

Digital Communication Systems

Chapter (1) Introduction

1. Background

The term digital communication covers broad area communications techniques, including digital transmission and digital radio. Digital transmission is the transmission of digital pulses between two or more points in a communication system. Digital radio is the transmission of digital modulated analog carriers between two or more points in a communication system.

2. Why Digital Communication

There are a many reasons that makes the digital communication is more interactive communication system as follows:

- A.** Ease generation of digital signals if compared with analog signals generation as shown in figure (1.1).

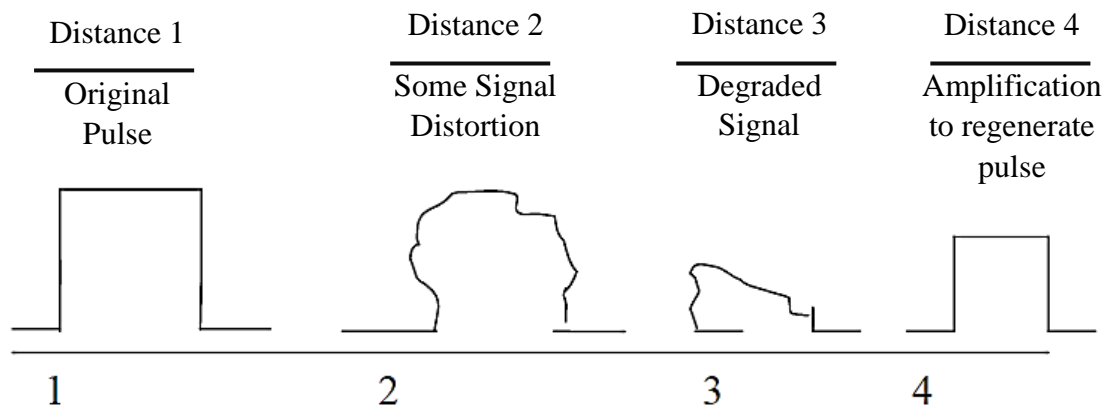


Figure (1.1) Pulses Forms.

The shape of the waveform is affected by two basic mechanisms:

- All transmission lines and circuits have some non-ideal frequency transfer function, which means there is a distorting effect on the ideal pulse.
- Unwanted electrical noise or other interference farther distorts the pulse waveform.

- B.** Digital circuits are less subject to distortion and interference than analog circuits. Because binary digital circuit operate in one of two states either one (fully on) or

zero (fully off) such two state operation facilitates signal regeneration and thus prevents noise and other disturbance from accumulating in transmission. Analog signals, however, are not two-state signals, they can take any infinite variety of shapes. Once the analog signal is distorted, the distortion cannot be removed by amplification,

- C. Digital circuits are more reliable and can be produced at a lower cost than analog circuits.
- D. Digital hardware is more flexible implementation than analog hardware (e.g microprocessor, digital switching and large scale integrated (LSI) circuit.
- E. Multiplexing of digital signals (TDM) is simpler than analog signals (FDM).
- F. Different type of digital signals (data, telegraph, telephone, television) can be treated as identical signals in transmission and switching.
- G. Digital techniques protect themselves against interference and jamming by using error correction codes, signal processing and cryptography.
- H. Data communication always is from computer to computer or from digital instruments or terminal to computer such digital terminations, are best served by digital communication link.

3. Disadvantages of Digital Communication

Although all this advantage mentioned above, there are some disadvantage as follows:

- A. Signal processing for digital communication is more **complex than** for analog communication.
- B. Synchronization for digital communication is **more complex than** analog communication.
- C. **Large System Bandwidth** so that digital transmission requires a large system bandwidth to communicate the same information in a digital format as compared to analog format.

4. Communication System Models

Generally, there are two types for communication system models, **base-band** model and **pass-band** model.

In **base-band** model, the spectrum of signal from zero to some frequency (i.e. carrier frequency=0). For transmission of base-band signal by a digital communication system, the information is formatted so that it is represented by digital symbols. Then, pulse waveforms are assigned that represented these symbols. This step referred to as **pulse** modulation or **base-band** modulation. These waveforms can be transmitted over a cable. Base-band signal also called **low-pass** signal.

In **pass-band** (or band-pass) signal, the signal has a spectral magnitude that is nonzero for frequency in some band concentrate about a frequency $f = \pm fc$ and negligible elsewhere, where fc is the carrier frequency need to be much greater than zero. For radio transmission, the carrier is covered to an electromagnetic (EM) filed for propagation to desired destination.

5. Elements of Digital Communication Systems

The figure (1.2) shows the functional elements of a digital communication system.

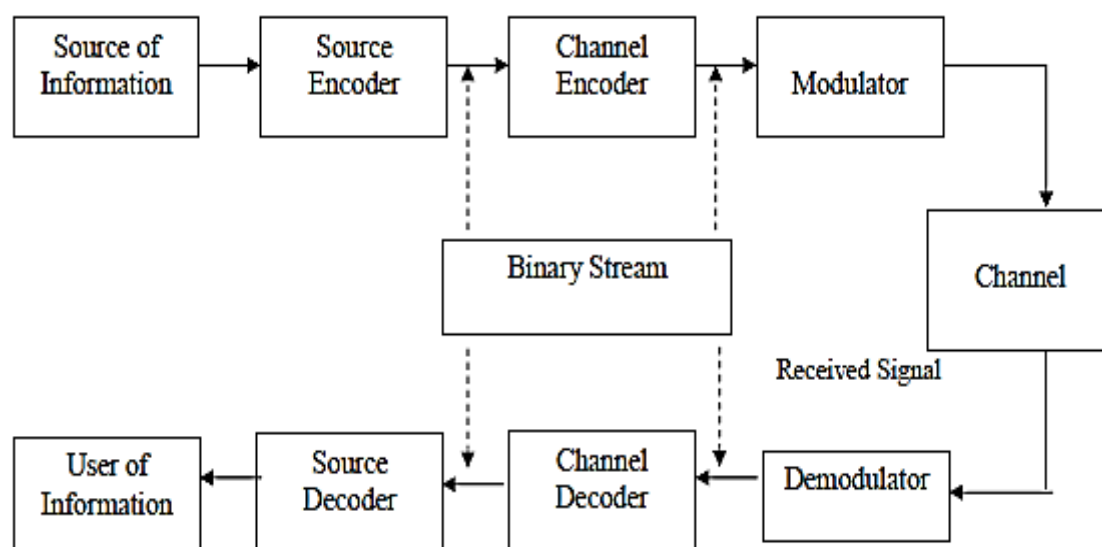


Figure (1.2) Block Diagram of Digital Communication System.

A. Source of Information: there are two types:

- Analog Information Sources → Microphone actuated by a speech, TV Camera scanning a scene, continuous amplitude signals.
- Digital Information Sources → These are teletype or the numerical output of computer which consists of a sequence of discrete symbols or letters.

An Analog information is transformed into a discrete information through the process of sampling and quantizing.

B. Source Encoder / Decoder: The Source encoder (or Source coder) converts the input i.e. symbol sequence into a binary sequence of 0's and 1's by assigning code words to the symbols in the input sequence. For example. :-If a source set is having hundred symbols, then the number of bits used to represent each symbol will be 7 because $2^7=128$ unique combinations are available. The important parameters of a source encoder are block size, code word lengths, average data rate and the efficiency of the coder (i.e. actual output data rate compared to the minimum achievable rate).

At the receiver, the source decoder converts the binary output of the channel decoder into a symbol sequence. The decoder for a system using fixed – length code words is quite simple, but the decoder for a system using variable – length code words will be very complex. Aim of the source coding is to remove the redundancy in the transmitting information, so that bandwidth required for transmission is minimized. Based on the probability of the symbol code word is assigned. Higher the probability, shorter is the codeword. Ex: Huffman coding.

C. Channel Encoder / Decoder: Error control is accomplished by the channel coding operation that consists of systematically adding extra bits to the output of the source coder. These extra bits do not convey any information but helps the receiver to detect and / or correct some of the errors in the information bearing bits.

There are two methods of channel coding:

1. **Block Coding:** The encoder takes a block of 'k' information bits from the source encoder and adds 'r' error control bits, where 'r' is dependent on 'k' and error control capabilities desired.
2. **Convolution Coding:** The information bearing message stream is encoded in a continuous fashion by continuously interleaving information bits and error control bits.

The Channel decoder recovers the information bearing bits from the coded binary stream. Error detection and possible correction is also performed by the channel decoder. The important parameters of coder / decoder are Method of coding, efficiency, error control capabilities and complexity of the circuit.

D. Modulator: The Modulator converts the input bit stream into an electrical waveform suitable for transmission over the communication channel. Modulator can be effectively used to minimize the effects of channel noise, to match the frequency spectrum of transmitted signal with channel characteristics, to provide the capability to multiplex many signals.

E. Demodulator: The extraction of the message from the information bearing waveform produced by the modulation is accomplished by the demodulator. The output of the demodulator is bit stream. The important parameter is the method of demodulation.

F. Channel: The Channel provides the electrical connection between the source and destination. The different channels are: Pair of wires, Coaxial cable, Optical fibre, Radio channel, Satellite channel or combination of any of these. The communication channels have only finite Bandwidth, non-ideal frequency response, the signal often suffers amplitude and phase distortion as it travels over the channel. Also, the signal power decreases due to the attenuation of the channel. The signal is corrupted by unwanted, unpredictable electrical signals referred to as noise. The important

parameters of the channel are Signal to Noise power Ratio (SNR), usable bandwidth, amplitude and phase response and the statistical properties of noise.

Some additional blocks as shown in the block diagram are used in most of digital communication system:

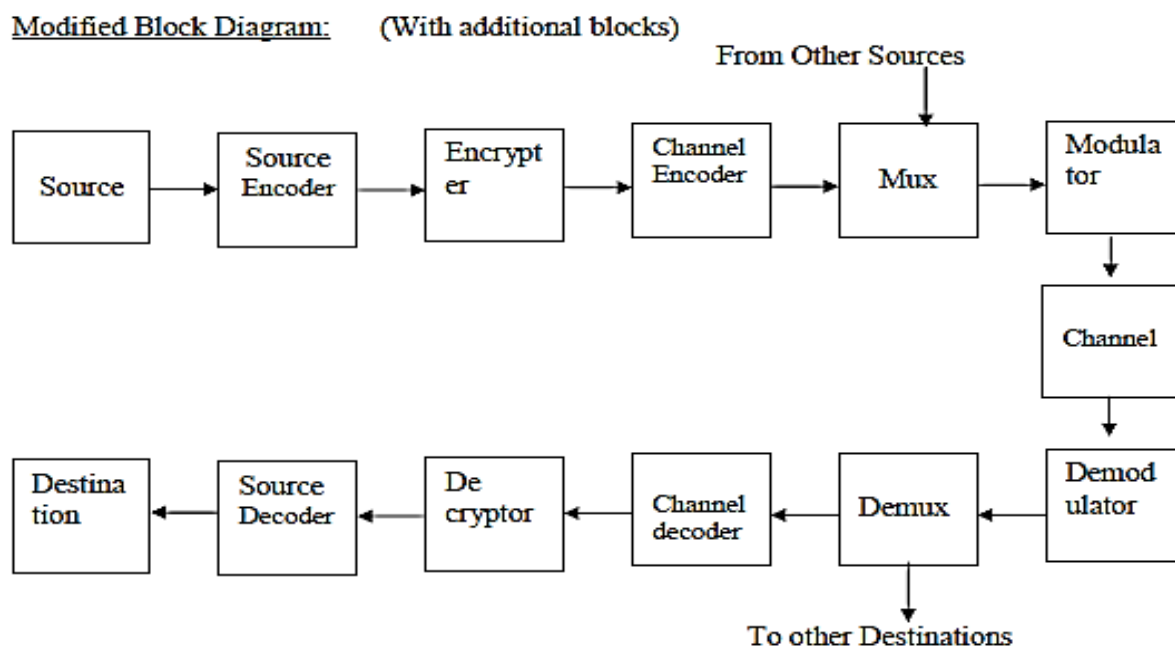


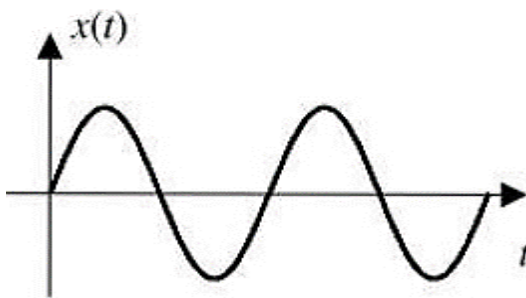
Figure (1.3) Additional Blocks to Digital Communication System.

- **Encryptor:** Encryptor prevents unauthorized users from understanding the messages and from injecting false messages into the system.
- **MUX :** Multiplexer is used for combining signals from different sources so that they share a portion of the communication system.
- **DeMUX:** DeMultiplexer is used for separating the different signals so that they reach their respective destinations.
- **Decryptor:** It does the reverse operation of that of the Encryptor.

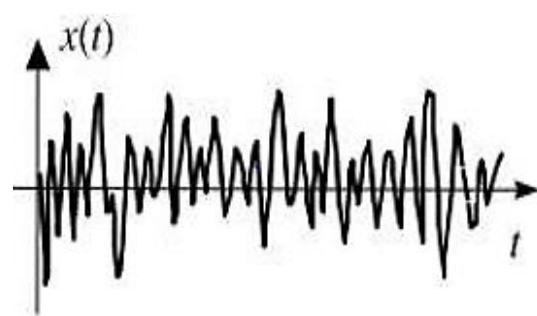
Last important matter is synchronization which involves the estimation of both time and frequency coherent systems need to synchronize their frequency reference with carrier in both frequency and phase.

6. Classification of Signals

A. Deterministic and Random Signals: Deterministic signals or waveforms are modeled by explicit mathematical expressions, such as $x(t) = 5 \cos 10t$. For a random waveform it is not possible to write such an explicit expression. However, when examined over a long period, a random waveform, also referred to as a random process, may exhibit certain regularities that can be described in terms of probabilities and statistical averages.



(a) Deterministic signal.



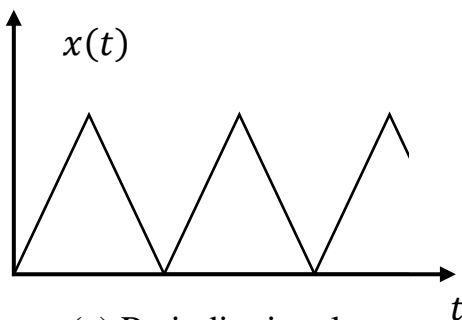
(b) Random signal.

Figure (1.4) Deterministic and Random Signals.

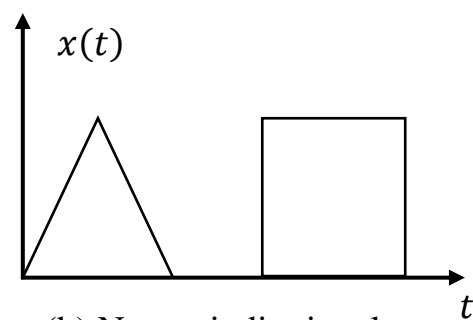
B. Periodic and Nonperiodic Signals: A signal $x(t)$ is called periodic in time if there exists a constant $T_0 > 0$ such that

$$x(t) = x(t + T_0) \quad \text{for} \quad -\infty < t < \infty$$

Where t denotes time. The smallest value of T_0 that satisfies this condition is called the period of $x(t)$. The period T_0 defines the duration of one complete cycle of $x(t)$. A signal for which there is no value of T_0 that satisfies above Equation is called a nonperiodic signal.



(a) Periodic signal.



(b) Nonperiodic signal.

Figure (1.5) Periodic and Nonperiodic signals.

C. Analog and Discrete Signals: An analog signal $x(t)$ is a continuous function of time; that is, $x(t)$ is uniquely defined for all t . An electrical analog signal arises when a physical waveform (e.g., speech) is converted into an electrical signal by means of a transducer. By comparison, a discrete signal $x(kT)$ is one that exists only at discrete times; it is characterized by a sequence of numbers defined for each time, kT , where k is an integer and T is a fixed time interval.



Figure (1.6) Analog and Discrete signals.

D. Analog and digital Signals: An analog signal has amplitude can take any value in a continuous range, which means that an analog signal amplitude can take on an infinite number of values through any time. However, a digital signal can take only a finite number of values through the time.

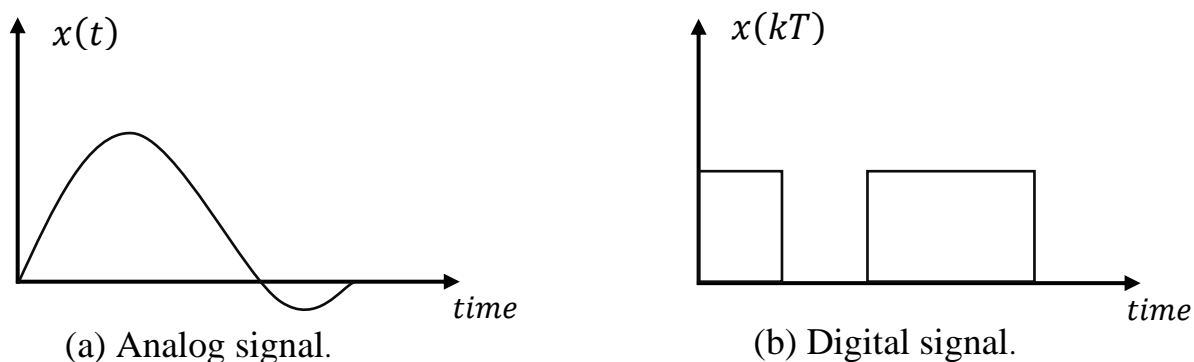


Figure (1.7) Analog and Digital signals

E. Energy and Power Signals: An electrical signal can be represented as a voltage $v(t)$ or a current $i(t)$ with instantaneous power $P(t)$ across a resistor R defined by

$$P(t) = \frac{v^2(t)}{R}$$

or

$$P(t) = i^2(t) R$$

In communication systems, power is often normalized by assuming R to be 1Ω , although R may be another value in the actual circuit. Therefore, regardless of whether

the signal is a voltage or current waveform, the normalization convention allows us to express the instantaneous power as

$$P(t) = x^2(t)$$

The energy dissipated during the time interval $(-T/2, T/2)$

$$E_x^T = \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t) dt$$

and the average power dissipated by the signal during the interval is

$$P_x^T = \frac{1}{T} E_x^T = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t) dt$$

- The performance of a communication system depends on the received signal energy, higher energy signals are detected more reliably (with fewer errors) than are lower energy signals.
- The power is the rate at which energy is delivered.
- The power determines the voltages that must be applied to the transmitter and the intensities of the electromagnetic field that one must contend with radio in systems.
- We classify $x(t)$ as an energy signal if and only if ,it has nonzero but finite energy $[0 < E_x < \infty]$ for all time where

$$E_x = \lim_{T \rightarrow \infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t) dt$$

$$E_x = \int_{-\infty}^{\infty} x^2(t) dt$$

- The periodic signal which is defined according to the following equation

$$x(t) = x(t + T_0)$$

- Thus, the periodic signal exist for all time and thus has infinite energy (i.e the periodic signal it is not an energy signal). In addition, a random signal has infinite energy it is not an energy signal.
- A signal is defined as a power signal if and only if it has finite but nonzero power $[0 < P_x < \infty]$. For all time

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t) dt$$

- The energy and power classifications are mutually exclusive. An energy signal has finite energy but zero average power, whereas power signal has finite average power but infinite energy.
- As general rule, Periodic signals and random signals are classified as power signals, but Determinisitic and nonperiodic signals are classified as energy signals.

7. The Unit Impulse Function

A useful function in communication theory is the unit impulse or Dirac delta function $\delta(t)$. The impulse function is an abstraction—an infinitely large amplitude pulse, with zero pulse width, and unity weight (area under the pulse), concentrated at the point where its argument is zero. The unit impulse is characterized by the following relationships:

$$\int_{-\infty}^{\infty} \delta^2(t) dt = 1 \quad \delta(t) = 0 \text{ for } t \neq 0$$

$$\delta(t) \text{ is unbounded at } t=0$$

$$\int_{-\infty}^{\infty} x(t) \delta(t - t_0) dt = x(t_0)$$

$$P(t) = \sum_{n=-\infty}^{+\infty} a \delta(t - nT_s)$$

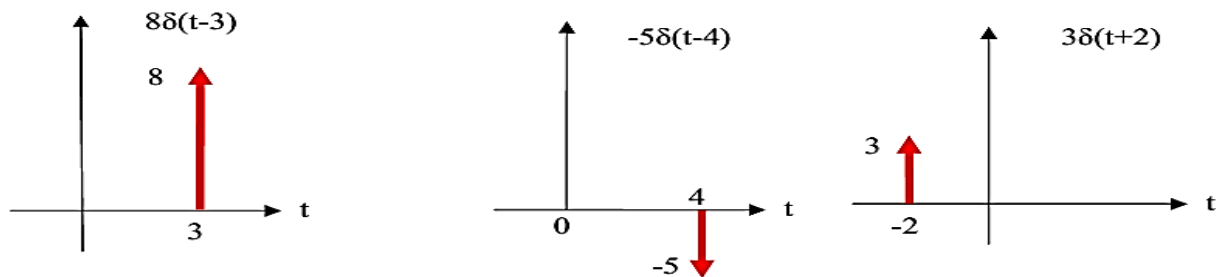


Figure (1.8) Impulse function.

