) + (j3)(4) + (j3)(j1)$$
$$= 8 + j2 + j12 - 3 = 5 + j14$$

In general ;

$$\boxed{(a+jb)(c+jd) = (ac-bd) + j(ad+bc)}$$

* Division

By way of illustration, let us consider the division of $\bar{V} = (5+j10)$ by $\bar{I} = (2+j1)$.

$$\Rightarrow \frac{5+j10}{2+j1}$$

* The first step is to multiply both numerator and denominator by $(2-j1)$ which is called complex conjugate of the denominator and usually denoted by asterisk,

$$\bar{I} = 2 + j1$$
$$\bar{I}^* = 2 - j1 \leftarrow \text{complex conjugate of } \bar{I}.$$

$$\boxed{(\bar{I})(\bar{I}^*) = \text{Real value}}$$

then ; for

$$\frac{5+j10}{2+j1} = \frac{(5+j10)(2-j1)}{(2+j1)(2-j1)} = \frac{20+j15}{5}$$

$$\therefore \frac{5+j10}{2+j1} = 4 + j3$$

In general :

$$\boxed{\frac{a+jb}{c+jd} = \frac{ac+bd}{c^2+d^2} + j\,\frac{bc-ad}{c^2+d^2}}$$

EEG

* The j operator

$$j = \sqrt{-1} \qquad \Rightarrow 90^\circ \text{ ccw rotation}$$

$$j^2 = -1 \qquad \Rightarrow 180^\circ \text{ ccw rotation}$$

$$j^3 = j^2 . j = -j \qquad \Rightarrow 270^\circ \text{ ccw rotation}$$

$$j^4 = j^2 . j^2 = +1 \qquad \Rightarrow 360^\circ \text{ ccw rotation}$$

and we have also:

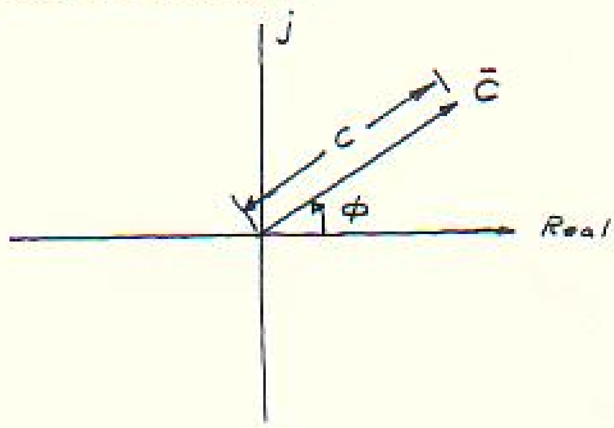
$$\boxed{\frac{1}{j} = -j}$$

Polar Form

In this form, the complex number is represented as

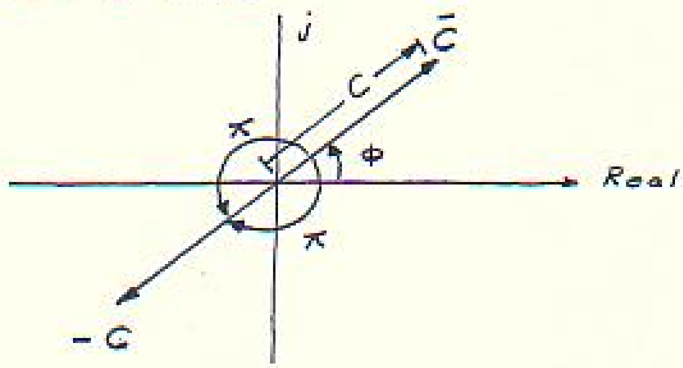
$$\bar{C} = C \angle \phi$$

where C is the magnitude only, and ϕ is always measured counter-clockwise



* A negative sign has the effect shown in the figure

$$-\bar{C} = -C\angle\phi = C\angle\phi \mp \pi$$



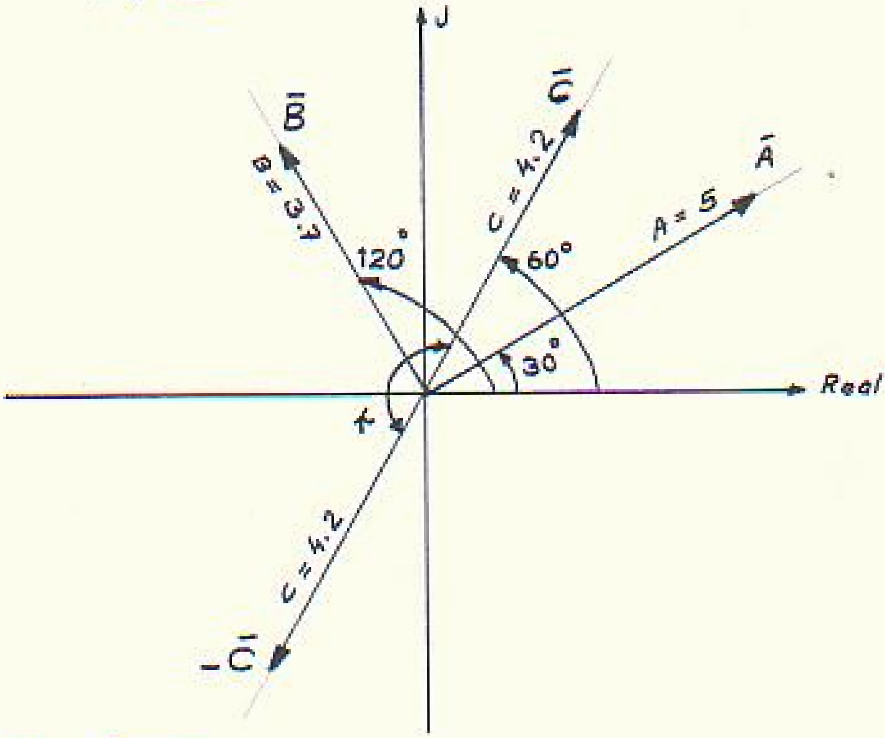
Example: Sketch the following complex numbers in the complex plane.

a. $\bar{A} = 5\angle 30^\circ$

$\bar{B} = 3.7\angle 120^\circ$

$\bar{C} = -4.2\angle 60^\circ$

Solution:



Conversion Between Forms

* Rectangular to Polar:

Example: Convert the following from rectangular to polar form.