8. Digital Coding

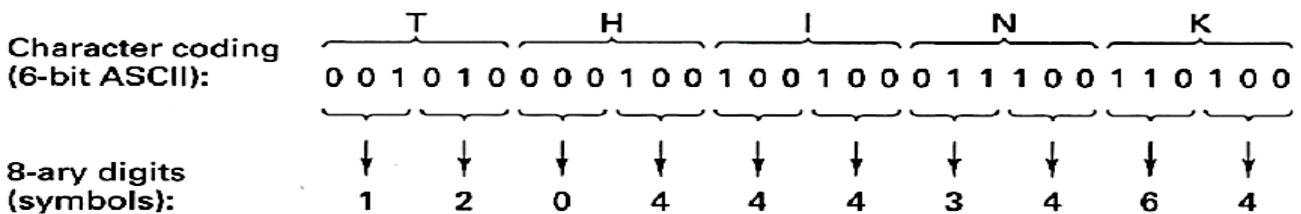
If the data consist of alphanumeric text, they will be character encoded with one of several standard formats; examples include the American Standard Code for Information Interchange (ASCII).

Bits				5	0	1	0	1	0	1	0	1
				6	0	0	1	1	0	0	1	1
				7	0	0	0	0	1	1	1	1
1	2	3	4									
0	0	0	0	NUL	DLE	SP	0	@	P	'	p	
1	0	0	0	SOH	DC1	!	1	A	Q	a	q	
0	1	0	0	STX	DC2	"	2	B	R	b	r	
1	1	0	0	ETX	DC3	#	3	C	S	c	s	
0	0	1	0	EOT	DC4	\$	4	D	T	d	t	
1	0	1	0	ENQ	NAK	%	5	E	U	e	u	
0	1	1	0	ACK	SYN	&	6	F	V	f	v	
1	1	1	0	BEL	ETB	'	7	G	W	g	w	
0	0	0	1	BS	CAN	(8	H	X	h	x	
1	0	0	1	HT	EM)	9	I	Y	i	y	
0	1	0	1	LF	SUB	*	:	J	Z	j	z	
1	1	0	1	VT	ESC	+	;	K	[k	{	
0	0	1	1	FF	FS	,	<	L	\	l		
1	0	1	1	CR	GS	-	=	M]	m	}	
0	1	1	1	SO	RS	.	>	N	^	n	~	
1	1	1	1	SI	US	/	?	O	-	o	DEL	

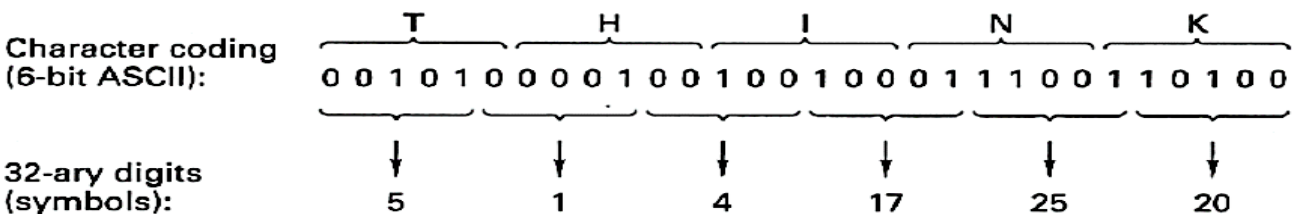
NUL	Null, or all zeros	DC1	Device control 1
SOH	Start of heading	DC2	Device control 2
STX	Start of text	DC3	Device control 3
ETX	End of text	DC4	Device control 4
EOT	End of transmission	NAK	Negative acknowledge
ENQ	Enquiry	SYN	Synchronous idle
ACK	Acknowledge	ETB	End of transmission
BEL	Bell, or alarm	CAN	Cancel
BS	Backspace	EM	End of medium
HT	Horizontal tabulation	SUB	Substitute
LF	Line feed	ESC	Escape
VT	Vertical tabulation	FS	File separator
FF	Form feed	GS	Group separator
CR	Carriage return	RS	Record separator
SO	Shift out	US	Unit separator
SI	Shift in	SP	Space
DLE	Data link escape	DEL	Delete

The textual message is word “THINK” using 6-bit ASCII character coding yields a bit stream comprising 30 bits. The symbol set size, M , has been chosen to be 8 (each symbol represents an 8- ary digit). The bits are therefore partitioned into groups of three ($k = \log_2 8$). The transmitter must have a repertoire of eight waveforms $S_i(t)$, where $i = 1, \dots, 8$. to represent the possible symbols, any one of which may be transmitted during a symbol time.

Message (text): “THINK”



8-ary waveforms: $s_1(t)$ $s_2(t)$ $s_0(t)$ $s_4(t)$ $s_4(t)$ $s_4(t)$ $s_3(t)$ $s_4(t)$ $s_6(t)$ $s_4(t)$
(a)



32-ary waveforms: $s_5(t)$ $s_1(t)$ $s_4(t)$ $s_{17}(t)$ $s_{25}(t)$ $s_{20}(t)$
(b)

9. Fourier Transform

It is a technique used to transform nonperiodic and periodic signal from time domain to frequency domain and vice versa. The basic Fourier transform formulas

$$X(w) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$$

$$\text{Or } X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft} dt \quad \text{sinc } w = 2\pi f$$

Invers Fourier Transform formulas

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(w)e^{j\omega t} dw$$

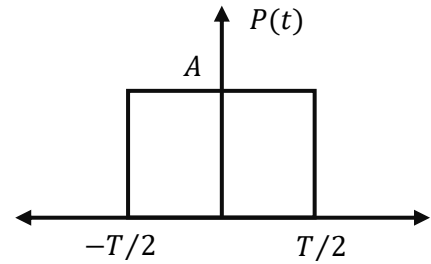
$$\text{Or } x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi t} df$$

Example: Obtain the Fourier transform of rectangular pulse of duration T and amplitude A shown below:

The rectangular pulse represented by:

$$\text{rect} \frac{t}{T} = \begin{cases} A & \text{for } -\frac{T}{2} < t < \frac{T}{2} \\ 0 & \text{elsewhere} \end{cases}$$

$$x(t) = A \text{rect} \left(\frac{t}{T} \right)$$



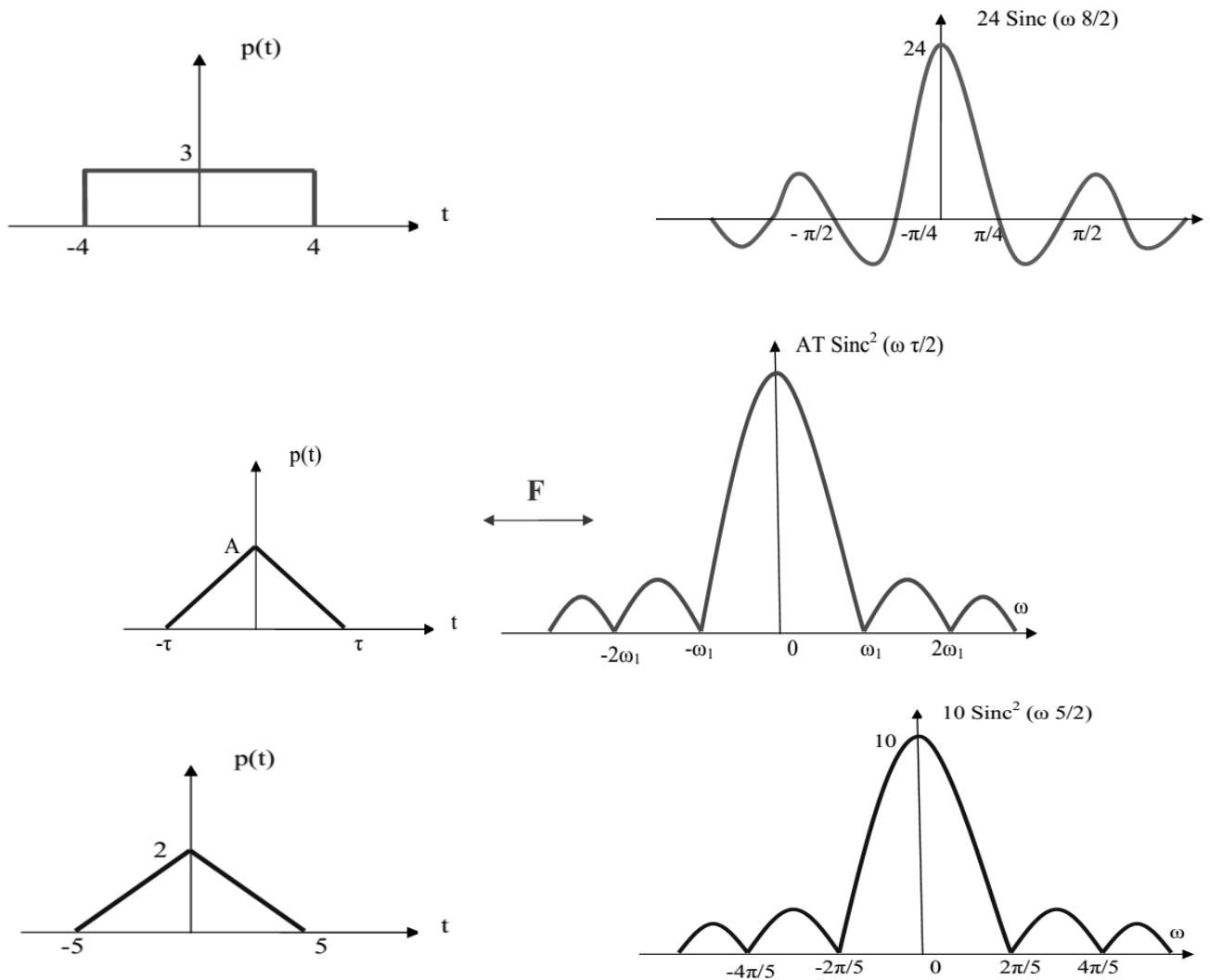
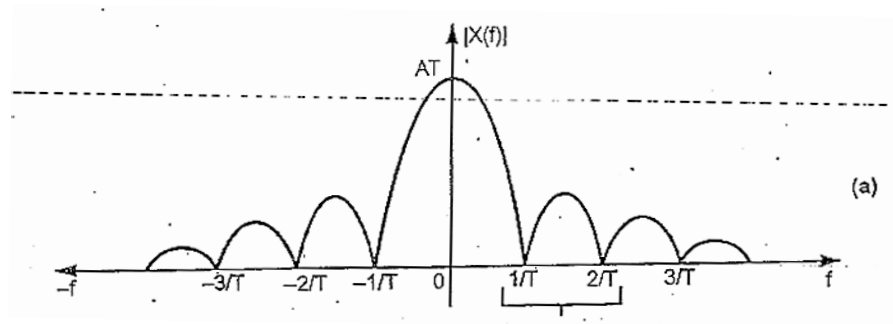
FT for $x(t)$

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft} dt$$

$$= \int_{-\frac{T}{2}}^{\frac{T}{2}} Ae^{-j2\pi ft} dt = \frac{A}{-j2\pi f} \left[e^{-j2\pi ft} \right]_{-\frac{T}{2}}^{\frac{T}{2}}$$

$$= \frac{A}{-j2\pi f} [e^{-j\pi fT} - e^{j\pi fT}] = \frac{A}{\pi f} \left[\frac{e^{-j\pi fT} - e^{j\pi fT}}{2j} \right]$$

$$= \frac{A}{\pi f} \sin(\pi fT) = AT \frac{\sin(\pi fT)}{\pi fT} \quad \Longrightarrow \quad \boxed{\therefore X(f) = AT \text{sinc}(\pi fT)}$$



10. Sampling theorem

Sampling of the signals is the fundamental operation in digital communication. A continuous time signal is first converted to discrete time signal by sampling process. Also it should be possible to recover or reconstruct the signal completely from its samples. The sampling theorem state that:

- 1- A band limited signal of finite energy, which has no frequency components higher than W Hz, is completely described by specifying the values of the signal at instant of time separated by $1/2W$ second.
- 2- A band limited signal of finite energy, which has no frequency components higher than W Hz, may be completely recovered from the knowledge of its samples taken at the rate of $2W$ samples per second.

Proof of sampling theorem: Let $x(t)$ the continuous time signal shown in figure below, its band width does not contain any frequency components higher than W Hz. A sampling function samples this signal regularly at the rate of f_s sample per second.

Assume an analog waveform, $x(t)$ with a Fourier transform, $X(f)$, which is zero outside the interval $(-f_m < f < f_m)$. The sampling of $x(t)$ can viewed as the product of $x(t)$ with periodic train of unit impulse function $x_\delta(t)$ defined as

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

The shifting property of unit impulse state that

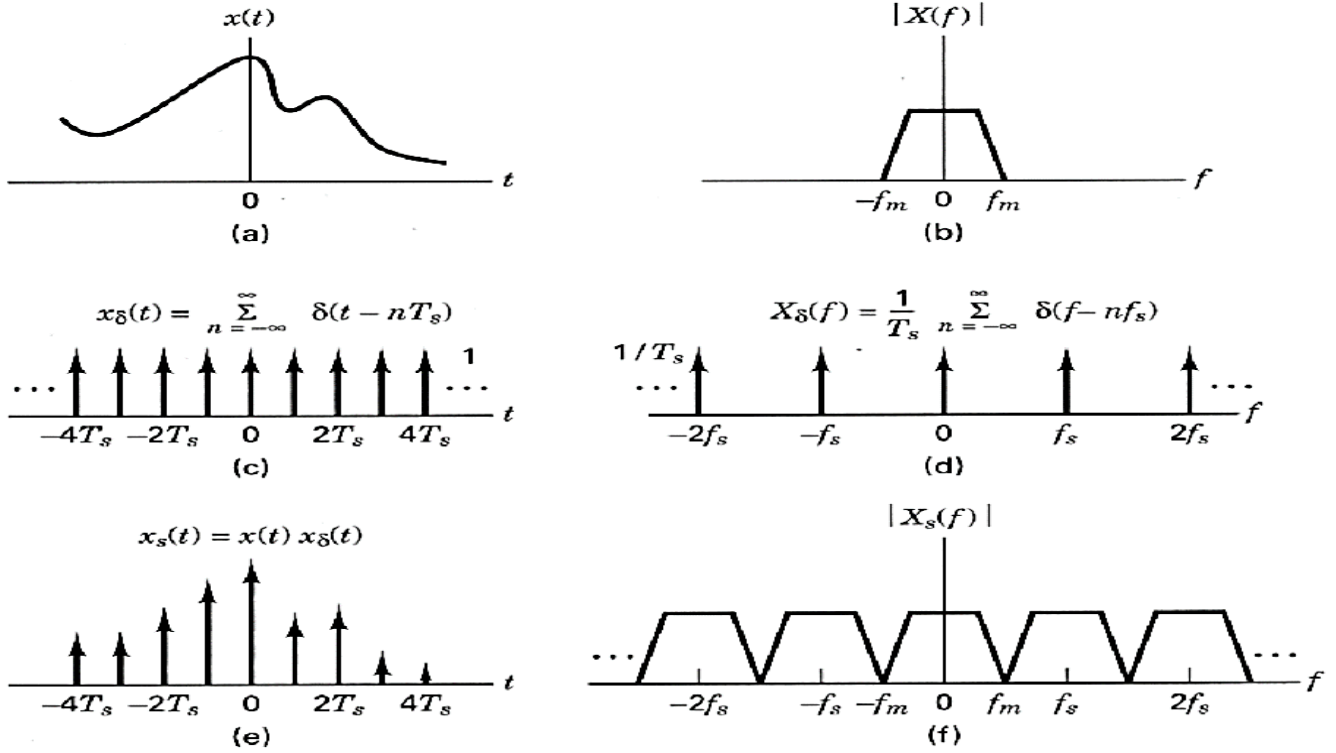
$$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0)$$

Using this property so that:

$$\begin{aligned} x_s(t) &= x(t)x_\delta(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t - nT_s) \\ &= \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s) \end{aligned}$$

Notice that the Fourier transform of an impulse train is another impulse train.

$$X_\delta(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$



Convolution with an impulse function simply shifts the original function:

$$X(f) * \delta(f - nf_s)$$

We can now solve for the transform $X_s(f)$ of the sampled waveform:

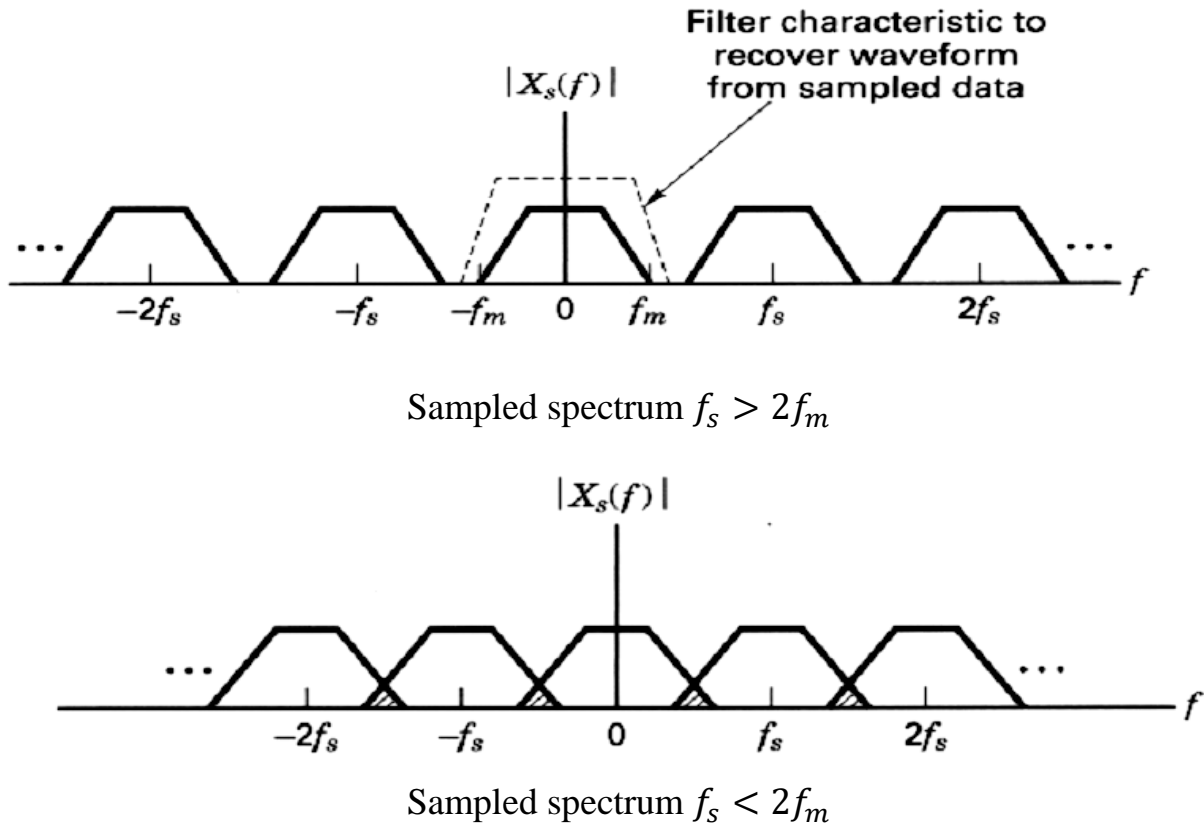
$$X(f) * \delta(f - nf_s) = X(f - nf_s)$$

So that

$$X_s(f) = X(f) * X_\delta(f) = X(f) * \left[\frac{1}{T_s} \sum_{n=-\infty}^{\infty} \delta(f - nf_s) \right] = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

When the sampling rate is chosen $f_s = 2f_m$ each spectral replicate is separated from each of its neighbors by a frequency band exactly equal to f_s hertz, and the analog waveform can theoretically be completely recovered from the samples, by the use of filtering. It should be clear that if $f_s > 2f_m$, the replications will be move farther apart in frequency making it easier to perform the filtering operation.

When the sampling rate is reduced, such that $f_s < 2f_m$, the replications will overlap, as shown in figure below, and some information will be lost. This phenomenon is called aliasing.



A bandlimited signal having no spectral components above f_m hertz can be determined uniquely by values sampled at uniform intervals of $T_s \leq \frac{1}{2f_m}$ sec. The sampling rate is $f_s = \frac{1}{T_s}$, So that $f_s \geq 2f_m$. The sampling rate $f_s = 2f_m$ is called Nyquist rate.

Example: Find the Nyquist rate and Nyquist interval for the following signals.

i- $m(t) = \frac{\sin(500\pi t)}{\pi t}$

ii- $m(t) = \frac{1}{2\pi} \cos(4000\pi t) \cos(1000\pi t)$

i- $\omega t = 500\pi t \quad \therefore 2\pi f = 500\pi \quad \rightarrow f = 250\text{Hz}$

$$\text{Nyquist interval} = \frac{1}{2f_{max}} = \frac{1}{2 \times 250} = 2 \text{ msec.}$$

$$\text{Nyquist rate} = 2f_{max} = 2 \times 250 = 500 \text{ Hz}$$

$$\begin{aligned} \text{ii- } m(t) &= \frac{1}{2\pi} \left[\frac{1}{2} \{ \cos(4000\pi t - 1000\pi t) + \cos(4000\pi t + 1000\pi t) \} \right] \\ &= \frac{1}{4\pi} \{ \cos(3000\pi t) + \cos(5000\pi t) \} \end{aligned}$$

Then the highest frequency is 2500Hz

$$\text{Nyquist interval} = \frac{1}{2f_{max}} = \frac{1}{2 \times 2500} = 0.2 \text{ msec.}$$

$$\text{Nyquist rate} = 2f_{max} = 2 \times 2500 = 5000 \text{ Hz}$$

H. W: Find the Nyquist interval and Nyquist rate for the following:

$$\text{i- } \frac{1}{2\pi} \cos(400\pi t) \cdot \cos(200\pi t)$$

$$\text{ii- } \frac{1}{\pi} \sin \pi t$$

Example: A waveform $[20 + 20\sin(500t + 30^\circ)]$ is to be sampled periodically and reproduced from these sample values. Find maximum allowable time interval between sample values, how many sample values are needed to be stored in order to reproduce 1 sec of this waveform?

Solution:

$$x(t) = 20 + 20 \sin(500t + 30^\circ)$$

$$\omega = 500 \rightarrow 2\pi f = 500 \rightarrow f = 79.58 \text{ Hz}$$

Minimum sampling rate will be twice of the signal frequency:

$$f_{s(\min)} = 2 \times 79.58 = 159.15 \text{ Hz}$$

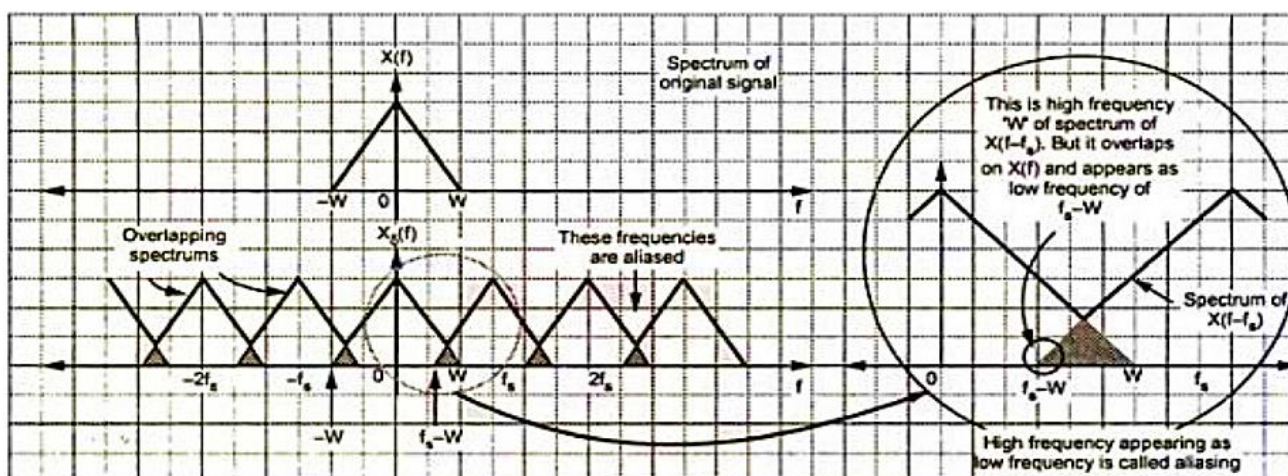
$$T_{s(\max)} = \frac{1}{f_{s(\min)}} = \frac{1}{159.15} = 6.283 \text{ msec.}$$

$$\text{Number of sample in 1sec} = \frac{1}{6.283 \text{ msec}} = 159.16 \approx 160 \text{ sample}$$

11. Effects of Undersampling (Aliasing)

While proving sampling theorem we considered that $f_s = 2W$. Consider the case of $f_s < 2W$, Then the spectrum of $X_\delta(f)$ shown in figure bellow will be modified as follows:

- The spectrums located at $X(f), X(f - f_s), X(f - 2f_s), \dots$ overlap on each other.
- Consider the spectrums of $X(f)$ and $X(f - f_s)$ shown as magnified in above figure. The frequencies from $(f_s - W)$ to W are overlapping in these spectrums.
- The high frequencies near W in $X(f - f_s)$ overlap with low frequencies $(f_s - W)$ in $X(f)$.



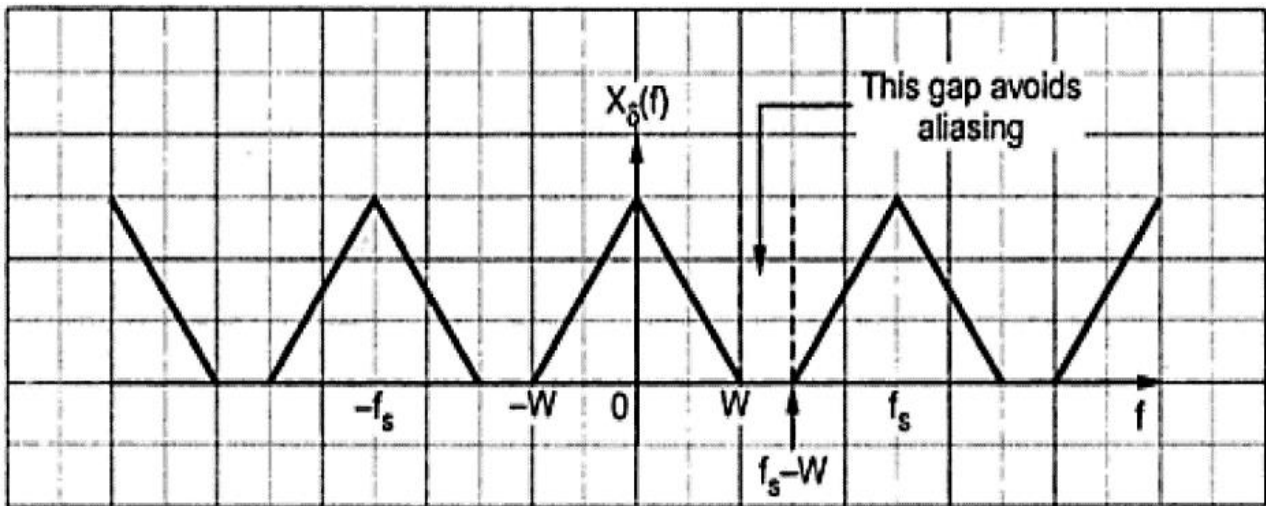
Definition of aliasing: When the high frequency interferes with low Frequency and appears as low frequency, then the phenomenon is called aliasing.

Effects of aliasing:

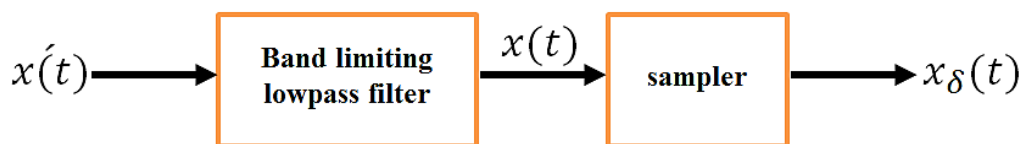
- Since high and low frequencies interfere with each other, distortion is generated.
- The data is lost and it cannot be recovered.

Avoid aliasing: There are a different ways to avoid aliasing in following:

a) Sampling rate $f_s \geq 2W$: When the sampling rate is made higher than $2W$, then the the spectrums will not overlap and there will be sufficient gap between the individual spectrums. This is shown in figure bellow. `



B) Bandlimiting the signal: The sampling rate is, $f_s = 2W$. Ideally speaking there should be no aliasing. But there can be few components higher than $2W$. The components create aliasing. Hence a lowpass filter is used before sampling the signals as shown in figure below. Thus the output of lowpass filter is strictly bandlimited and there are no frequency components higher than W . Then there will be no aliasing.

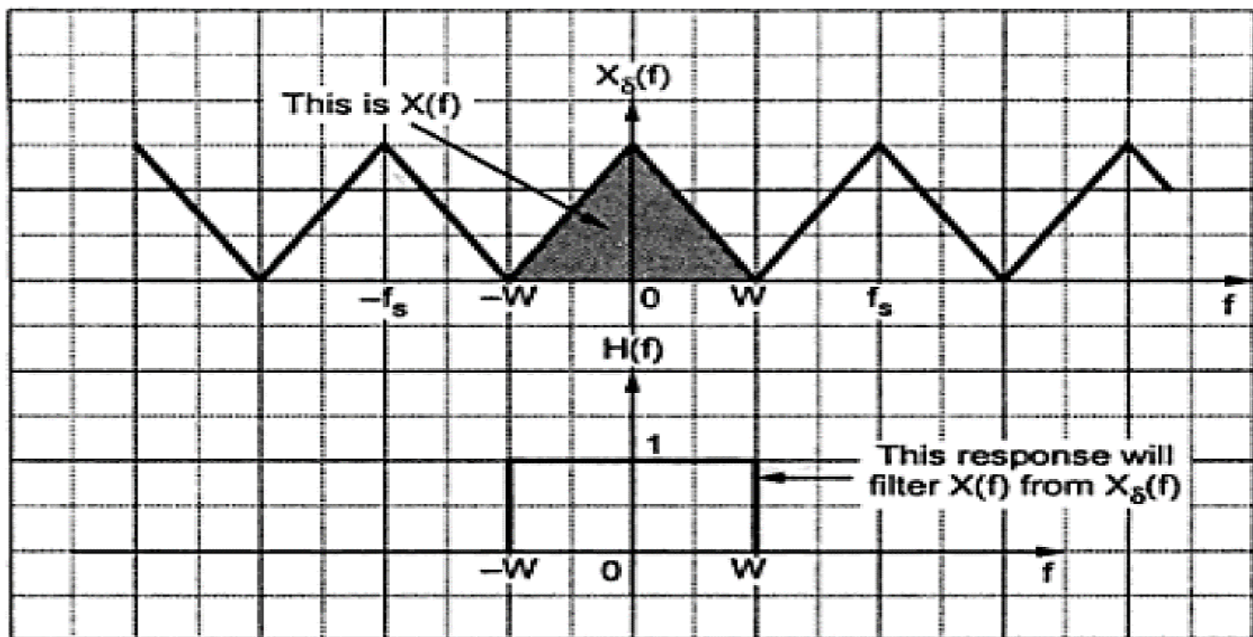


12. Reconstruction Filter (Interpolation Filter)

We have shown that the reconstructed signal is the succession of sinc pulses weighted by $x(nT_s)$. These pulses are interpolated with the help of a lowpass filter. It is also called reconstruction filter or interpolation filter.

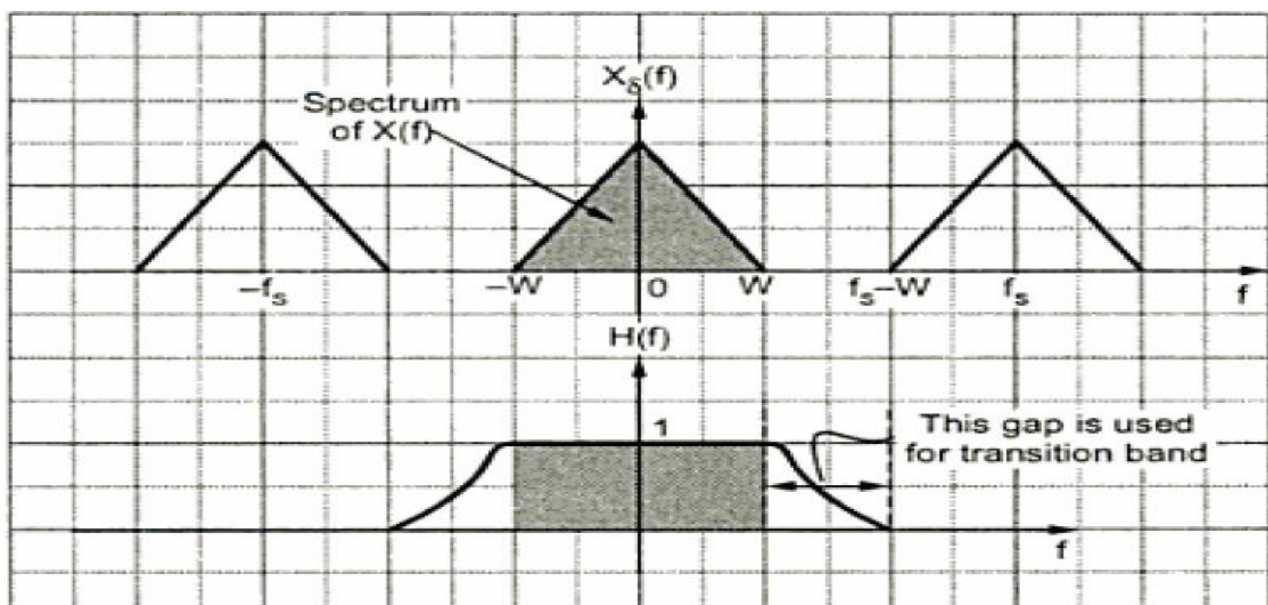
a) Ideal filter

figure below shows the spectrum of sampled signal and frequency response of required filter. When the sampling frequency is exactly $2W$, then the spectrums just touch each other. The spectrum of original signal, $X(f)$ can be filtered by an ideal filter having passband from $-W \leq f \leq W$.



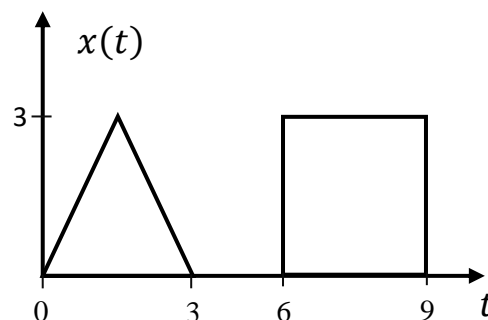
b) Non-ideal filter

As discussed above, an ideal filter of bandwidth W filters out an original signal. But practically ideal filter is not realizable. It requires some transition band. Hence f_s must be greater than $2W$, It creates the gap between adjacent spectrums of $X_s(f)$. This gap can be used for the transition band of the reconstruction filter. The spectrums of $X(f)$ is then properly filtered out from $X_s(f)$. Hence the sampling frequency must be greater than $2W$ to ensure sufficient gap for transition band.



Tutorial Chapter One

- 1-State four of advantage and disadvantage of digital communication?
- 2-Draw the block diagram of digital communication system?
- 3-Explain the elements of digital communication system?
- 4-What is the Encryptor, Decryptor, Multiplexer and Demultiplexer?
- 5-What is the sampling theorem?
- 6- Explain the aliasing, Effect of aliasing, and how can avoid the aliasing?
- 7-if we have $x_1(t) = \sin\left(\frac{\pi}{2}t\right)$ and $x_2(t) = e^{-\pi t}$, draw the two signals and explain whether it is deterministic or random, periodic or nonperiodic, and analog and discrete?
- 8-Determine whether the signal $x(t) = e^{-a|t|}$ is power or energy signals or neither?
- 9-Obtain the Fourier transform of the signal shown in figure?



- 10- An analogue signal is given as

$$x(t) = \sin(3000t) + 2 \cos(350t) + \sin(200t) \cos(20t)$$

where t is the time in seconds, determine the required minimum sampling frequency for the signal and calculate the time interval between any two adjacent samples.

- 11- Consider an analog signal given by $s(t) = 2 \cos(2\pi 100t) \cos(2\pi 10t)$ is sampled at the rate of 250 samples per seconds. Determine the maximum frequency component present in the signal, Nyquist rate and cut-off frequency of the ideal reconstruction filter so as to recover the signal from its sampled version. Draw the spectrum of the resultant sampled signal also.

- 12- Correct the false sentence with the reason in the following.

- a) It is easy to operate with analog mode than digital mode.
- b) Data transmission in digital communication is more security than analog signal.
- c) Modulator adding extra bits to the output of the source coder to help the receiver to detect or correct the errors in the information.
- d) Source encoder is used for combining signals from different sources.
- e) Periodic signal means that signals appears different shape at constant time.
- f) An energy signal if and only if, it has zero energy.
- g) Periodic signal exists finite energy.
- h) Nyquist interval is the number of samples per second.
- i) Aliasing appears when the sampling process used frequency double or more than the information frequency.
- j) Demultiplexer is used to reconstruct the sampled signal.

13- State the stable words in the blanks.

- 1) In digital communication, the _____ is more complex than analog communication.
- 2) Source coding operate to remove _____, so that bandwidth required for transmission is _____.
- 3) The Channel provides _____ between the source and destination.
- 4) The performance of a communication system depends on _____.
- 5) Periodic signals and random signals are classified as _____, but Deterministic and nonperiodic signals are classified as _____.
- 6) _____ is converted continuous time signal to discrete time signal.
- 7) When the sampling rate is _____, such that _____ the replications will overlap.
- 8) _____ determines the voltages that must be applied to the transmitter and the intensities of the _____ that one must contend with radio in systems.
- 9) _____ exists only at discrete times at fixed time interval.

10) _____converts the binary output of the channel decoder into a symbol sequence.

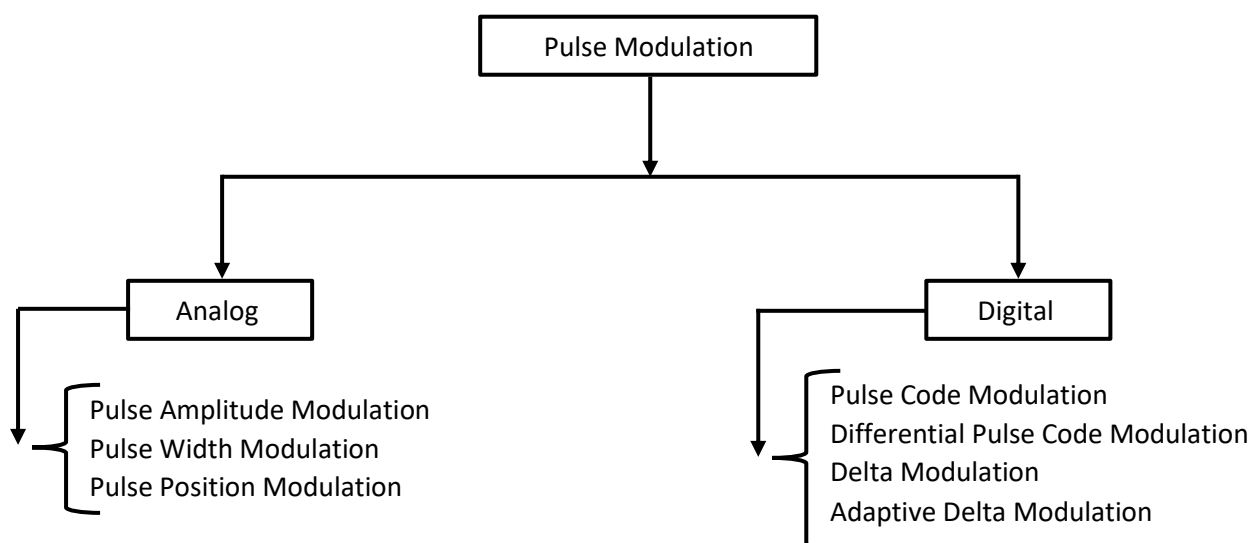
Digital Communication Systems

Chapter (2) Pulse Modulation

1. Background:

In pulse modulation some parameter of a pulse train is varied in accordance with the message signal. Two families of pulse modulation may be distinguished: **analog pulse modulation and digital pulse modulation**. In analog pulse modulation, a periodic pulse train is used as the carrier wave, and some characteristics features of each pulse (e.g. Amplitude, Position, and Width) is varied in a continuous manner in accordance with the corresponding sample value of the message signal. Thus in analog pulse modulation, information is transmitted basically in analog form, but the transmission takes place at discrete times. The analog Pulse modulation techniques are mainly classified into Pulse Amplitude Modulation (**PAM**), Pulse Duration Modulation (**PDM**)/Pulse Width Modulation (**PWM**), and Pulse Position Modulation (**PPM**).

In digital pulse modulation, on the other hand, the message signal is represented in a form that is discrete in both time and amplitude, thereby permitting its transmission in digital form as a sequence of coded pulses. The digital pulse modulation techniques are mainly classified into Pulse Code Modulation (**PCM**), Differential Pulse Code Modulation (**DPCM**), Delta Modulation (**DM**) and **Adaptive Delta Modulation**.