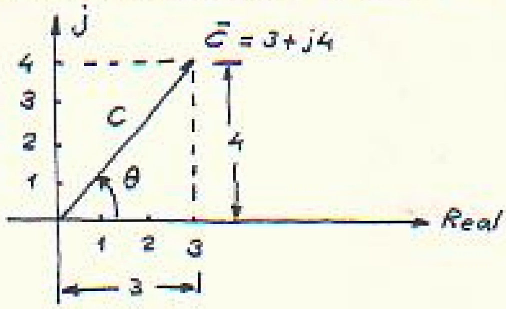
$\bar{C} = 3 + j4$

The magnitude $C = \sqrt{3^2 + 4^2}$

$= 5$

The angle $\phi = \tan^{-1}\left(\frac{4}{3}\right)$

$= 53.13^\circ$



$\therefore \bar{C}$ in the polar form is $\bar{C} = 5\angle 53.13^\circ$

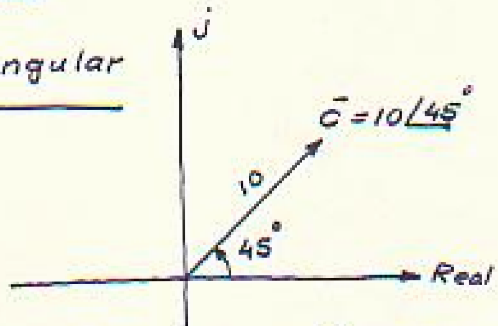
* Polar to Rectangular

Example: convert the following from polar to rectangular form:

$\bar{C} = 10\angle 45^\circ$

$\bar{C} = \text{Real} + j\,\text{Imaginary}$

$\therefore \bar{C} = 7.07 + j\,7.07$



$\text{Real} = 10\cos 45^\circ = 7.07$

$\text{Imaginary} = 10\sin 45 = 7.07$

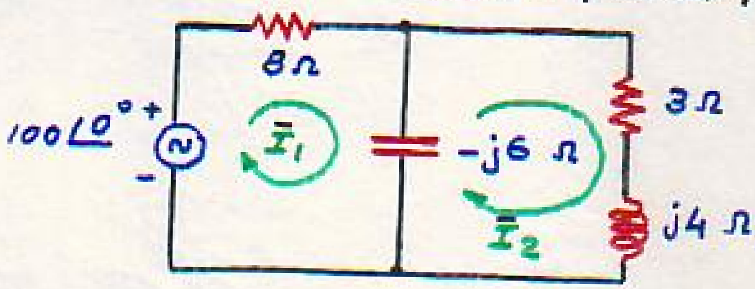
EE3
TS3

AC Network Analysis

Demonstrative Examples :

* Mesh (Loop) Analysis :

Example : Find the power output of the voltage source in the circuit shown; prove that this power equals the power in the circuit resistors. Use mesh analysis in your solution.



Solution : * Loop 1 :

$$100\angle 0^\circ = \bar{I}_1(8-j6) - \bar{I}_2(-j6)$$

$$\Rightarrow 100\angle 0^\circ = \bar{I}_1(8-j6) + \bar{I}_2(j6) \quad \text{—— (1)}$$

* Loop 2 :

$$0 = \bar{I}_2(3+j4-j6) - \bar{I}_1(-j6)$$

$$= \bar{I}_1(j6) + \bar{I}_2(3-j2) \quad \text{—— (2)}$$

$$\bar{I}_1 = \frac{\Delta_1}{\Delta} \quad \text{and} \quad \bar{I}_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta = \begin{vmatrix} (8-j6) & j6 \\ j6 & (3-j2) \end{vmatrix} = (8-j6)(3-j2) - (j6)^2$$

$$\Rightarrow \Delta = 58.703 \angle -35.33^\circ$$

Similarly

$$\Delta_1 = \begin{vmatrix} 100\angle 0^\circ & j6 \\ 0 & (3-j2) \end{vmatrix} = (300-j2) = 360\angle -33.69^\circ$$

and

$$\Delta_2 = \begin{vmatrix} (8-j6) & 100\angle 0^\circ \\ j6 & 0 \end{vmatrix} = j600 = 600\angle 90^\circ$$

$$\therefore\ \bar{I}_1 = \frac{\Delta_1}{\Delta} = \frac{360\angle -33.69^\circ}{58.703\angle -35.33^\circ} = 6.133\angle 1.64^\circ$$

$$\bar{I}_2 = \frac{\Delta_2}{\Delta} = \frac{600\angle 90^\circ}{58.703\angle -35.33^\circ} = 10.22\angle 125.33^\circ$$