2. Pulse Amplitude Modulation (PAM):

The amplitude of the pulse change according to amplitude of modulation signal at the sampling instant. There are many *Types of PAM* depending upon the shape of the pulse of PAM:

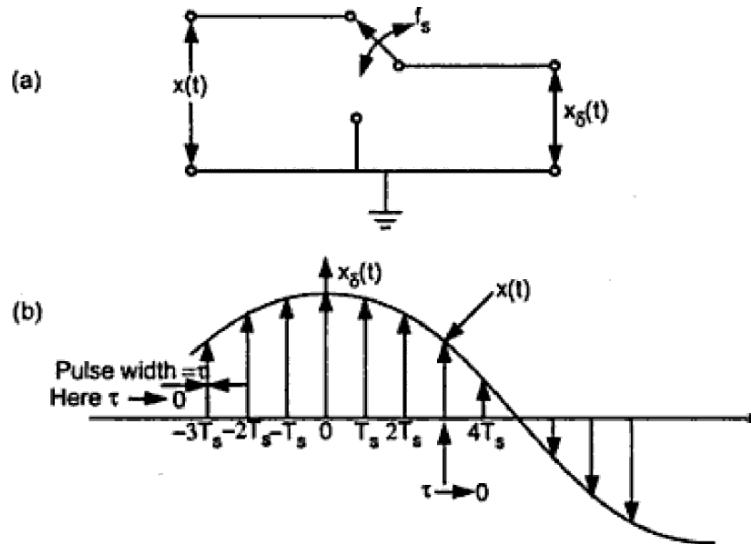
2.1. Ideally or Instantaneously Sampled PAM.

2.2. Naturally Sampled PAM.

2.3. Flat top Sampled PAM.

2.1. Ideal Sampling or Instantaneous Sampling or Impulse Sampling:

Ideal sampling is same as instantaneous sampling, figure below (a) shows the switching sampler. If dosing time t of the switch approaches zero the output $x_\delta(t)$ gives only instantaneous value. The waveforms are shown in same figure (b). Since the width of the pulse approaches zero, the instantaneous sampling gives train of impulses in $x_\delta(t)$. The area of each impulse in the sampled version is equal to instantaneous value of input signal $x(t)$.



We know that the train of impulses can be represented mathematically as,

$$s_\delta(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

This is called sampling function. The sampled signal $x_\delta(t)$ is given by multiplication of $x(t)$ and $s_\delta(t)$.

Therefore,

$$x_\delta(t) = x(t) s_\delta(t)$$

$$x_\delta(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \delta(t - nT_s)$$

2.2 Natural Sampling or Chopper Sampling:

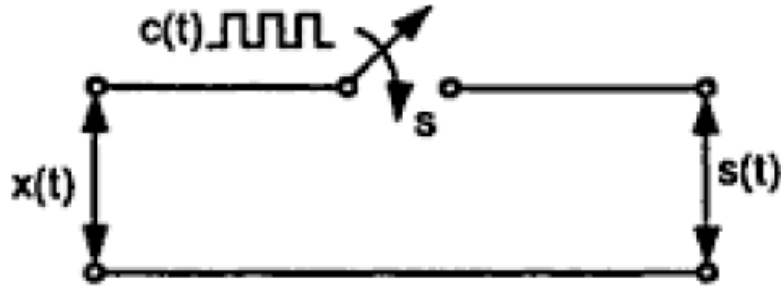
In natural sampling the pulse has a finite width τ . Natural sampling is some times called chopper sampling because the waveform of the sampled signal appears to be chopped off from the original signal waveform.

Let us consider an analog continuous time signal $x(t)$ to be sampled at the rate of f_s Hz and f_s is higher than Nyquist rate such that sampling theorem is satisfied. A sampled signal $s_\delta(t)$ is obtained by multiplication of a sampling function and signal $x(t)$. Sampling function $c(t)$ is a train of periodic pulses of width τ and frequency equal to f_s Hz. Figure below shows a functional diagram of natural sampler. When $c(t)$ goes high, a switch 's' is closed. Therefore,

$$s(t) = x(t) \quad \text{when} \quad c(t) = A$$

$$s(t) = 0 \quad \text{when} \quad c(t) = 0$$

Here A is amplitude of $c(t)$.



The waveforms of $x(t)$, $c(t)$ and $s(t)$ are shown in figure below (a), (b) and (c) respectively. Signal $s(t)$ can also be defined mathematically as,

$$s(t) = c(t) \cdot x(t)$$

Here, $c(t)$ is the periodic train of pulses of width τ and frequency f_s . We can find spectrum of naturally sampled signal as follow;

Exponential Fourier Series for a periodic waveform is given as

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n t / T_0}$$

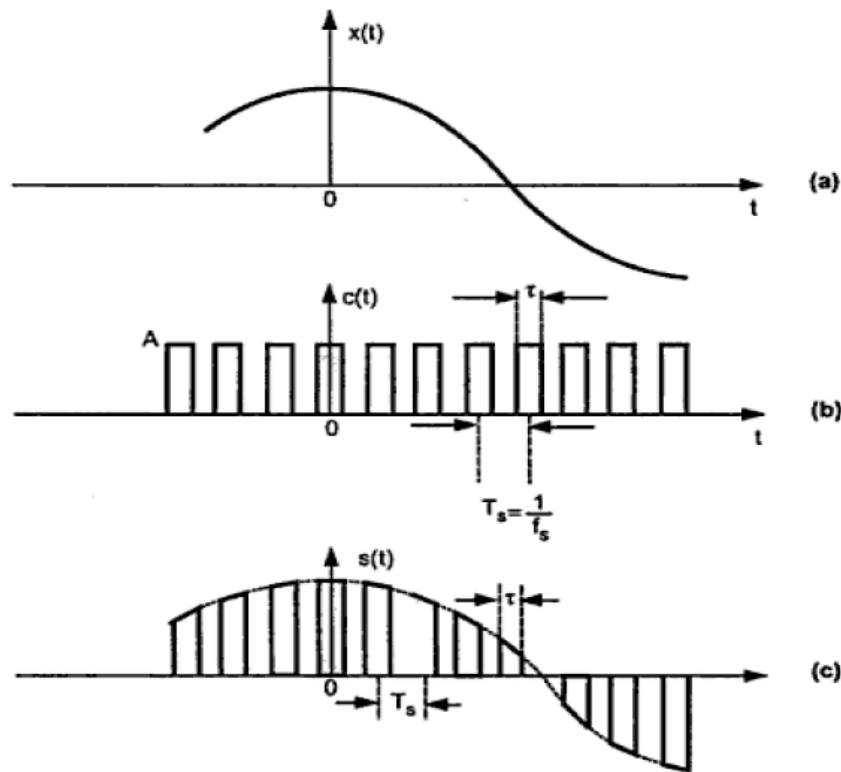
$c(t)$ is a rectangular pulse train. C_n , for this waveform is given as:

$$C_n = \frac{TA}{T_0} \text{sinc}(f_n T)$$

Here $T = \text{pulse width} = \tau$

And $f_n = \text{Harmonic frequency}$. Here $f_n = n f_s$ or $f_n = \frac{n}{T_0} = f_s$

$$C_n = \frac{\tau A}{T_s} \text{sinc}(f_n \tau)$$

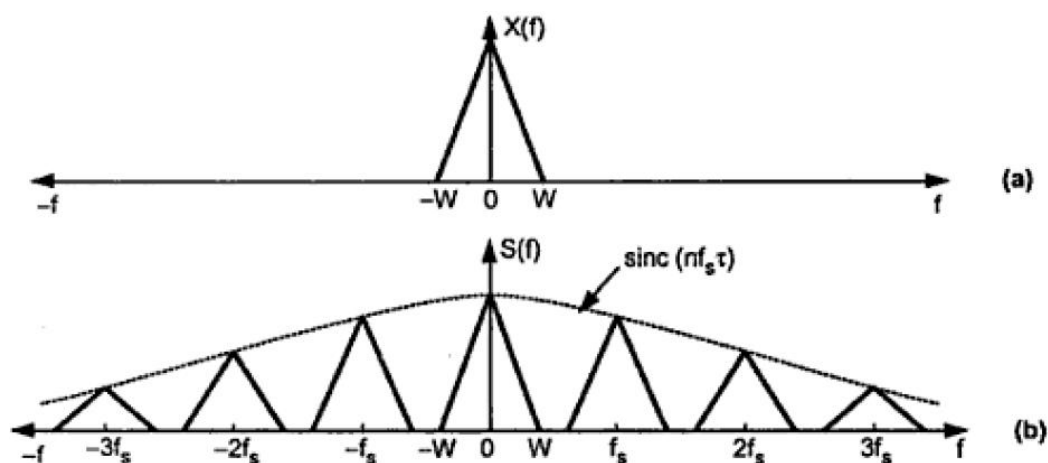


We know that $f_n = nf_s$ harmonic frequency, then above equation becomes,

Spectrum of Naturally Sampled Signal:

$$S(f) = \frac{\tau A}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}(nf_s \tau) X(f - nf_s)$$

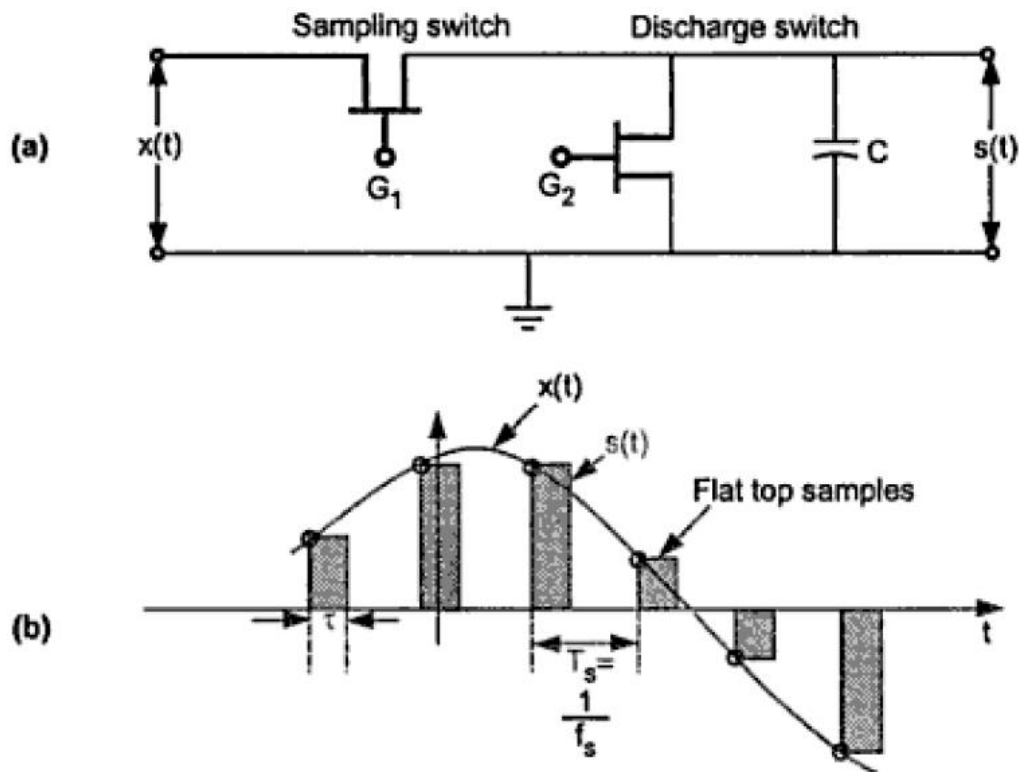
$X(f)$ are periodic in f_s and are weighed by the *sinc* function, figure (a) bellow shows some arbitrary spectra for $x(t)$ and corresponding spectrum $S(f)$ is shown in figure (b). Thus unlike the spectrum of instantaneously sampled signal given previously, the spectrum of naturally sampled signal is weighted by *sinc* function. However, the spectrum of instantaneously sampled signal remains constant (as in sampling theorem lecture) throughout the frequency range.



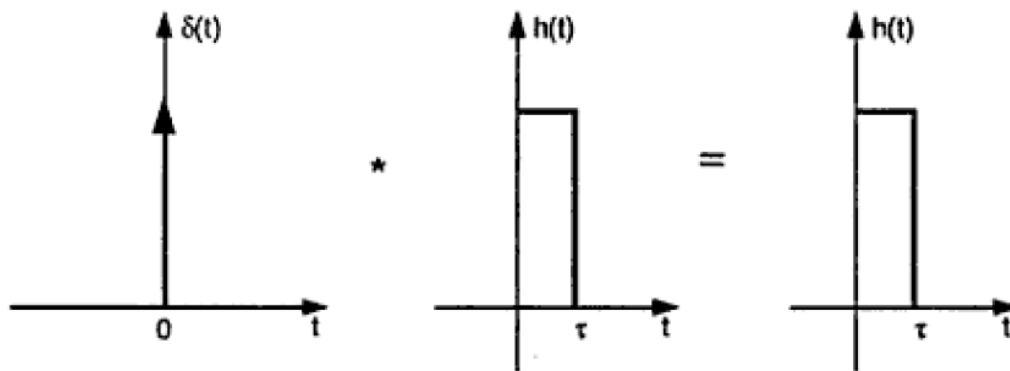
2.3 Flat Top Sampling or Rectangular Pulse:

This is also a practically possible sampling method. Natural sampling is little complex, but it is very easy to get flat top samples. The top of the samples remains constant and equal to instantaneous value of baseband signal $x(t)$ at the start of sampling. The duration of each sample is τ and sampling rate is equal to $f_s = \frac{1}{T_s}$.

Generation of flat top samples: Figure (a) below shows the functional diagram of sample and hold circuit generating flat top samples and figure (b) shows waveforms. Normally the width of the pulse in flat top sampling and natural sampling is increased as far as possible to reduce the transmission bandwidth.



Here we can see from figure (b) above that only starting edge of the pulse represents instantaneous value of the baseband signal $x(t)$. The flat top pulse of $s(t)$ is mathematically equivalent to the convolution of instantaneous sample and pulse $h(t)$ as shown in figure below.



That is width of the pulse in $s(t)$ is determined by width of $h(t)$, and sampling instant is determined by delta function. In the waveforms shown in previous figure (b), the starting edge of pulse represents the point where baseband signal is sampled and width is determined by function $h(t)$. Therefore $s(t)$ will be given as,

$$s(t) = x_{\delta}(t) * h(t)$$

By taking Fourier transform of both sides of above equation,

$$S(f) = X_{\delta}(f) H(f)$$

Convolution in time domain is converted to multiplication in frequency domain.

$X_{\delta}(f)$ is given as,

$$X_{\delta}(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

Then Spectrum of Flat Top Sampled Signal :

$$S(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) H(f)$$

3. Transmission Bandwidth of PAM Signal

The pulse duration τ is supposed to be very small compared to time period T_s between the two samples. If the maximum frequency in the signal $x(t)$ is W then by sampling theorem, f_s should be higher than Nyquist rate or,

$$f_s \geq 2W$$

$$T_s \leq \frac{1}{2W}$$

And

$$t \ll T_s \ll \frac{1}{2W}$$

If ON and OFF time of the pulse is same, then frequency of the PAM pulse becomes

$$f = \frac{1}{\tau + \tau} = \frac{1}{2\tau}$$

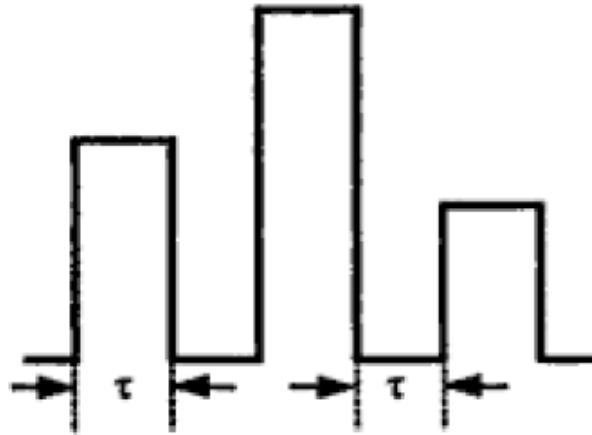


Figure above shows that if ON and OFF times of PAM signal are same, then maximum frequency of PAM signal is given by,

$$f_{max} = \frac{1}{2\tau}$$

Bandwidth required for transmission of PAM signal will be equal to maximum frequency f_{max} given by above equation. This bandwidth gives adequate pulse resolution i.e.,

$$B_T \geq f_{max}$$

$$B_T \geq \frac{1}{2\tau}$$

$$\text{Since } \tau \ll \frac{1}{2W} \quad B_T \geq \frac{1}{2\tau} \gg W$$

Transmission bandwidth of PAM signal: $B_T \gg W$

Thus the transmission bandwidth B_T of PAM signal is very large compared to highest frequency in the signal $x(t)$.

4. Disadvantages of PAM

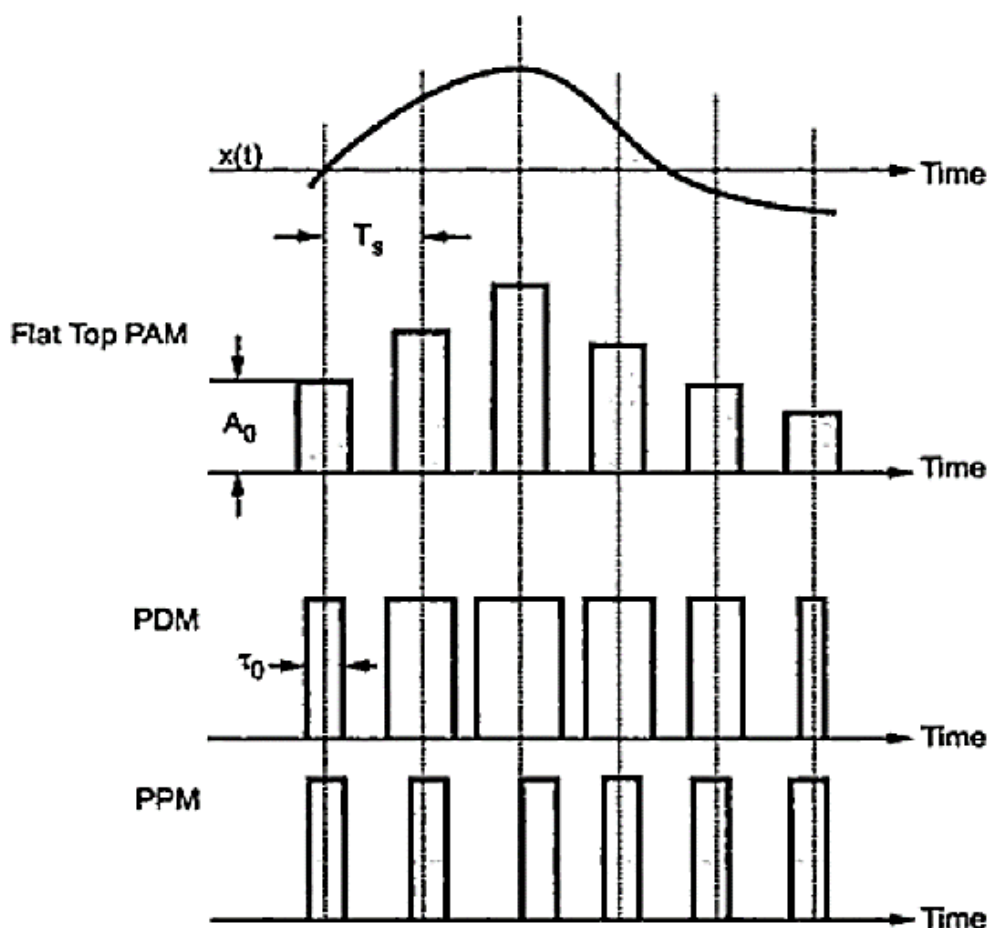
1. As we have seen just now, the bandwidth needed for transmission of PAM signal is very large compared to its maximum frequency content.

2. The amplitude of PAM pulses varies according to modulating signal. Therefore, the interference of noise is maximum for the PAM signal and this noise cannot be removed easily.
3. Since amplitude of PAM signal varies, this also varies the peak power required by the transmitter with modulating signal.

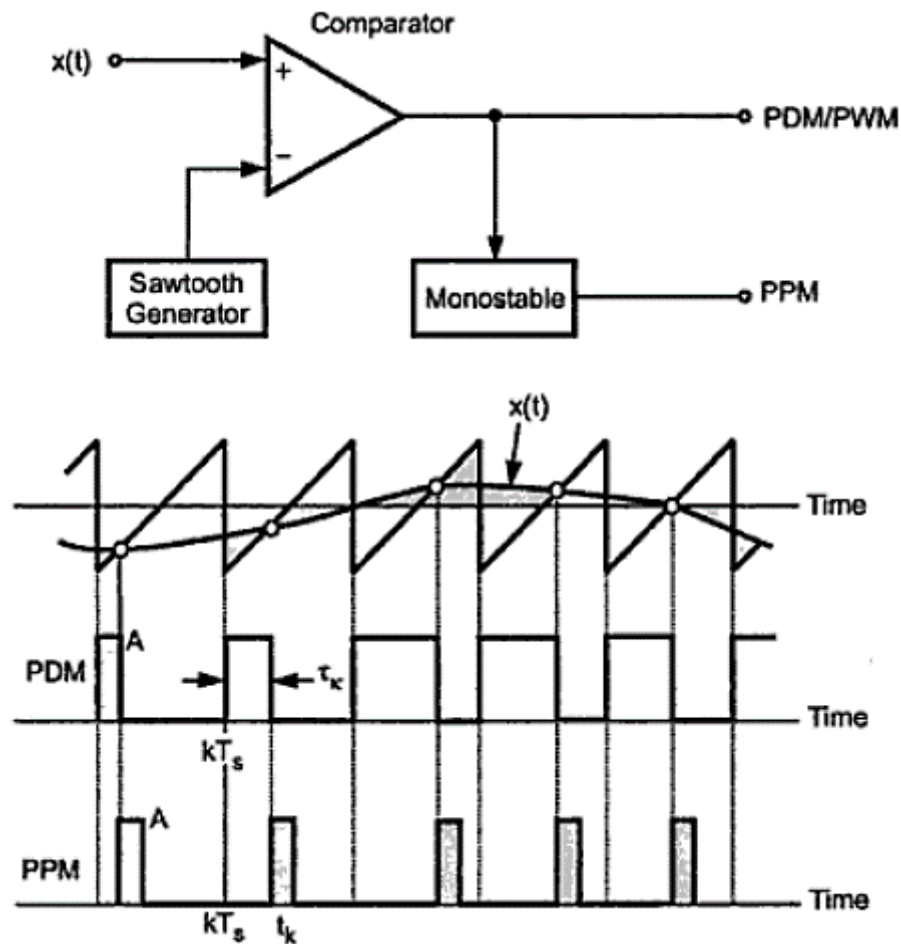
5. Pulse Duration and Pulse position Modulation (PDM & PPM):

Pulse Duration Modulation (PDM): In this technique, the width of pulse changes according to the amplitude of the modulating signal at sample instant.