$\therefore$ Total Power output = total power absorbed by resistors

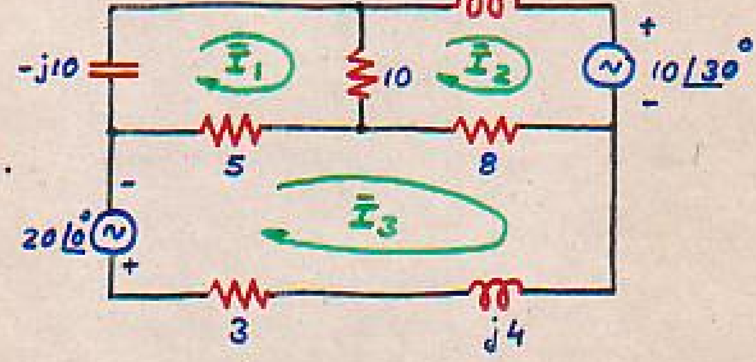
$$\therefore\ VI_1 = I_1^2(8) + I_2^2(3)$$

$$\boxed{(100)(6.133)} = \boxed{300.4} + \boxed{313.3}$$

$$\therefore\ 613.3\ W \doteq 613.7\ W$$

Example :

Write the mesh current equations for the circuit shown.



Solution

: The three mesh current equations are

*** Loop 1**

$$0 = (15 - j10)\bar{I}_1 - 10\,\bar{I}_2 - 5\,\bar{I}_3$$

*** Loop 2**

$$-10\angle 30^\circ = -10\,\bar{I}_1 + (18 + j5)\bar{I}_2 - 8\,\bar{I}_3$$

*** Loop 3**

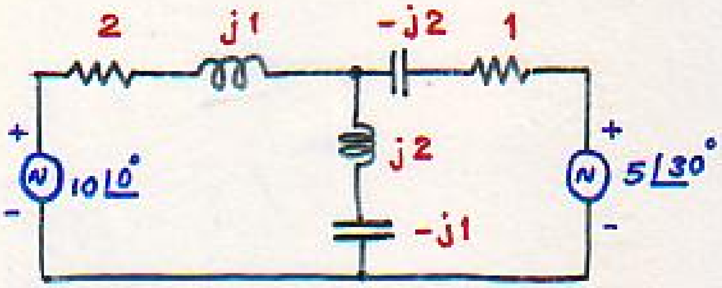
$$-20\angle 0^\circ = -5\,\bar{I}_1 - 8\,\bar{I}_2 + (16 + j4)\bar{I}_3$$

$$\therefore\ \Delta = \begin{vmatrix} (15-j10) & -10 & -5 \\ -10 & (18+j5) & -8 \\ -5 & -8 & (16+j4) \end{vmatrix} = ?$$

Then you can find $\bar{I}_1 = \dfrac{\Delta_1}{\Delta}$, $\bar{I}_2 = \dfrac{\Delta_2}{\Delta}$ and $\bar{I}_3 = \dfrac{\Delta_3}{\Delta}$

Homework :

For the cct. shown, determine the branch voltages and currents and the power delivered by the source using mesh analysis.



* All resistance and reactance values are in Ohms .

HW

* Active, Reactive and Apparent Power (The Power Triangle):

* The Apparent Power:
It is the product of the rms values of the applied voltage and the circuit current

$$\Rightarrow \text{Apparent power} = S = VI = (IZ).I = I^2Z$$

وحدات قياس
volt-ampere (VA)

* The Active Power:
It is the power which is actually dissipated in the circuit resistance.

$$\Rightarrow \text{Active power} = P = I^2R = VI\cos\phi \qquad \text{watts (W)}$$

* The Reactive Power:
It is the power developed in the inductive reactance of the circuit.

$$\Rightarrow \text{Reactive Power} = Q = I^2X_L = I^2(Z\sin\phi)$$

$$\therefore Q = VI\sin\phi$$

وحدات قياس
Volt-amper reactive (VAR)

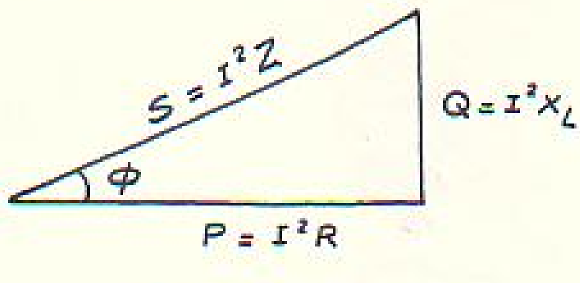
These three powers are shown in the power triangle:

From the power triangle:

$$S^2 = P^2 + Q^2$$

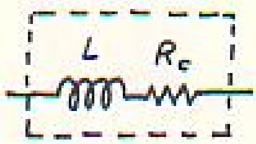
or

$$S = \sqrt{P^2 + Q^2}$$



The power triangle

* The quality factor of the Coil:
It is defined as the reciprocal of the power factor of the coil. Hence:



coil

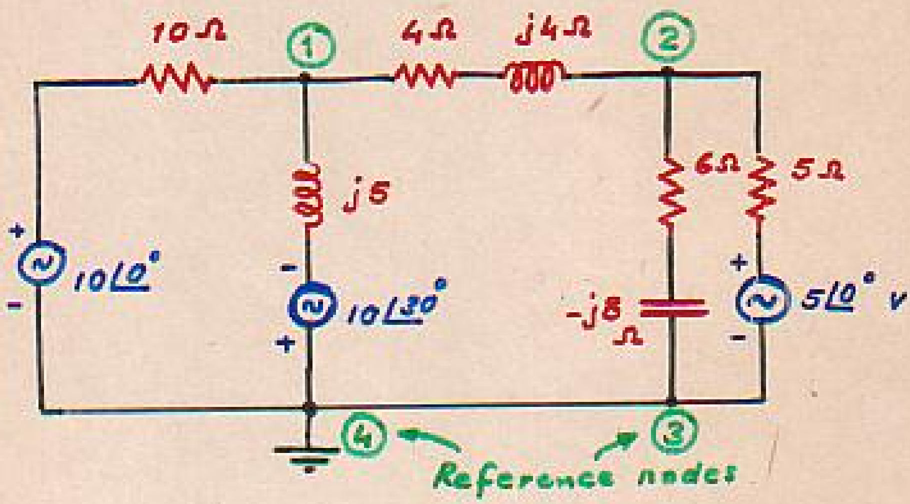
$$\Rightarrow Q_{factor} = \frac{1}{\text{power factor}} = \frac{1}{\cos\phi} = \frac{Z}{R_c} = \frac{\sqrt{R_c^2 + X_L^2}}{R_c}$$