Pulse position Modulation (PPM): In this technique, the position of pulse changes according to the amplitude of the modulating signal at sample instant as shown in figure below.



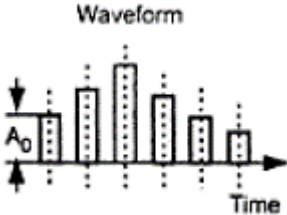
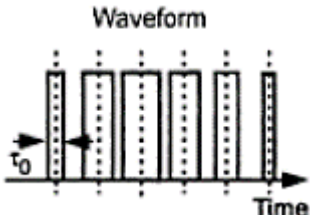
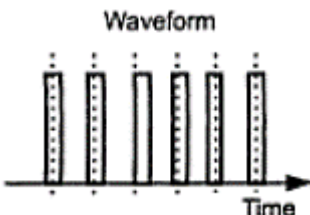
Pulse Duration Modulation (PDM) or Pulse Width Modulation (PWM) and PPM are both modulate the time parameter of the pulses. PPM has fixed width whereas the width of PWM is varied. The generation of PWM and PPM shown in figure below.



The modulating signal $x(t)$ is applied to the noninverting input of the comparator. The output is high only when instantaneous value of $x(t)$ is higher than that of the sawtooth waveform. Thus the leading edge of the PWM signal occurs on the fixed time period(KT_s), the trailing edge of the output of comparator depends on the amplitude of signal $x(t)$. The trailing edge of output of comparator PWM is modulated by the signal $x(t)$. To generate PPM, the trailing edge of PWM is used to switch on the monostable with fixed period then goes low. The pulse is delayed from sampling time KT_s depending on the amplitude of signal $x(t)$ at KT_s .

The rise time should be very less than T_s i.e., $t_r \ll T_s$, and transmission bandwidth of PWM and PPM should be, $B_T \geq \frac{1}{2t_r}$.

The comparison between PAM, PWM and PPM is listed in the following table

No.	Pulse Amplitude Modulation (PAM)	Pulse Width (Duration) Modulation (PWM/PDM)	Pulse Position Modulation (PPM)
1.			
2.	Amplitude of pulse is proportional to amplitude of modulating signal	Width of pulse is proportional to amplitude of modulating signal	The relative position of pulse is proportional to amplitude of modulating signal
3.	Bandwidth of transmission channel depends on width of the pulse	Bandwidth of transmission channel depends on rise time of the pulse	Bandwidth of transmission channel depends on rising time of the pulse
4.	Instantaneous power of transmitter varies with amplitude of pulses	Instantaneous power of transmitter varies with (amplitude with width) of pulses	Instantaneous power of transmitter remains constant with width of pulses
5.	High Noise interference	Minimum Noise interference	Minimum Noise interference
6.	Complex system	Simple system	Simple system
7.	Similar to (AM)	Similar to (FM)	Similar to (PM)

Example: The voice signal with maximum frequency of $3kHz$, is to be transmitted using sampling frequency $f_s = 8kHz$, and pulse duration $\tau = 0.1 T_s$, determine the required bandwidth of PAM, PWM and PPM if the rise time $t_r = 1\%$ of pulse duration.

Solution:
$$T_s = \frac{1}{f_s} = \frac{1}{8 \times 10^3} = 0.125 \text{ msec}$$

$$\tau = 0.1 T_s = 0.1 \times 0.125 = 0.0125 \text{ msec}$$

For PAM the bandwidth:
$$B_T \geq \frac{1}{2\tau}$$

$$B_T \geq \frac{1}{2 \times 0.0125 \times 10^{-3}} = 40kHz$$

For PDM and PPM the bandwidth $B_T \geq \frac{1}{2t_r}$

$$t_r = 0.01\tau = 0.01 \times 0.0125 \text{ msec} = 0.125 \mu\text{sec}$$

$$B_T \geq \frac{1}{2 \times 0.125 \times 10^{-6}} = 4MHz$$

H.W: If the bandwidth of PAM system not exceed $4kHz$ is used to transmit voice signal sampled at Nyquist frequency. Calculate the bandwidth required to transmit the same signal using PPM system with rise time of 2% of pulse duration.

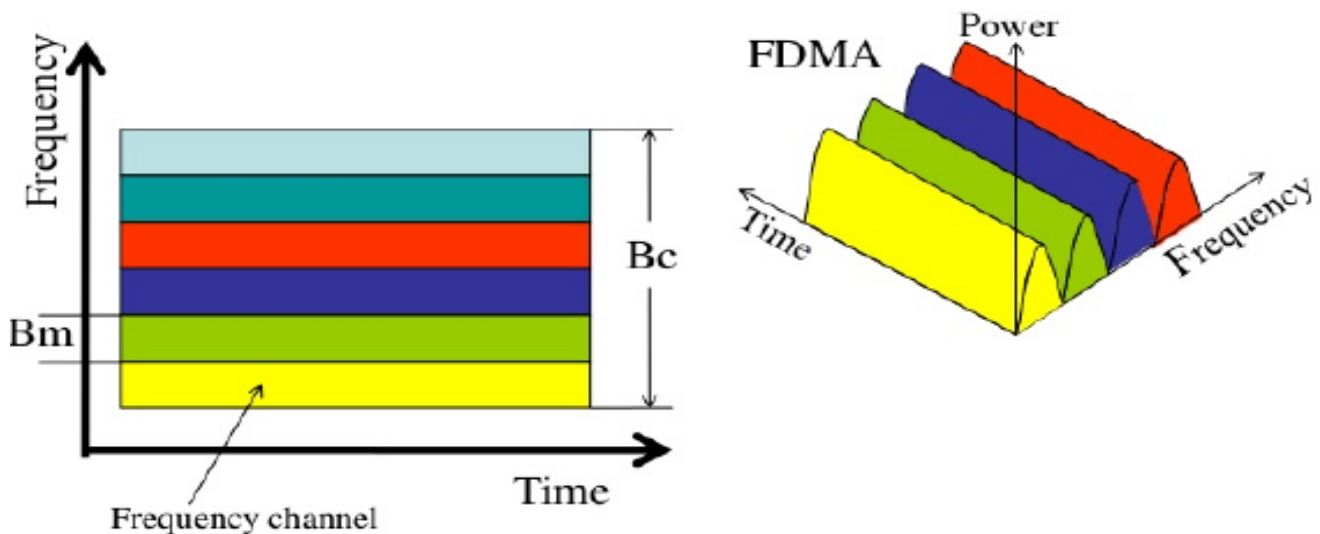
6. Multiplexing:

Multiplexing is the transmission of information (either voice or data) from more than one source to more than one destination on the same transmission medium. Two most common methods are used, *frequency division multiplexing (FDM) and time division multiplexing (TDM)*.

6.1. Frequency Division Multiplexing (FDM):

In FDM multiple sources that originally occupied the same frequency spectrum are each converted to a different frequency band and transmitted simultaneously over a

single transmission medium. FDM is an analog multiplexing scheme. Figure below shows the frequency-time plane.



6.2. Time Division Multiplexing (TDM):

The principle of TDM system is that the transmission from multiple sources occurs on the same transmission medium but not at the same time. Transmission from various sources is interleaved in time domain. Figure below shows the time-frequency plan in TDM system, the same communication resources is shared by assigning each of N symbols or users the full spectral occupancy of the system for a short duration of time called time slot. The unused time regions between slot assignments, called guard times, act as buffer zone to reduce interference.

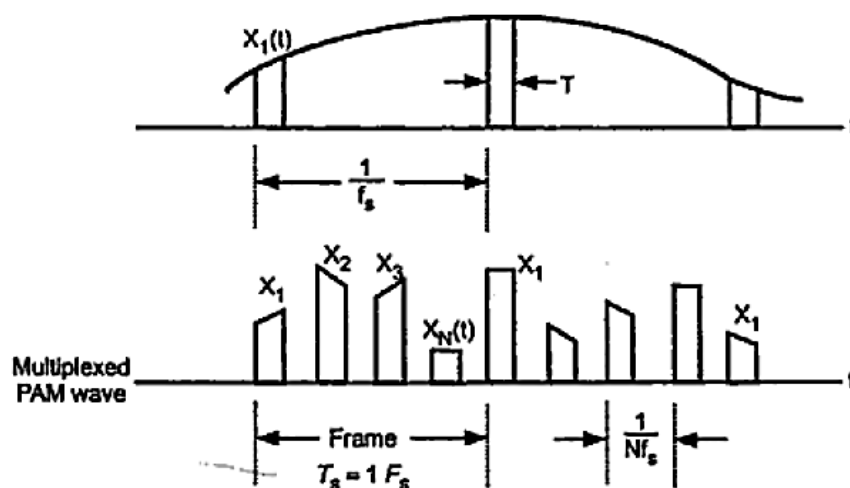
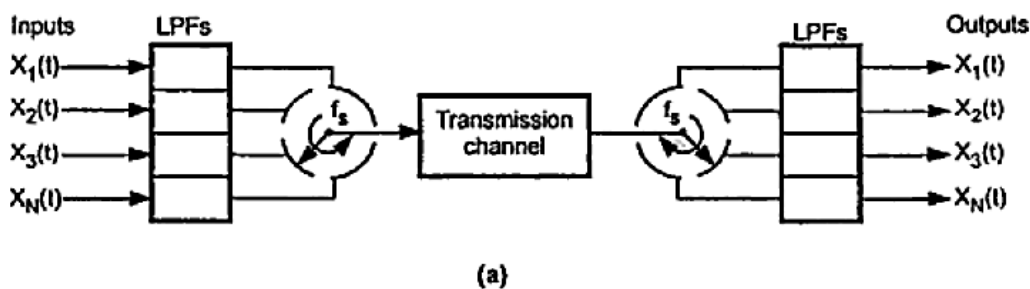


6.3. PAM / TDM System:

Figure (a) below shows the block diagram of a simple TDM system and figure (b) shows the waveforms of the system. The system shows the time division multiplexing of 'N' PAM channels. Each channel to be transmitted is passed through the lowpass filter.

The outputs of the low pass filters are connected to the rotating sampling switch or commutator. It takes the sample from each channel per revolution and rotates at the rate of f_s .

Thus the sampling frequency becomes f_s . The single signal composed due to multiplexing of input channels is given to the transmission channel. At the receiver the decommutator separates (decodes) the time multiplexed input channels. These channel signals are then passed through lowpass reconstruction filters.



If the highest signal frequency present in all the channels is 'W' then by sampling theorem the sampling frequency f_s should be,

$$f_s \geq 2W$$

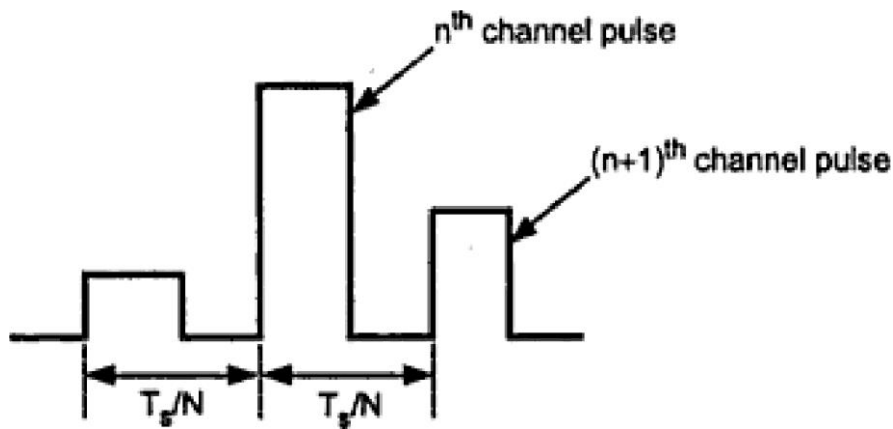
Therefore, the time space between successive samples from any one input will be,

$$T_s = \frac{1}{f_s}$$

$$T_s \leq \frac{1}{2W}$$

Thus, the time interval T_s contains one sample from each input. This time interval is called frame. Let there be N input channels. Then in each frame, there will be one sample from each of the N channels. That is one frame of T_s seconds contain total N samples. Therefore pulse to pulse spacing between two samples in the frame will be equal to $\frac{T_s}{N}$.

$$\text{Spacing between two samples} = \frac{T_s}{N}$$



From the above figure, we can very easily calculate the number of pulses per second or pulse frequency as,

$$\begin{aligned} \text{Number of pulses per second} &= \frac{1}{\text{Spacing between two samples}} \\ &= \frac{1}{T_s/N} \end{aligned}$$

$$= \frac{N}{T_s}$$

$$\text{Number of pulses per second} = \frac{N}{1/f_s} = Nf_s$$

These number of pulses per second is also called signalling rate of TDM signal and is defined by,

$$\text{Signalling rate} = r = Nf_s$$

$$f_s \geq 2W$$

$$\text{Signalling rate in PAM/TDM system: } r \geq 2NW$$

The RF transmission of TDM needs modulation. That is TDM signal should modulate some carrier. Before modulation, the pulsed signal in TDM is converted to baseband signal. That is pulsed TDM signal is converted to smooth modulating waveform $x_b(t)$; the baseband signal that modulates the carrier. The baseband signal $x_b(t)$ passes through all the individual sample values baseband signal is obtained by passing pulsed TDM signal through lowpass filter. The bandwidth of this lowpass filter is given by half of the signalling rate. i.e.,

$$B_b = \frac{1}{2}r = \frac{1}{2}Nf_s$$

Transmission bandwidth of TDM channel will be equal to bandwidth of the lowpass filter,

$$B_T = \frac{1}{2}Nf_s$$

If sampling rate becomes equal to Nyquist rate i.e

$$f_s = \text{Nyquist rate} = 2W$$

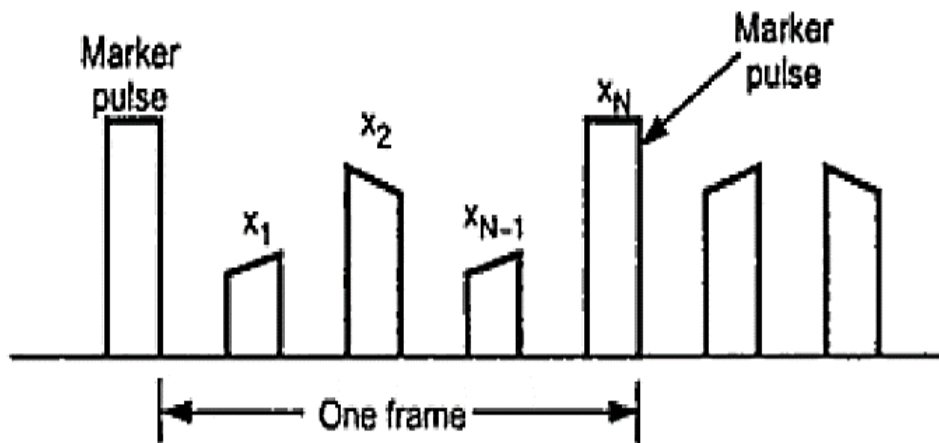
$$B_T = \frac{1}{2}N * 2W$$

Minimum transmission bandwidth of TDM channel: $B_T = NW$

This equation shows that if there are total 'N' channel in TDM which are bandlimited to W Hz, then minimum bandwidth of the transmission channel will be equal to NW.

6.4. Synchronization in TDM System:

From the discussion of TDM system it is clear that the receiver should operate in perfect synchronization with the transmitter normally markers are inserted to indicate the separation between the frames. Figure below shows the TDM signals with markers.



The above figure shows that a marker pulse is inserted at the end of the frame. Because of the marker pulse, synchronization is obtained but number of channels to be multiplexed is reduced by one (i.e. N-1 channels can be multiplexed)

Example: Twelve different message signals, each of bandwidth 10 kHz are to be multiplexed and transmitted. Determine the minimum bandwidth required for PAM/TDM system.

Solution:

Here the number of channels $N = 12$.

Bandwidth of each channel $W = 10 \text{ kHz}$

Minimum channels bandwidth to avoid crosstalk in PAM /TDM system is,

$$B_T = NW$$

$$B_T = 12 * 10 \text{ kHz}$$

$$B_T = 120 \text{ kHz}$$

Example: Twenty four voice signals are sampled uniformly and then time division multiplexed. The highest frequency component for each voice signal is 3.4 kHz.

1. If the signals are pulse amplitude modulated using Nyquist rare sampling, what is the minimum channel bandwidth required?
2. Determine signaling rate of TDM signal and bit rate of an individual channel.

Solution :

1. We know that if N channels are time division multiplexed, then minimum transmission bandwidth is given as,

$$B_T = NW$$

Here W is the maximum frequency in the signals

$$B_T = 24 * 3.4 \text{ kHz}$$

$$B_T = 81.6 \text{ kHz}$$

2. The signalling rate of the system is given as,

$$r = 2NW$$

$$r = 2B_T = 2 * 81.6$$

$$r = 163.2 * 10^3 \text{ bits/sec}$$

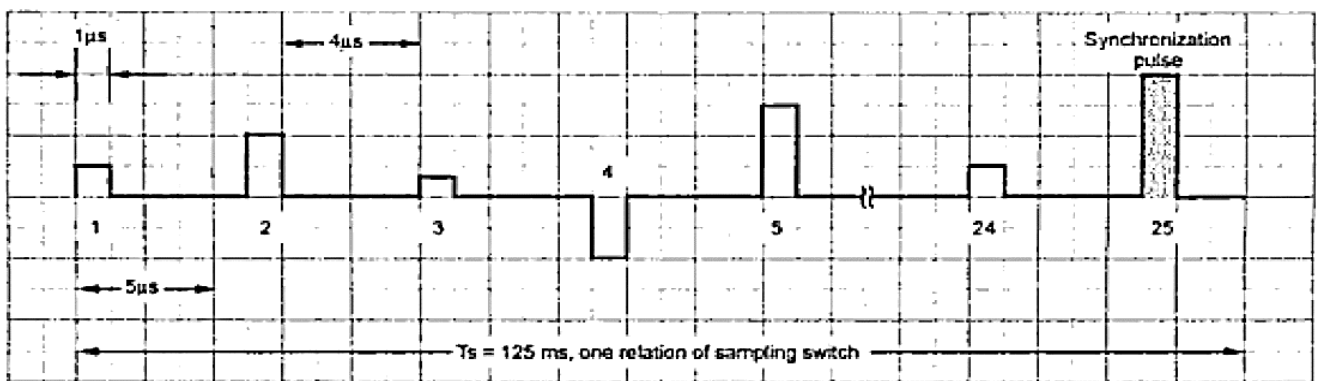
Since there are 24 channels, the bit rate of an individual channel is,

$$r(\text{one channel}) = \frac{163.2 * 10^3}{24}$$

$$r(\text{one channel}) = 6800 \text{ bits/sec}$$

Example: Twenty four voice signals are sampled uniformly and then time division multiplexed. The sampling operation uses flat samples with 1 μsec duration. The multiplexing operation provides for synchronization by adding an extra pulse of 1 μsec duration. Assuming sampling rate of 8 kHz, calculate spacing between successive pulses of multiplexed signal and setup a scheme for accomplishing a multiplexing requirement.

Solution: There are 24 voice signal pulses plus one synchronization pulse, Hence there are total 25 pulses, Sampling rate is 8 kHz. Hence duration of one frame will be illustrated in figure below,

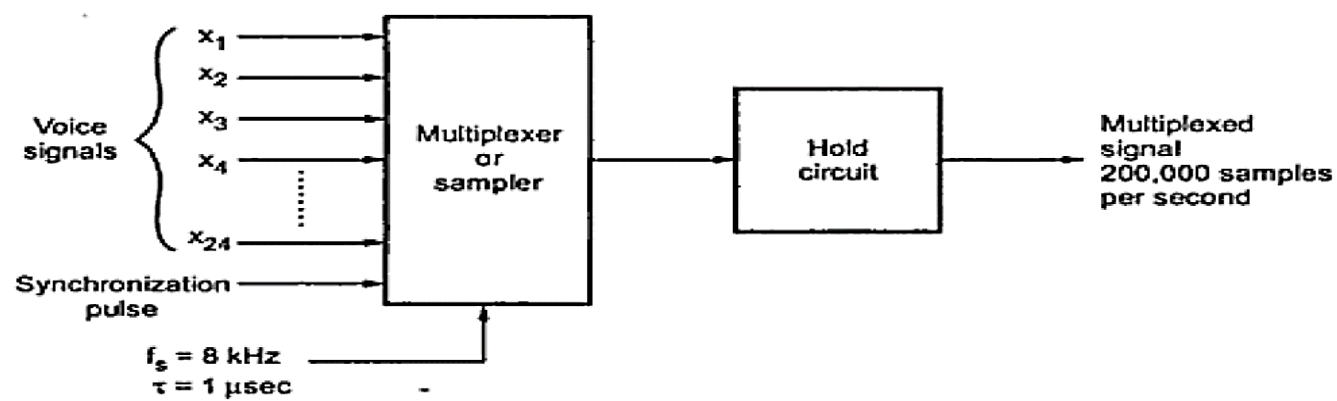


$$T_s = \frac{1}{f_s} = \frac{1}{8000} = 125 \mu\text{sec}$$

Thus in 125 μsec time there are 25 pulses at uniform distances. Where the pulses are separated by $\frac{125 \mu\text{sec}}{25} = 5 \mu\text{sec}$. Width of the pulse is 1 μsec , hence;

$$\text{spacing between pulses} = 5 - 1 = 4 \mu\text{sec}$$

Figure shows the multiplexing scheme.



Digital Communication Systems

Chapter (3) Digital Pulse Modulation

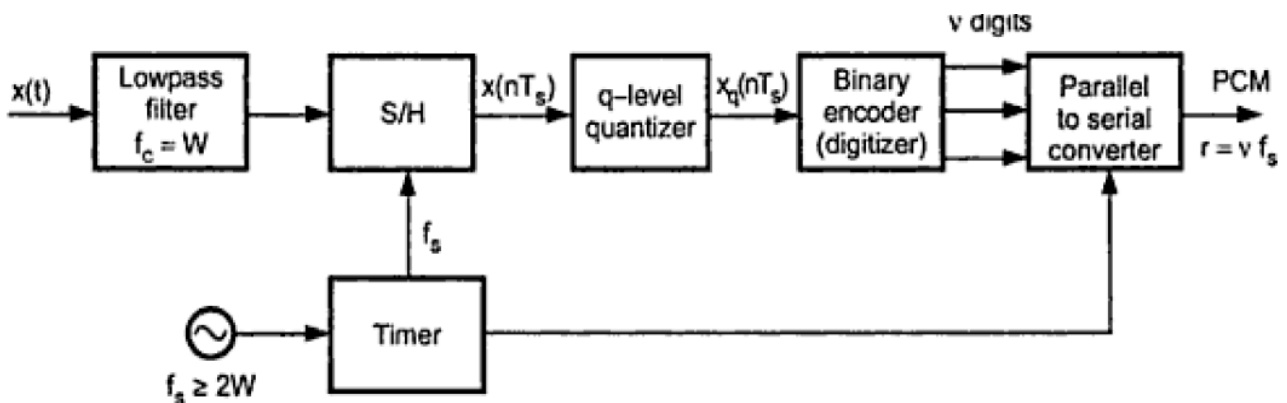
1. Background

As we expressed in previous chapter, Digital Pulse Modulation means that the message signal is represented in a digital form (discrete in both time and amplitude). Therefore, the information signal is processed in digital form as a sequence of coded pulses. In this chapter, we express the digital pulse modulation techniques, which are Pulse Code Modulation (PCM), Differential Pulse Code Modulation (DPCM), Delta Modulation (DM) and Adaptive Delta Modulation.

2. Pulse Code Modulation (PCM)

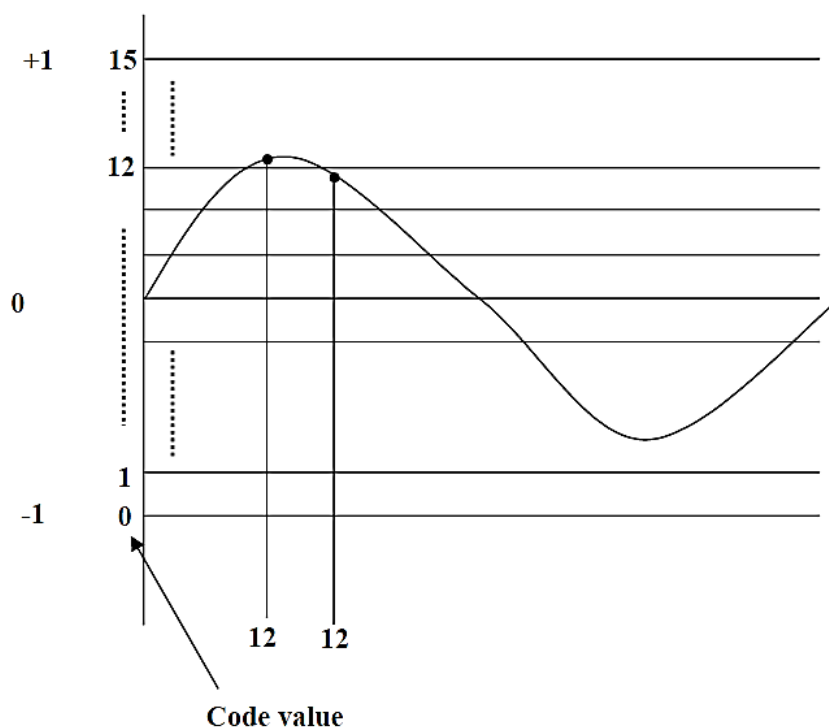
Pulse code modulation (PCM) is used to digitally represent sampled analog signals. It is the standard form of digital audio in computers, compact discs, digital telephony and other digital audio applications. In a PCM stream, the amplitude of the analog signal is sampled regularly at uniform intervals, and each sample is quantized to the nearest value within a range of digital steps. Figure below shows the PCM generator.

The pulse code modulator technique samples the input signal, $x(t)$ at frequency $f_s \geq 2W$. This sampled 'Variable amplitude' pulse is then digitized by the analog to digital converter. The parallel bits obtained are converted to a serial bit stream. Figure below shows the PCM generator.

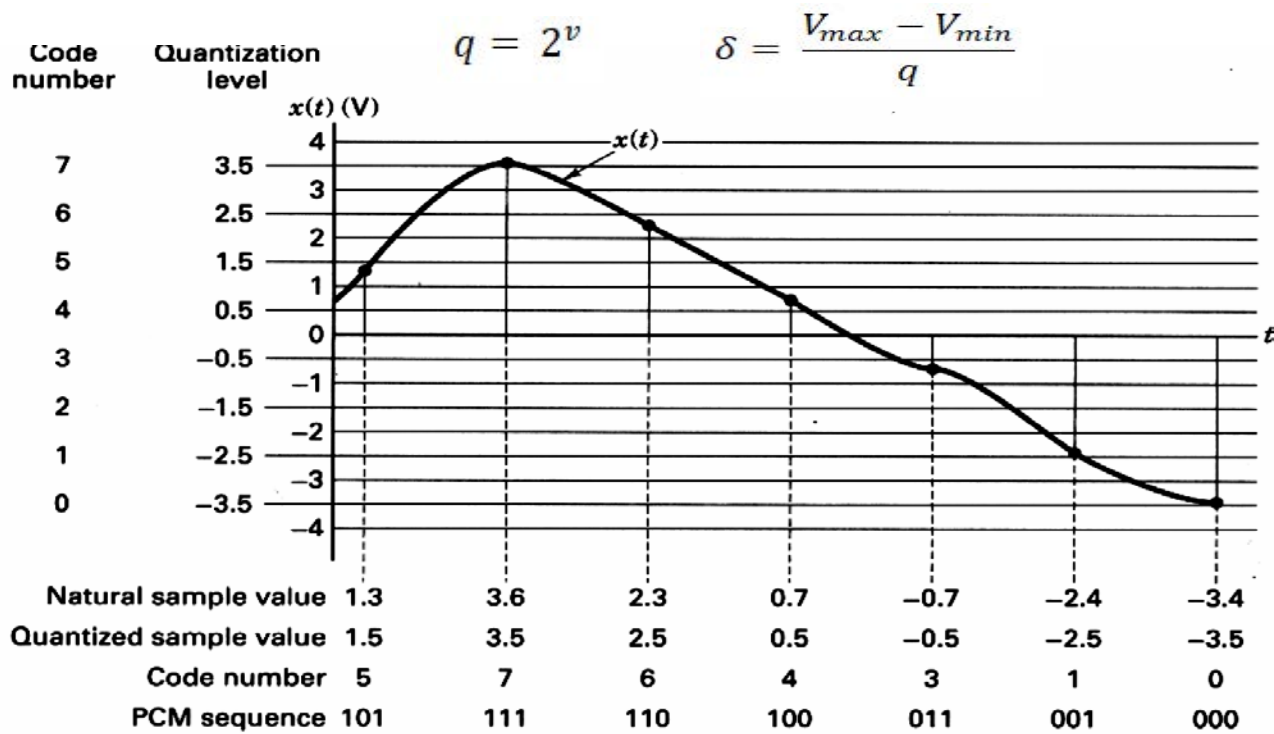


3. Quantization

The objective of the quantization step in PCM process is to represent each sample by a fixed number of bits. For example, if the amplitude of PAM resulting from sampling process ranges between (-1V and +1V), there can be infinite values of voltage between (-1 and +1). For instance, one value can be -0.27689V. To assign a different binary sequence to each voltage value, we would have to construct a code of infinite length.



Therefore, we can take a limit number of voltage values between (-1V and +1V) to represent the original signal and these values must be discrete. Assume that the quantization steps were in 0.1V increment, and the voltage measurement for one sample is 0.58V. That would have to be rounded off to 0.6V, the nearest discrete value. Note that there is a 0.02V error, the difference between 0.58V and 0.6V. See figure above. Take step 12 in the curve, for example, the curve is passing through a maximum and is given two values of 12. For the first value, the actual curve is above 12 and for second value below 12. That error from the true value to the quantum value is called quantization distortion. This distortion is the major source of imperfection in PCM system.



The more quantization level, the better quality the system will deliver. However, increasing the number of quantization level has two major costs:-

- 1) The cost of designing a system with large binary code size needed.
- 2) The time it takes to process this large number of quantizing steps by the coder.

The performance of a PCM system is influenced by two major sources of noise.

- 1) Channel noise.
- 2) Quantization noise.

4. Transmission Bandwidth in PCM

Let the quantizer use v number of binary digits to represent each level. Then the number of levels that can be represented by v digits will be

$$q = 2^v$$

Here q represents total number of digital levels of q -level quantizer. For example if $v = 3$ bits, then total number of levels will be,

$$q = 2^3 = 8 \text{ levels}$$

Each sample is converted to v binary bits. i.e. Number of bits per sample = v We know that, Number of samples per second = f_s . Therefore number of bits per second is given by,

Number of bits per second

$$\begin{aligned} &= (\text{Number of bits per samples}) * (\text{Number of samples per second}) \\ &= v \text{ bits per sample} * f_s \text{ samples per second} \end{aligned}$$

The number of bits per second is also called signaling rate of PCM and is denoted by r ,

$$\text{signaling rate in PCM} = r = v f_s$$

Bandwidth needed for PCM transmission will be given by half of the signaling rate i.e.,

Transmission Bandwidth of PCM:

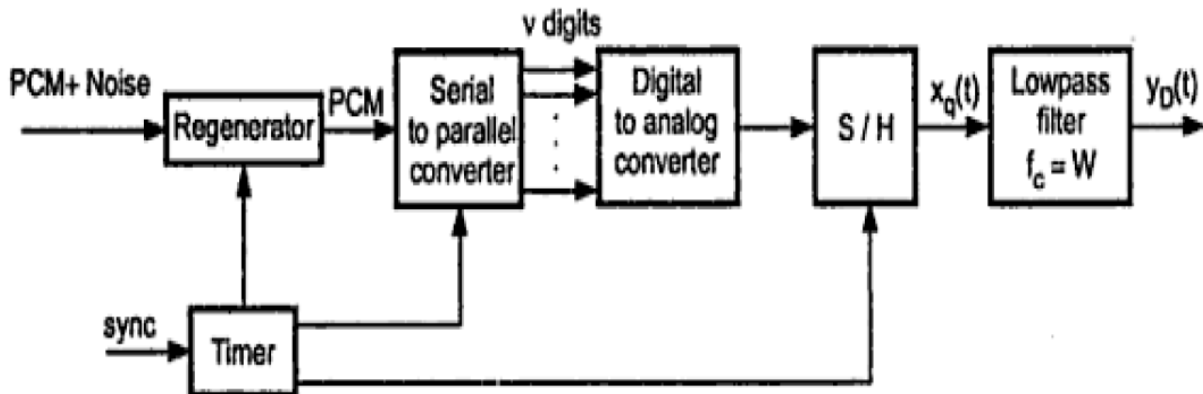
$$B_T \geq \frac{1}{2} r$$

$$B_T \geq \frac{1}{2} v f_s \quad \text{since } f_s \geq 2w$$

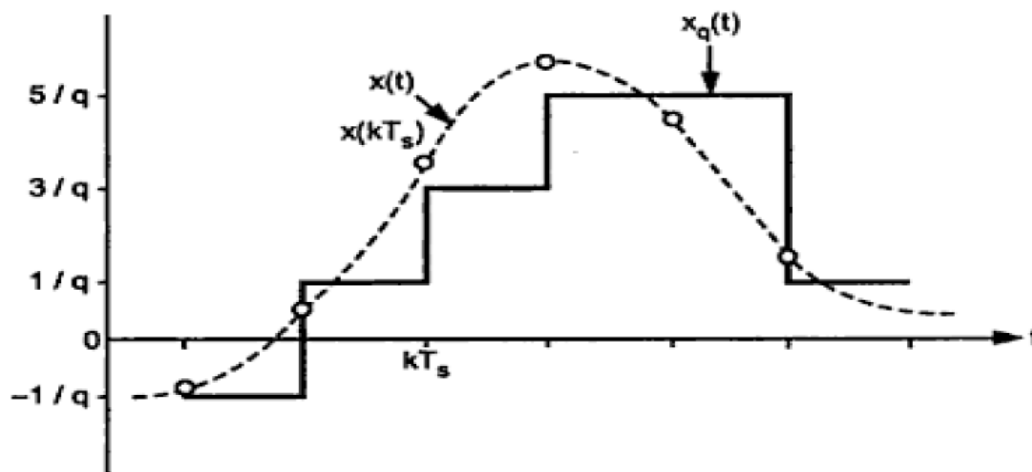
$$B_T \geq v w$$

5. PCM Receiver

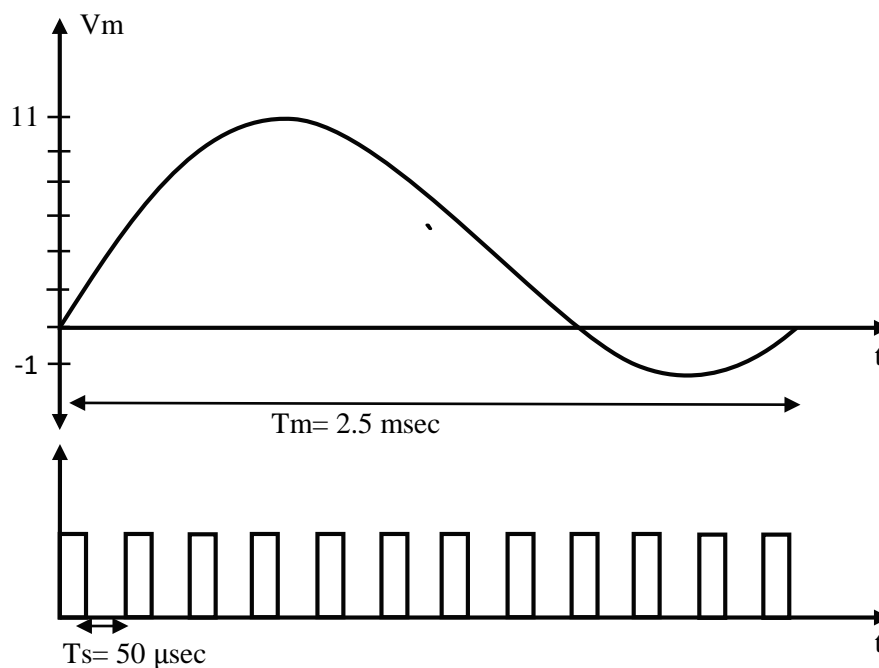
Figure below shows the block diagram of PCM receiver. The regenerator is to reshapes the pulse and removes the noise. The signal is then converted into parallel digital words for each sample.



The digital word is converted to its analog value $x_q(t)$ along with sample and hold (S/H), then passed through lowpass reconstruction filter to get $y_D(t)$. There is quantization error between reconstructed signal $x(kT_s)$ and original signal $x(t)$ as shown in figure below. This can be reduced by increasing bits 'v', but this increases the bandwidth.



H.W1: Find the 3-bit PCM signal for the message wave and sampling signal that shown below.



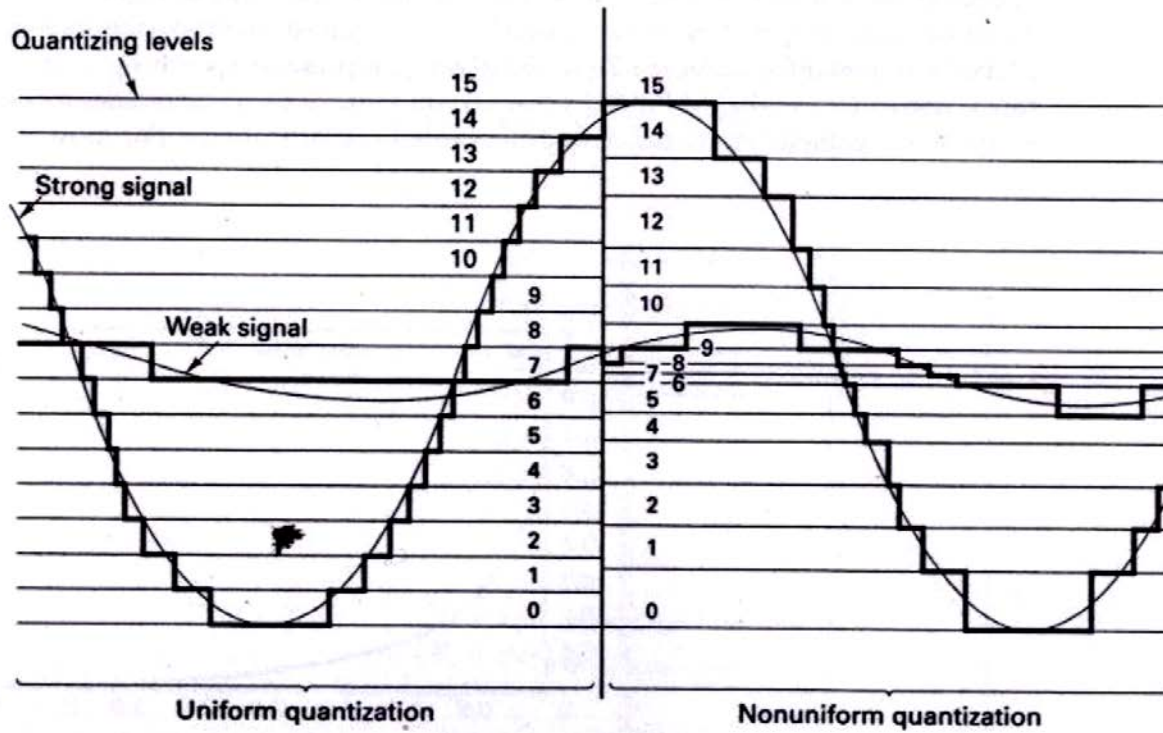
H.W2: Analog signal is inserted to the input of PCM transmitter system, the sampler output is $(-1, 0.3, 1.4, 2.9, 2.7, 4.5, -1.6, 6.7, 9.2)$ with sample time $50\mu s$ after that the signal is quantizing with 8 level, the range of upper and lower limit of analog signal 10 to -2 . Find:

1. Step size.
2. The output after quantizing process.
3. Binary representation of each sample.
4. Sampling rate.
5. Bit rate of PCM system.

6. Uniform and Nonuniform Quantization

From the above discussion it can be seen that the quantization noise depends on the step size. When the steps have uniform size the quantization called as uniform quantization. For uniform quantization, the quantization noise is the same for all signal magnitudes. Therefore, with uniform quantization the signal to noise ratio (SNR) is worse for low level signals than for high level signals.

Nonuniform quantization can provide fine quantization of the weak signal and coarse quantization of the strong signal. Thus in the case of nonuniform quantization, quantization noise can be made proportional to signal size. The effect is to improve the overall SNR by reducing the noise for the predominant weak signals, at the expense of an increase in noise for the rarely occurring strong signals. Figure below compares the quantization of strong signal versus a weak signal for uniform and nonuniform quantization.