if R_c is too small compared with X_L, then:

$$\boxed{Q_{factor} = \frac{X_L}{R_c}}$$

* Nodal Analysis :

Example : Write the nodal equations for the circuit shown ;



Solution : We have two independent node 1 and 2, so two equations have to be written :

Node 1 :

$$\bar{V}_1\left(\frac{1}{10}+\frac{1}{(4+j4)}+\frac{1}{j5}\right)-\bar{V}_2\left(\frac{1}{4+j4}\right)=\frac{10\angle 0^\circ}{10}-\frac{10\angle 30^\circ}{j5}$$

Node 2 :

$$\bar{V}_2\left(\frac{1}{4+j4}+\frac{1}{5}+\frac{1}{6-j8}\right)-\bar{V}_1\left(\frac{1}{4+j4}\right)=\frac{5\angle 0^\circ}{5}$$

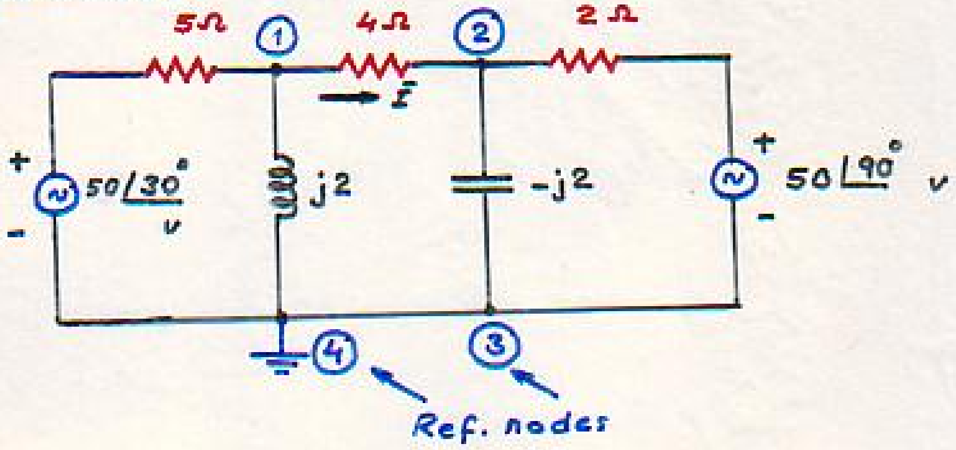
From these two equations, the unknown voltages $\bar{V}_1$ and $\bar{V}_2$ must be determined.

HW

Example

: For the circuit shown in the Fig. below, determine the current flowing through the branch of 4-Ω resistance using :

(a). Nodal Analysis
(b). Thevenin's theorem.
(c). Mesh method



Solution

(a). By Nodal Analysis Method

Node 1

$$\bar{V}_1\left(\frac{1}{5}+\frac{1}{4}+\frac{1}{j2}\right)-\bar{V}_2\left(\frac{1}{4}\right)=\frac{50\angle 30^\circ}{5}$$

$$\therefore \Rightarrow \bar{V}_1(9-j10)-5\bar{V}_2 = 200\angle 30^\circ \quad —(1)$$

Node 2

$$\bar{V}_2\left(\frac{1}{4}+\frac{1}{2}+\frac{1}{-j2}\right)-\bar{V}_1\left(\frac{1}{4}\right)=\frac{50\angle 90^\circ}{2}$$

$$\therefore \Rightarrow \bar{V}_2(3+j2)-\bar{V}_1 = j100 \quad —(2)$$

Solving Equ. 1 and 2 ; we find:

$$\bar{V}_1 = j27.26 \quad \text{volts}$$

and

$$\bar{V}_2 = 19.58+j29.36$$

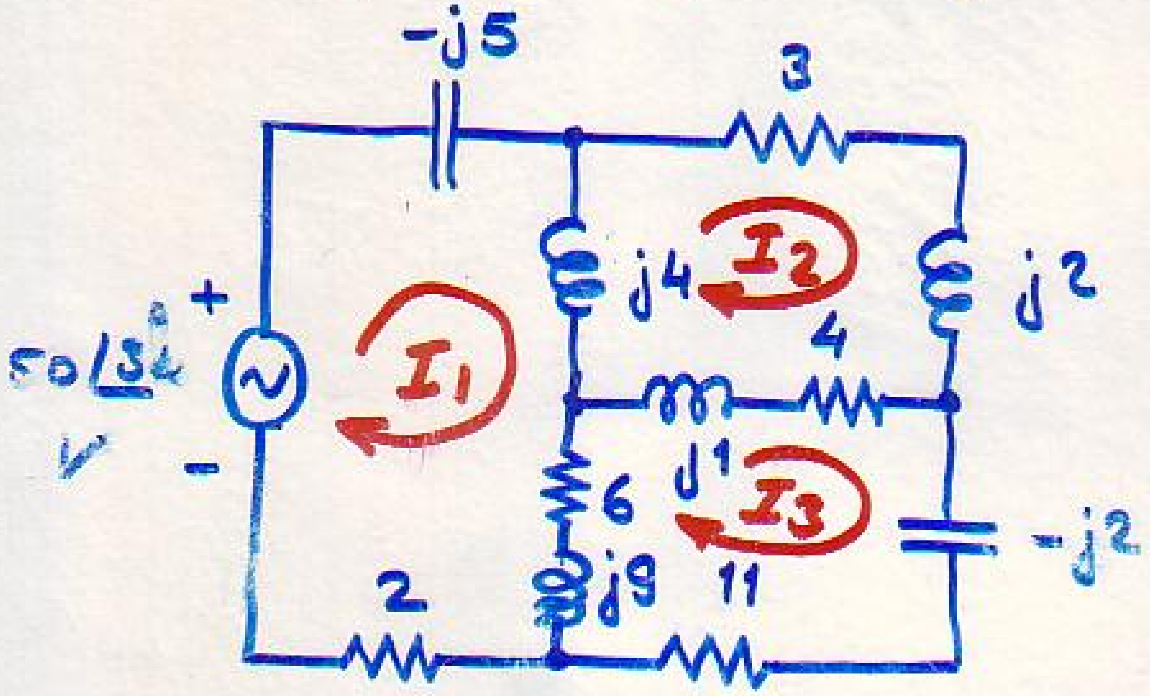
The current $\bar{I} = \dfrac{V_1-V_2}{4} = \dfrac{j27.26-19.58-j29.36}{4}$

$$= \frac{19.69\angle 186.12^\circ}{4} = 4.92\angle 186.12^\circ \ A$$

(*) Try to solve again using the methods of (b), and (c) and getting the same results.

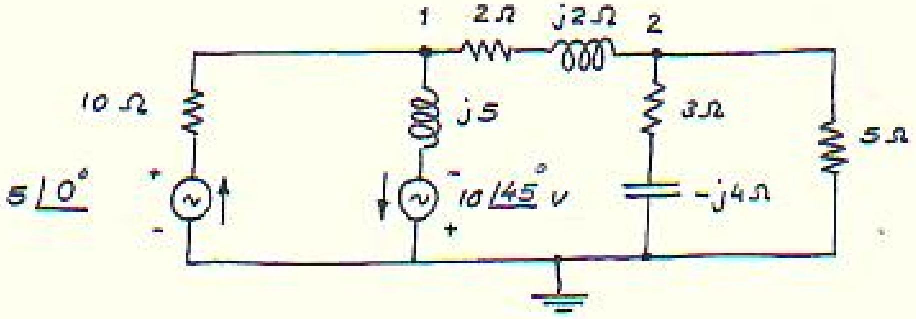
Homework (B)

For the cct. shown, find the current I_1 using the loop Method



TS9

Example: Write the nodal voltage equations for the circuit shown.



Solution: The nodal voltage equations are ;

For Node 1:

$$\left(\frac{1}{10} - \frac{1}{j5} + \frac{1}{2+j2}\right)\bar{V}_1 - \left(\frac{1}{2+j2}\right)\bar{V}_2 = \frac{5\angle 0^\circ}{10} - \frac{10\angle 45^\circ}{j5}$$

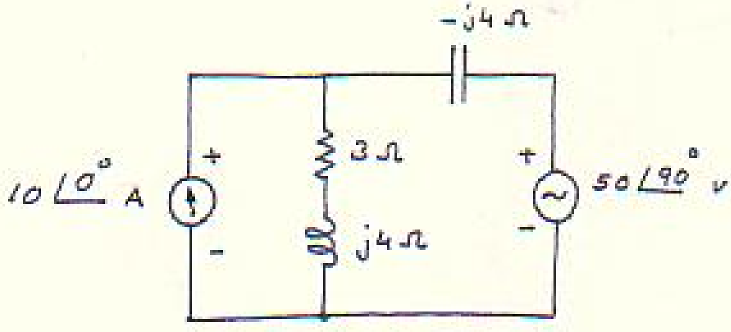
and

For Node 2:

$$-\frac{1}{(2+j2)}\Big)\bar{V}_1 + \left(\frac{1}{2+j2} + \frac{1}{3-j4} + \frac{1}{5}\right)\bar{V}_2 = 0$$

Solving the above equations to determine $\bar{V}_1$ & $\bar{V}_2$.

Example: Apply the superposition theorem to determine the voltage drop across the $(3+j4)\,\Omega$ impedance, in the circuit shown.

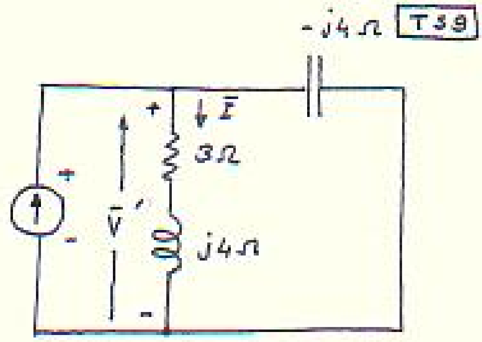


Solution:

* The effect of $(10\angle 0^\circ\,A)$ source alone ; Removing the $50\angle 90^\circ\,V$ source and substitute it by a short circuit, then :

⟹

* using the current divider rule, then:



$$\bar{I} = \bar{I}_T \frac{\bar{Z}_1}{\bar{Z}_1 + \bar{Z}_2}$$

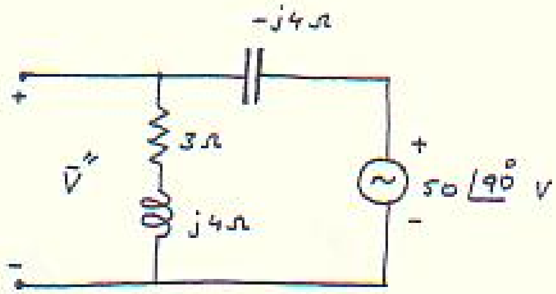
$$= (10\angle 0^\circ)\ \frac{(-j4)}{(-j4) + (3+j4)}$$

$$= (10\angle 0^\circ)\ \frac{(-j4)}{3} = -j\frac{40}{3}$$

$$\therefore\ V' = \bar{I}(3+j4) = -j\frac{40}{3}(3+j4) = \underline{53.3 - j40}$$

* The effect of ($50\angle 90^\circ$ v) voltage source; the circuit will be in this case as:

using the voltage divider rule, then:



$$\bar{V}'' = V_T \frac{\bar{Z}_2}{\bar{Z}_2 + \bar{Z}_1}$$

$$= (50\angle 90^\circ)\left(\frac{(3+j4)}{(3+j4)+(-j4)}\right)$$

$$= -66.7 + j50$$

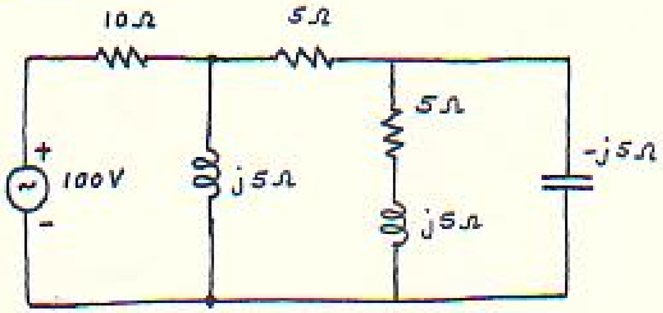
$$\therefore\ \bar{V} = \bar{V}' + \bar{V}'' = (53.3 - j40) + (-66.7 + j50)$$

$$= -13.4 + j10 = 16.72\angle 143.27^\circ$$

Example: For the network shown, determine the voltage across the capacitor, using:

(a). Thevenin's theorem.

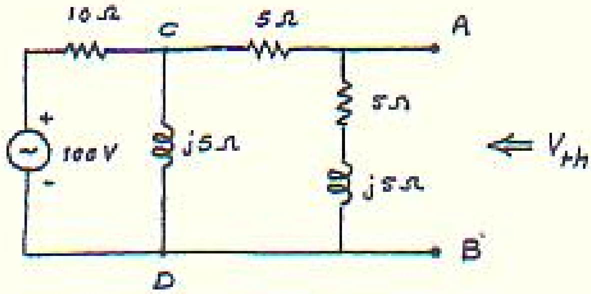
(b). Mesh current method.



⇒

Solution: (a). Using Thevenin's theorem

⊛ $V_{th} = ?$



voltage divider rule ⇒

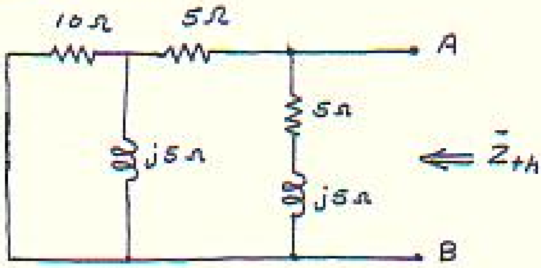
$$\bar{V}_{th} = \bar{V}_{AB}$$
$$= \bar{V}_{CD}\frac{(5+j5)}{(5+j5)+(5)}$$
$$= \bar{V}_{CD}\frac{(5+j5)}{(10+j5)}$$

$$\bar{V}_{CD} = V_T\frac{(j5)}{10+j5} = 100\frac{(j5)}{10+j5}$$
$$\doteq 20+j40$$

check

$$\therefore \bar{V}_{th} = 21.1\angle 71.57^\circ \text{ V}$$
$$= 6.67 + j20$$

⊛ $\bar{Z}_{th} = ?$ when looking through terminals A and B, the voltage source removed; the equivalent impedance $\bar{Z}_{th}$ is determined from the circuit:

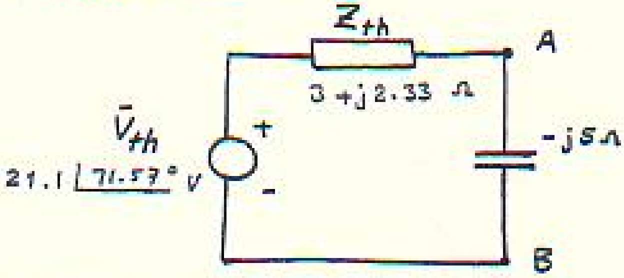


$$\bar{Z}_{th} = [(10 // j5) + 5] // (5+j5)$$

check

$$\therefore \bar{Z}_{th} = 3 + j2.33 \ \Omega$$

⊛ Thevenin's equivalent circuit will be as shown:

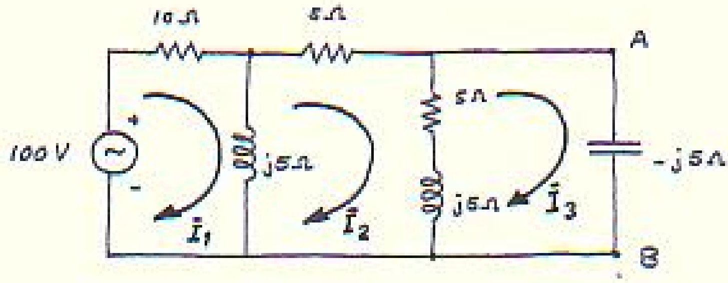


∴ Total impedance

$$\bar{Z}_T = (3+j2.33) + (-j5)$$
$$= 3 - j2.67 = 4.02\angle -41.67^\circ \ \Omega$$

$$\therefore I = \frac{V_{th}}{\bar{Z}_T} = \frac{21.1\angle 71.57^\circ}{4.02\angle -41.67^\circ} = 5.25\angle 113.24^\circ \text{ A}$$

(b). Using the mesh method



$\bar{I}_3 = ?$

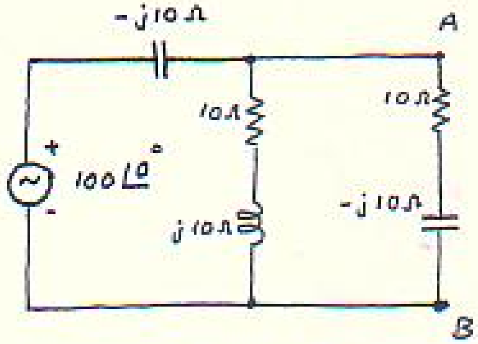
$$\bar{I}_3 = \frac{\Delta_3}{\Delta}$$

$$\therefore \quad \bar{I}_3 = \frac{\begin{vmatrix} 10+j5 & -j5 & 100 \\ -j5 & 10+j10 & 0 \\ 0 & -(5+j5) & 0 \end{vmatrix}}{\begin{vmatrix} 10+j5 & -j5 & 0 \\ -j5 & 10+j10 & -(5+j5) \\ 0 & -(5+j5) & 5 \end{vmatrix}} = \frac{3535 \angle 135^\circ}{673 \angle 21.8^\circ}$$

$$\Rightarrow \quad \bar{I}_3 = 5.25 \angle 113.2^\circ \ \text{A}$$

which is the same result.

Example: Use Norton's theorem to find the current in the load connected across terminals A and B of the circuit shown.

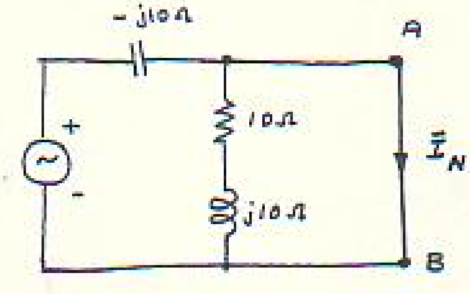


Solution: $\bar{I}_N = ?$ ⇒ the current in the short circuit terminals A & B

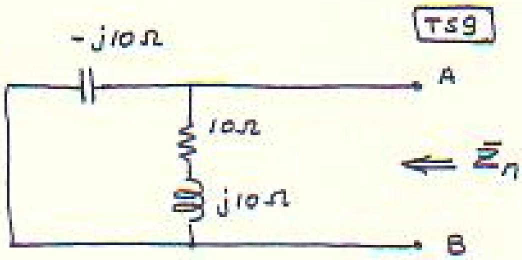
$$\therefore \quad \bar{I}_N = \frac{\bar{V}_T}{\bar{Z}_T} = \frac{100\angle 0^\circ}{-j10}$$

$$= \frac{100\angle 0^\circ}{-10\angle 90} = \frac{100\angle 0^\circ}{10\angle -90^\circ}$$

$$= 10\angle 90 \ \text{A}$$



* $\bar{Z}_n = \bar{Z}_{th} = ?$



$$\bar{Z}_n = (-j10) // (10 + j10)$$

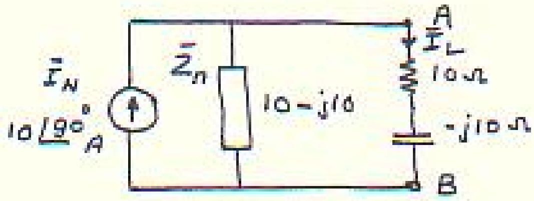
$$= \frac{(-j10)(10+j10)}{(-j10) + (10+j10)}$$

$$\therefore \ \bar{Z}_n = 10 - j10$$

* The Norton's equivalent circuit is :

$$\bar{I}_L = \frac{\bar{I}_N}{2} = \frac{10\angle 90}{2}$$

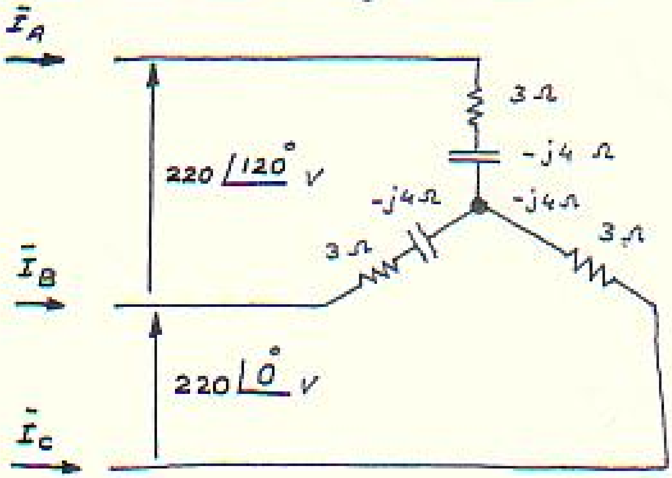
$$= 5\angle 90 \quad A$$



Since $\bar{I}_N$ is equally divided between the two equal impedances.