7. Quantization Noise in PCM:

The quantization error is expressed as:

$$\varepsilon = x_q(nT_s) - x(nT_s)$$

The range of amplitude of input $x(nT_s)$ is $-x_{max}$ to $+x_{max}$ and it is mapped into q levels. So that total amplitude range $2x_{max}$ is divided into q levels with step size δ .

$$\delta = \frac{2x_{max}}{q}$$

We have the maximum quantization error is $\pm \frac{\delta}{2}$, or $\varepsilon_{max} = \left| \frac{\delta}{2} \right|$

The mean square value of quantization error is:

$$E(\varepsilon^2) = \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \varepsilon^2 f_{\varepsilon}(\varepsilon) d\varepsilon = \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \varepsilon^2 \frac{1}{\delta} d\varepsilon = \frac{1}{\delta} \left[\frac{\varepsilon^3}{3} \right]_{-\frac{\delta}{2}}^{\frac{\delta}{2}}$$

$$E(\varepsilon^2) = \frac{1}{\delta} \left[\frac{\left(\frac{1}{\delta}\right)^3}{3} + \frac{\left(\frac{1}{\delta}\right)^3}{3} \right] = \frac{1}{3\delta} \left[\frac{\delta^3}{8} + \frac{\delta^3}{8} \right] = \frac{\delta^2}{12}$$

The noise power = $\frac{V_{noise}^2}{R}$

Assume $R=1$, then the noise power (normalized) = $\frac{V_{noise}^2}{1}$

$$E(\varepsilon^2) = \frac{\delta^2/12}{1} = \frac{\delta^2}{12}$$

The maximum signal power to quantization noise ratio:

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{\delta^2/12}$$

We have $q = 2^v$, so:

$$\delta = \frac{2x_{max}}{q} = \frac{2x_{max}}{2^v}$$

Substituting in above $\frac{S}{N}$ equation:

$$\frac{S}{N} = \frac{\text{Normalized signal power}}{\frac{\left(\frac{2x_{max}}{2^v}\right)^2}{12}}$$

Let normalized signal power as P

$$\frac{S}{N} = \frac{P}{\left(\frac{2x_{max}}{2^v}\right)^2 / 12} = \frac{3P}{(x_{max}^2)} \times 2^{2v}$$

This equation shows that the signal to noise power ratio of quantizer increases exponentially with increasing bits per sample. For normalized x_{max}

$$\frac{S}{N} = 3 \times 2^{2v} \times P$$

$$\left(\frac{S}{N}\right) dB = 10 \log_{10} \left(\frac{S}{N}\right) dB = 10 \log_{10}(3 \times 2^{2v}) = (4.8 + 6v) dB$$

For normalized values of power the destination signal power 'P' is less than 1

So that

$$\left(\frac{S}{N}\right) dB \leq (4.8 + 6v) dB$$

Ex1: A television signal with bandwidth of 4.2 MHz is transmitted using binary PCM. The number of quantization levels is 512. Calculate:

- i- Code word length
- ii- Transmission bandwidth
- iii- Final bit rate
- iv- Output signal to quantization noise ratio

Solution:

$$i- \quad q = 2^v \rightarrow 512 = 2^v$$

$$\log 512 = v \times \log 2 = 9 \text{ bits}$$

Thus the code word length is 9 bits

$$ii- \quad B_T \geq vW \rightarrow B_T \geq 9 \times 4.2 \times 10^6$$

$$\therefore B_T \geq 37.8 \text{ MHz}$$

$$iii- \quad \text{The signaling rate } r = v \times f_s = v \times 2W = 9 \times 2 \times 4.2 \times 10^6$$

$$r = 75.6 \text{ Mbps}$$

Also we have $B_T \geq \frac{1}{2}r$ or $B_T \geq 0.5 \times 75.6 = 37.8 \text{ MHz}$ which is same value obtained earlier

$$iv- \quad \left(\frac{S}{N}\right) dB = (4.8 + 6v) dB = 4.8 + 6 \times 9 = 58.8 \text{ dB}$$

Ex2: The bandwidth of signal input to the PCM is restricted to 4 kHz. The input varies from -3.8V to +3.8 V and has the average power of 30 mW. The required signal to noise ratio is 20 dB.

- i- Calculate the number of bits required per sample.
- ii- Outputs of 20 such PCM coder are time multiplexed. What is the minimum required transmission bandwidth for the multiplexed signal?

Solution:

$$\left(\frac{S}{N}\right) dB = 10 \log_{10} \left(\frac{S}{N}\right) dB = 20 dB$$

$$\therefore \frac{S}{N} = 100$$

$$i- \quad \frac{S}{N} = \frac{3P}{(x_{max}^2)} \times 2^{2v} \rightarrow 100 = \frac{3 \times 30 \times 10^{-3} \times 2^{2v}}{(3.8)^2}$$

$$2^{2v} = \frac{1444}{0.06} = 24066.67$$

$$2v \log 2 = \log(24066.67)$$

$$v \cong 7 \text{ bits}$$

- ii- $B_T \geq vW$ and for 20 multiplexed signals

$$B_T \geq 20 \times 7 \times 4 \text{ kHz} \geq 840 \text{ kHz}$$

And signaling rate

$$r = 2B_T = 2 \times 840 = 1680 \text{ kbps}$$

Ex3: The information in an analog signal voltage waveform is to be transmitted over a PCM system with an accuracy of $\pm 0.1\%$ (full scale). The analog signal has a bandwidth of 100Hz and an amplitude range of -10 to +10 volts. Determine:

- i- The number of levels required for such accuracy.
- ii- The code word length.

- iii- The minimum bit rate required.
- iv- The bandwidth required for PCM signal.

Solution:

- i- The maximum quantization error should be $\mp 0.1\%$, so:

$$\varepsilon_{max} = \mp 0.001$$

$$\text{But } \varepsilon_{max} = \left| \frac{\delta}{2} \right|$$

$$\left| \frac{\delta}{2} \right| = 0.001 \rightarrow \text{the step size } \delta = 0.002$$

$$\text{We have } \delta = \frac{2x_{max}}{q}, \text{ and } |x_{max}| = 10 \text{ volts}$$

$$\text{Then } 0.002 = \frac{2 \times 10}{q} \rightarrow q = 10000$$

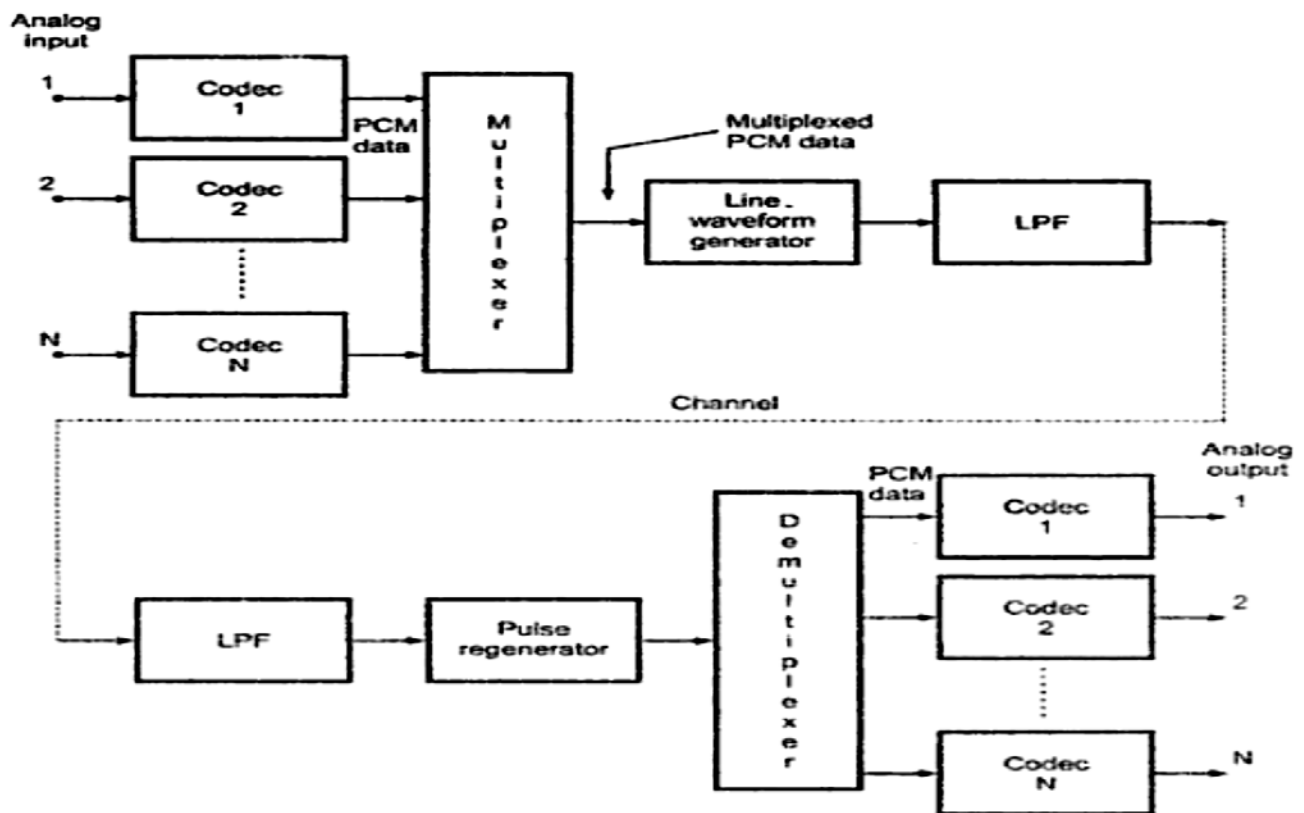
So that the number of levels are 10000.

- ii- We have $q = 2^v \rightarrow 10000 = 2^v \rightarrow \log 10000 = v \log 2$
 $\therefore \text{the code word length } v = 13.288 \cong 14 \text{ bits}$
- iii- We have the bit rate $r = v f_s = v \times 2 \times W = 14 \times 2 \times 100 = 2800 \text{ bps}$
- iv- The bandwidth required $B_T \geq \frac{1}{2} r \geq \frac{1}{2} \times 2800 = 1400 \text{ Hz}$

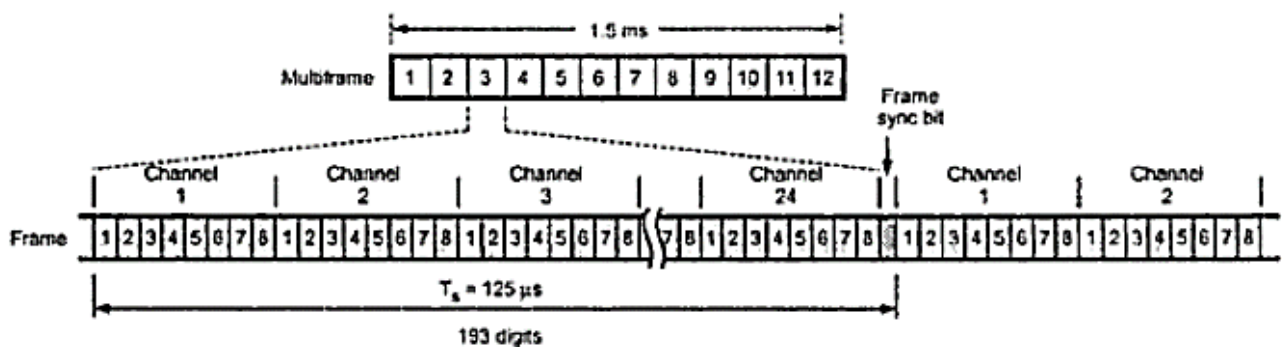
8. PCM TDM System

The PCM-TDM system uses many codecs as shown in figure below. The codec is basically a PCM encoder (transmitter) and decoder (receiver). Codec generates serial stream of PCM data. At the receiver side, codec receives serial PCM data and generates analog signal. The sampling frequency of PCM can be selected by external clock given to the codec. One codec per channel is used. The outputs from various codecs are combined by the multiplexer into single bit stream. This bit stream is converted to baseband waveform by line waveform generator. The low pass filter (LPF) bandlimits the baseband signal. The waveform regenerator is used at the receiver to construct the

input noisy waveform to clear digital signal. The Demux then detects individual channel signals and separates them. The codecs then recover the required analog signal.



The multiple channel alignment is very important in TDM/PCM system. TDM frame format of most widely used T1 system.



As shown in the figure above, this system contains a multiframe of 12 frames. The duration of the multiframe is 1.5 msec. Each frame consists of samples from 24 channels. Thus, the samples of 24 channels are Time Division Multiplexed. Each

channel sample is encoded into 8 bits. Thus, the total bits of 24 channels will be $24 \times 8 = 192$ bits. This indicates the start of the next frame, the frame sync bit or 'S' bit is transmitted at the beginning of each frame. Thus the total bits in one frame are $(24 \times 8) + 1 = 193$ bits.

Example: The T1 carrier system used in digital telephony multiplexes 24 voice channels based on 8 bit PCM. Each voice signal is usually put through a lowpass filter with cutoff frequency 3.4 kHz. The filtered signal is sampled at 8 kHz. In addition a single bit is added at the end of frame for the purpose of synchronization. Calculate

- i) The duration of each bit.
- ii) The resultant transmission rate.
- iii) Minimum required transmission bandwidth.

Solution:

i) Hence the time between any two successive samples of the same channel will be $\frac{1}{8000} = 125 \mu s$.

The total bits in one frame = $24 \frac{\text{channels}}{\text{frame}} \times 8 \frac{\text{bits}}{\text{channel}} + 1 \text{ frame sync bit} = 193 \text{ bits}$.

$$\begin{aligned} \text{Time duration of one bit} &= \frac{\text{Time duration of one frame}}{\text{bits per frame}} \\ &= \frac{125 \mu s}{193} = 0.6476 \mu s/\text{bit} \end{aligned}$$

ii. The transmission rate is the bit rate which is the reciprocal of duration of one bit i.e.,

$$R_b = \frac{1}{\text{Duration of one bit}}$$

$$R_b = \frac{1}{0.6476 \mu s/\text{bit}} = 1.544 * 10^6 \text{ bits/sec}$$

iii) Transmission Bandwidth

$$B_T \geq \frac{1}{2} R_b$$

$$\geq \frac{1}{2} * 1.544 * 10^6$$

$$B_T \geq 772 \text{ kHz}$$

Example: Twenty four voice channels of 4 kHz bandwidth each sampled at Nyquist rate and encoded into 8 bit PCM are time division multiplexed with 1 bit/frame as synchronization bit. What is bit rate at the output of multiplexers?

Solution:

The total bits in one frame are = $24 * 8 + 1 \text{ frame sync bit} = 193 \text{ bits}$.

$$\text{Nyquist rate} = 2W = 2 * 4 \text{ kHz} = 8 \text{ kHz}$$

$$\text{Bit rate} = \text{number of bits/frame} * \text{number of frames/sec.}$$

$$\text{Bit rate} = 193 * 8000 = 1.544 * 10^6 \text{ bits/sec}$$

\therefore Thus, the bit rate is 1.544 Mbps.

9. Advantages of PCM

- (1) Effect of channel noise and interference is reduced.
- (2) Multiplexing of various PCM signals is easily possible.
- (3) Encryption or decryption can be easily incorporated for security purpose.

10. Limitations of PCM

- (1) PCM systems are complex compared to analog pulse modulation methods.
- (2) The channel bandwidth is also increased because of digital coding of analog pulses.

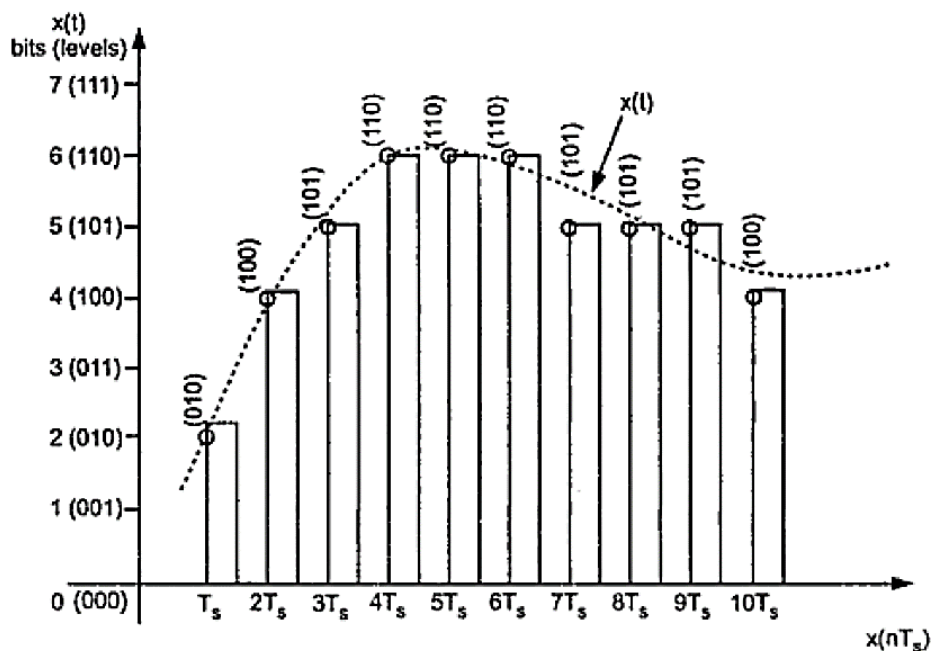
11. Differential Pulse Code Modulation

The samples of a signal are highly correlated with each other. This is because any signal does not change fast. That is its value from present sample to next sample does not differ by large amount. The adjacent samples of the signal carry the same information with little difference. When these samples are encoded by standard PCM system, the resulting encoded signal contains redundant information as shown in figure below.

We can see that the samples taken at $4T_s$, $5T_s$ and $6T_s$ are encoded to same value of (110). This information can be carried only by one sample. But three samples are carrying the same information means it is redundant.

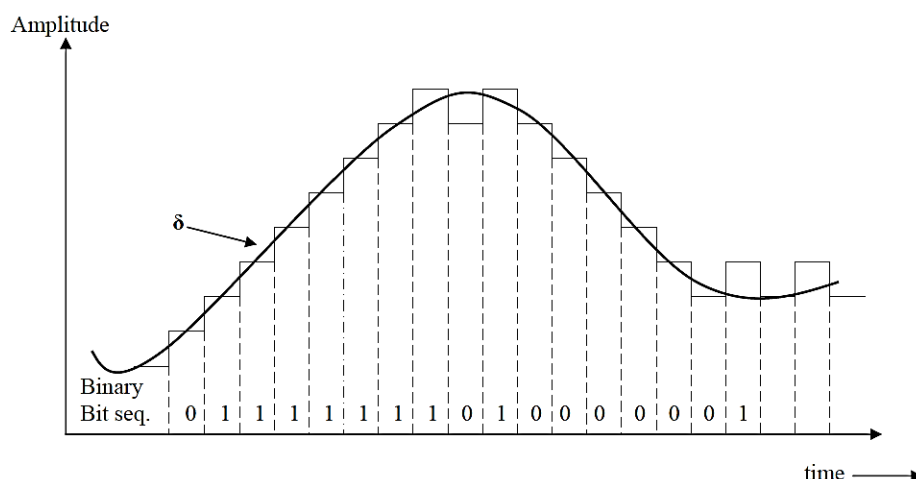
Consider another example of samples taken at $9T_s$ and $10T_s$. The difference between these samples is only due to last bit and first two bits are redundant since they do not change.

If this redundancy is reduced, then overall bit rate will decrease and number of bits required to transmit one sample will also be reduced.



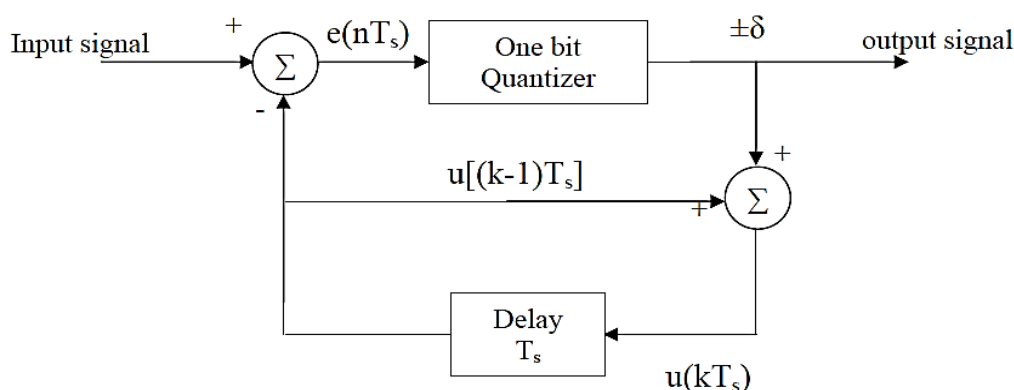
12. Delta Modulation (DM)

Delta modulation transmits only one bit per sample. That is the present sample value is compared with the previous sample value and the indication, whether the amplitude is increased or decreased is sent. The input signal is approximated to step signal by the delta modulator. This step size is fixed. The difference between the input signal $x(t)$ and the staircase approximated signal confined to two levels, i.e. $+\delta$, $-\delta$. If the difference is positive then the approximated signal is increased by $+\delta$, if the difference is negative then the approximated signal is decreased by $-\delta$. A zero is transmitted for $-\delta$ and a one is transmitted for $+\delta$. Figure below shows a delta modulation waveform.