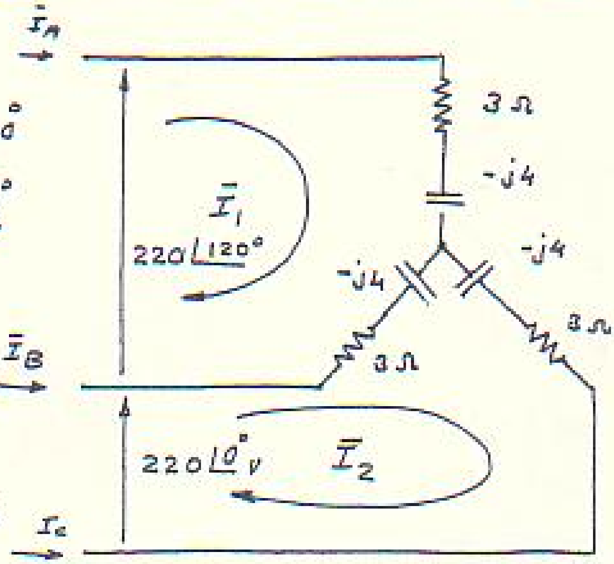
TS9

Example: For the circuit shown, determine the currents $\bar{I}_A$, $\bar{I}_B$ and $\bar{I}_C$, using the mesh current analysis method.



Solution

Using the mesh method, we have to determine the currents $\bar{I}_1$ and $\bar{I}_2$, then:



__Mesh Equations__

$$(6-j8)\bar{I}_1 - (3-j4)\bar{I}_2 = 220\angle 120^\circ$$

and

$$-(3-j4)\bar{I}_1 + (6-j8)\bar{I}_2 = 220\angle 0^\circ$$

$$\therefore \quad \bar{I}_1 = \frac{\begin{vmatrix} 220\angle 120^\circ & -(3-j4) \\ 220\angle 0 & (6-j8) \end{vmatrix}}{\begin{vmatrix} (6-j8) & -(3-j4) \\ -(3-j4) & (6-j8) \end{vmatrix}}$$

$$= \frac{1905\angle 36.9^\circ}{75\angle -106.2^\circ} = 25.4\angle 143.1^\circ \text{ A}$$

and

$$\bar{I}_2 = \frac{\begin{vmatrix} (6-j8) & 220\angle 120^\circ \\ -(3-j4) & 220\angle 0^\circ \end{vmatrix}}{\Delta} = \frac{1905\angle -23.2^\circ}{75\angle -106.2} = 25.4\angle 83^\circ \text{ A}$$

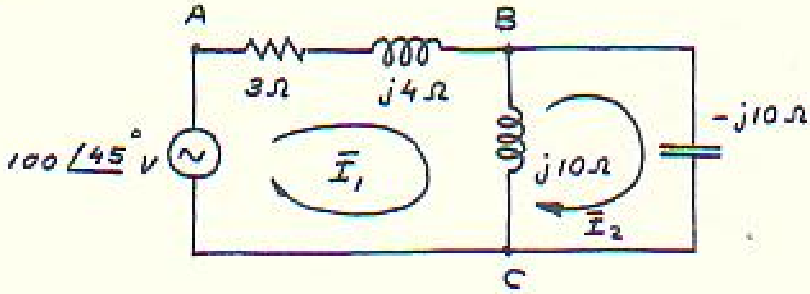
$$\therefore \bar{I}_A = \bar{I}_1 = \underline{\underline{25.4\angle 143}} \text{ A},$$

$$\bar{I}_B = \bar{I}_2 - \bar{I}_1 = 25.4\angle 83^\circ - 25.4\angle 143.1^\circ = \underline{\underline{25.4\angle 23.1^\circ}} \text{ A}$$

$$\bar{I}_c = -I_2 = -(25.4\angle 83^\circ) = \underline{\underline{25.4\angle -97^\circ}} \text{ A}$$

TS9

Example: Determine the voltage drops $\bar{V}_{AB}$ and $\bar{V}_{BC}$ in the circuit shown.



Solution: using the mesh current method, the currents $\bar{I}$ and $\bar{I}_2$ must be determined.

The 2-mesh current equations are:

$$(3+j4)\bar{I}_1 - j10\bar{I}_2 = 100\angle 45^\circ$$

and

$$-j10\bar{I}_1 + 0 = 0$$

$$\therefore \bar{I}_1 = \frac{\begin{vmatrix} 100\angle 45^\circ & -j10 \\ 0 & 0 \end{vmatrix}}{\begin{vmatrix} 3+j4 & -j10 \\ -j10 & 0 \end{vmatrix}} = \frac{0}{100} = 0$$

and

$$\bar{I}_2 = \frac{\begin{vmatrix} 3+j4 & 100\angle 45^\circ \\ -j10 & 0 \end{vmatrix}}{\begin{vmatrix} 3+j4 & -j10 \\ -j10 & 0 \end{vmatrix}} = \frac{1000\angle 135^\circ}{100} = 10\angle 135^\circ$$

$$\therefore \bar{V}_{AB} = \bar{I}_1(3+j4) = 0(3+j4) = 0$$

and

$$\bar{V}_{BC} = \bar{I}_2(-j10) = (10\angle 135^\circ)(10\angle -90^\circ) = 100\angle 45^\circ$$

For check: $\bar{V}_T = \bar{V}_{AB} + \bar{V}_{BC}$

$$= 0 + 100\angle 45^\circ = 100\angle 45^\circ = \text{Applied Voltage}$$