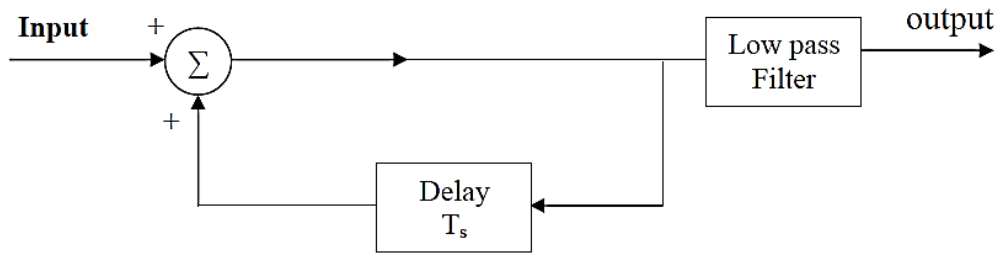
13. Delta Modulation Transmitter

The summer in the accumulator adds quantizer output ($\pm\delta$) with the previous sample approximation. The previous sample approximation $u[(n-1)T_s]$ is restored by delaying one sample period T_s . The sampled input signal $x(nT_s)$ and staircase approximated signal is subtracted to get the error signal $e(nT_s)$.



14. Delta Modulation Receiver

The accumulator generates the staircase approximated signal output and is delayed by one sampling period T_s . It is then added up to the input signal. If the input is '1', then $+\delta$ is added to the previous output (which is delayed). If the input is binary '0'. Then one-step is subtracted from the delayed signal.



Example: Consider a sine wave of frequency f_m and amplitude A_m applied to a delta modulator of step size δ . Show that the slope overload distortion will occur if

$$A_m > \frac{\delta}{2\pi f_m T_s}$$

Solution: Let the sine wave be represented as,

$$x(t) = A_m \sin(2\pi f_m t)$$

Slope of $x(t)$ will be maximum when derivative of $x(t)$ with respect to 't' will be maximum. The maximum slope of delta modulator is given as,

$$\begin{aligned} \text{Max. slope} &= \frac{\text{step size}}{\text{sampling period}} \\ &= \frac{\delta}{T_s} \end{aligned}$$

Slope overload distortion will take place if slope of sine wave is greater than slope of delta modulator i.e.

$$\max \left| \frac{d}{dt} x(t) \right| > \frac{\delta}{T_s}$$

$$\max \left| \frac{d}{dt} A_m \sin(2\pi f_m t) \right| > \frac{\delta}{T_s}$$

$$\max |A_m 2\pi f_m \cos(2\pi f_m t)| > \frac{\delta}{T_s}$$

$$A_m 2\pi f_m > \frac{\delta}{T_s} \quad \text{so that} \quad A_m > \frac{\delta}{2\pi f_m T_s}$$

To avoid the slope overload distortion

$$A_m \leq \frac{\delta}{2\pi f_m T_s}$$

Example: A delta modulator system is designed to operate at five times the Nyquist rate for a signal with 3 kHz bandwidth. Determine the maximum amplitude of a 2 kHz input sinusoid for which the delta modulator does not have slope over load. Quantizing step size is 250 mV.

Solution: Slope overload will not occur if,

$$A_m \leq \frac{\delta}{2\pi f_m T_s}$$

The maximum frequency in the signal is,

$$W = 3 \text{ kHz}$$

$$\text{Nyquist rate} = 2W = 2 * 3 \text{ kHz} = 6 \text{ kHz}$$

$$\text{Sampling frequency} = 5 \text{ times Nyquist rate}$$

$$f_s = 5 * 6 \text{ kHz} = 30 \text{ kHz}$$

$$\text{Sampling interval } T_s = \frac{1}{f_s} = \frac{1}{30 \text{ kHz}}$$

$$\text{Step size } \delta = 250 \text{ mV} = 0.25 \text{ V}$$

$$f_m = 2 \text{ kHz}$$

$$\therefore A_m \leq \frac{0.25 V}{2\pi * 2 \text{ kHz} * \frac{1}{30 \text{ kHz}}}$$

$$A_m \leq 0.6 \text{ volts}$$

Example: In a single integration DM scheme the voice signal is sampled at a rate of 64 kHz. The maximum signal amplitude is 2 volts. Voice signal bandwidth is 3.5 kHz.

- i- Determine the minimum value of step size to avoid slope over load.
- ii- Calculate granular noise power.
- iii- Determine the postfiltered output SNR for the signal.

Solution:

$$f_s = 64 \text{ kHz} \quad \therefore \quad T_s = \frac{1}{64 * 10^3}$$

$$A_m = 2 V$$

$$f_m = W = 3.5 \text{ kHz}$$

i.

$$A_m \leq \frac{\delta}{2\pi f_m T_s}$$

$$2 \leq \frac{\delta}{2\pi * 3.5 \text{ kHz} * \frac{1}{64 * 10^3}}$$

$$\delta \geq 0.687 \text{ volts}$$

ii.

$$\begin{aligned} \text{Noise power} &= \frac{W T_s \delta^2}{3} = \frac{3.5 \text{ kHz} * \frac{1}{64 * 10^3} * (0.687)^2}{3} \\ &= 8.6 \text{ mW} \end{aligned}$$

iii.

$$\begin{aligned}\frac{S}{N} &= \frac{3}{8\pi^2 W f_m^2 T_s^3} \\ &= \frac{3}{8\pi^2 * 3.5 \text{ kHz} * (3.5 \text{ kHz})^2 * (\frac{1}{64 * 10^3})^3} = 232.3 \\ \text{or } \left(\frac{S}{N}\right)_{dB} &= 10 \log_{10} \frac{S}{N} = 23.66 \text{ dB}\end{aligned}$$

Homework: A DM system designed to operate at 3 times the Nyquist rate for a signal with a 3 kHz bandwidth. The quantizing step size is 250 mV

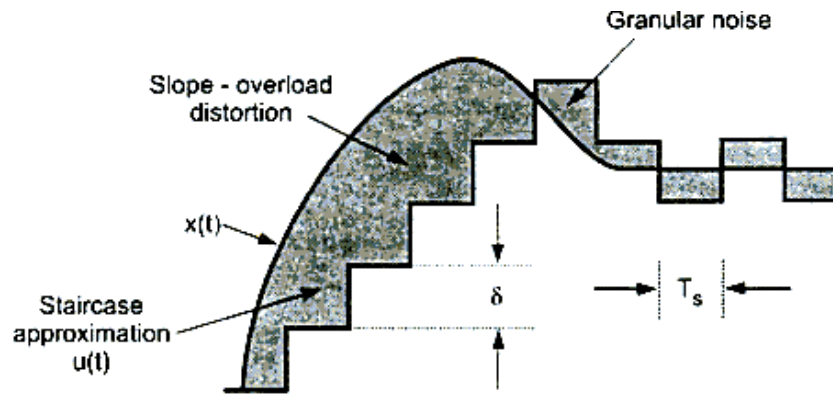
- i- Determine the maximum amplitude of a 1 kHz input sinusoid for which the delta modulator does not show slope overload.
- ii- What is the corresponding SNR?

15. Advantages of Delta Modulation

1. DM transmit only one bit for one sample. Thus, the signaling rate and transmission channel bandwidth is quite small for DM.
2. The transmitter and receiver is very simple to implement.

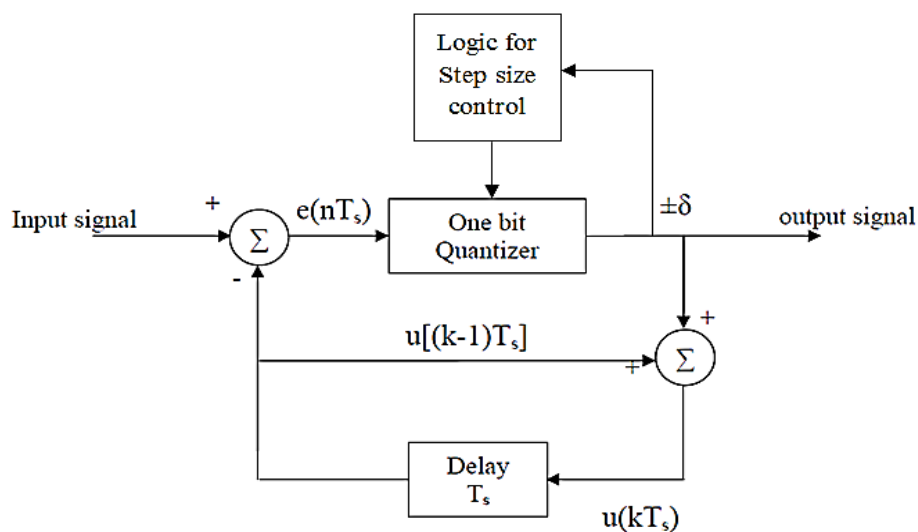
16. Disadvantages of Delta Modulation

1. Slope overload distortion: This distortion arises because of large dynamic range of input signal. In this case the step size δ is too small for staircase signal $u(t)$ to follow the steep segment of $x(t)$. Thus, there is large error between those signals. This error called slope overload distortion. To reduce this error the step size should be increased when slope of signal $x(t)$ is high. However, since the step size is fixed it is called Linear Delta Modulation (LDM).
2. Granular Noise (Hunting): It is occur when the step size is too large compared to small variation in the input signal $x(t)$ which can be considered flat, while the staircase signal is oscillated by $\pm\delta$ around it. The error in this case is called granular noise, so that step size should be small to reduce this error.

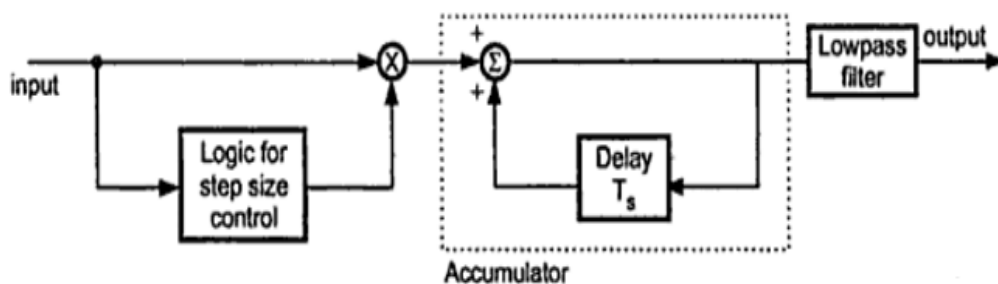


17. Adaptive Delta Modulation

The large step size is required to reduce slope overload while small steps are required to reduce granular noise. Adaptive DM shown in figure below is a modification of LDM to overcome these errors. The step size increases with steep segment of input signal and reduces with small variation.



At the receiver, the logic for step size control is added.



18. Line Coding

1. Introduction:

A line code is the code used for data transmission of a digital signal (a sequence of binary bits) over a transmission line. In other word, the representative techniques of digital sequences by pulse waveforms suitable for baseband transmission. This process of coding is chosen to avoid overlap and distortion of signal such as inter-symbol interference.

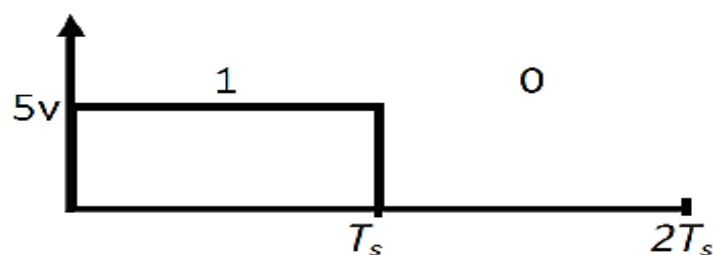
2. Properties of Line Coding:

- ❖ Transmission bandwidth: as small as possible.
- ❖ Power is efficiently: as small as possible for a given BW and probability of error.
- ❖ Error detection and correction capability.
- ❖ Power density is much suitable: DC component can be eliminated.
- ❖ The timing content is adequate: extract timing from pulses for timing recovery.
- ❖ Transparency: Long strings of 1s and 0s is avoided which can cause loss of transmission.

3. Types of Line Coding: There are many types of line coding as following:

1-Unipolar Nonreturn-to-Zero (NRZ) Signaling:

Symbol 1 is represented by transmitting a pulse of constant amplitude for the entire duration of bit interval, and symbol 0 is represented by no pulse. NRZ indicates that the assigned amplitude level is maintained throughout the entire bit interval.



Advantages:

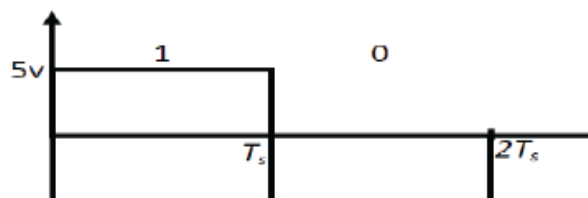
- ✓ It is simple.
- ✓ A lesser bandwidth is required.

Disadvantages:

- ☒ No error correction done.
- ☒ Presence of low frequency components may cause the signal droop.
- ☒ No clock is present.
- ☒ Loss of synchronization is likely to occur (especially for long strings of 1s and 0s).

2-Bipolar NRZ Signaling:

Symbols 1 and 0 are represented by pulses of equal positive and negative amplitudes. In either case, the assigned pulse amplitude level is maintained throughout the bit interval.

**Advantages:**

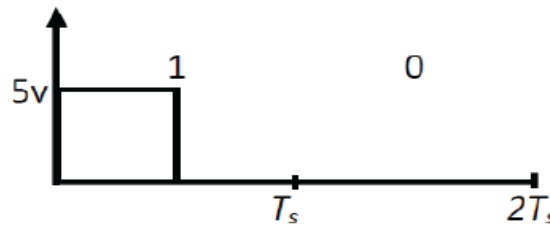
- ✓ It is simple.
- ✓ No DC component.

Disadvantages:

- ☒ No error correction.
- ☒ No clock is present.
- ☒ The signal droop is caused.

3- Unipolar Return-to-Zero (RZ) Signaling:

Symbol 1 is represented by a positive pulse that returns to zero before the end of the bit interval, and symbol 0 is represented by the absence of pulse.



Advantages:

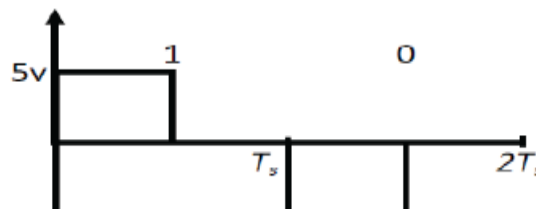
- ✓ It is simple.
- ✓ The spectral line present at the symbol rate can be used as a clock.

Disadvantages:

- ☒ No error correction.
- ☒ Occupies twice the bandwidth as unipolar NRZ.
- ☒ The signal droop is caused.

4-Bipolar RZ Signaling:

Positive and negative pulses of equal amplitude are used for symbols 1 and 0, respectively. In either case, the pulse returns to 0 before the end of the bit interval.



Advantages:

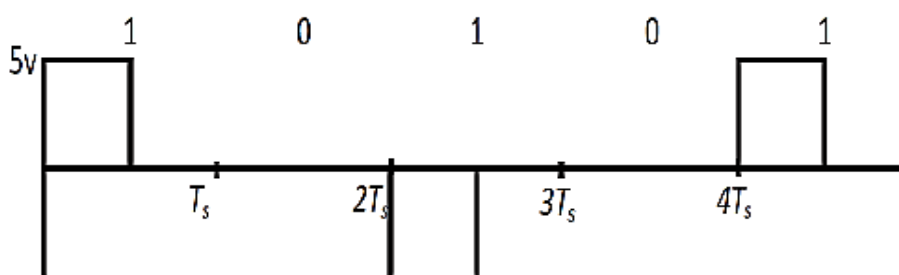
- ✓ It is simple.
- ✓ No DC component.

Disadvantages:

- ☒ No error correction.
- ☒ No clock is present.
- ☒ Occupies twice the bandwidth of Polar NRZ.
- ☒ The signal droop is caused.

5- Alternate Mark Inversion (AMI) RZ Signaling:

Positive and negative pulses (of equal amplitude) are used alternately for symbol 1, and no pulse used for symbol 0. In either case the pulse returns to 0 before the end of the bit interval.

**Advantages:**

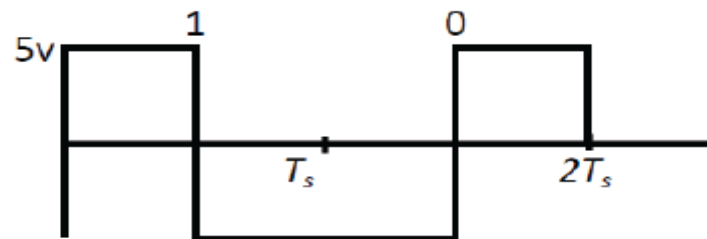
- ✓ No DC component.
- ✓ Occupies low bandwidth than unipolar and polar NRZ schemes.
- ✓ This technique is suitable for transmission over AC coupled lines, as signal drooping doesn't occur here.
- ✓ A single error detection capability is present in this.

Disadvantages:

- ☒ Long strings of data causes loss of synchronization. It is not transparent.
- ☒ Clock is not present.

6- Split- Phase (Manchester) Signaling:

Symbol 1 is represented by a positive pulse followed by a negative pulse, with both pulses being of equal amplitude and half-bit duration; for symbol 0, the polarities of these pulses are reversed.



Advantages:

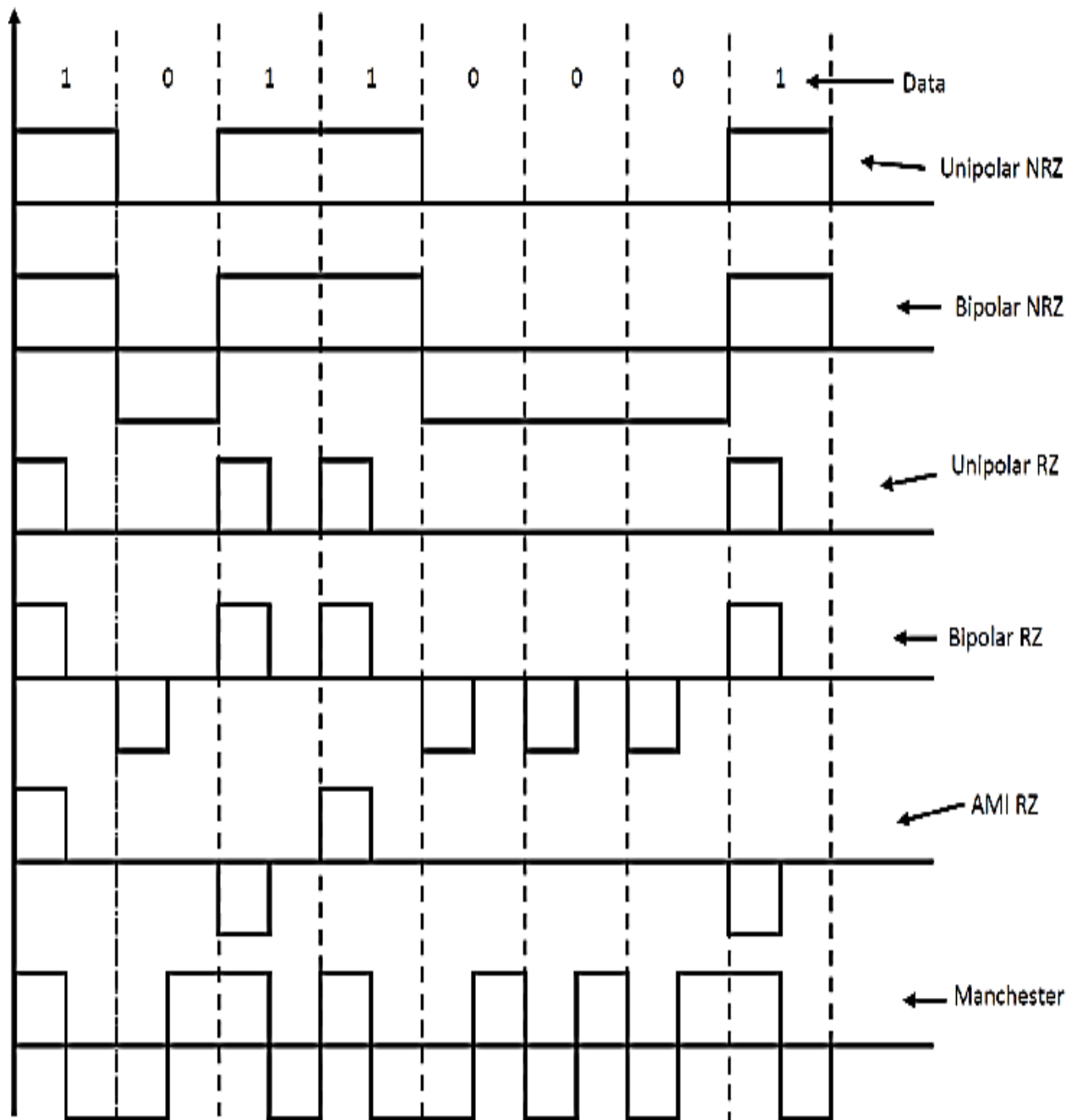
- ✓ No DC components.
- ✓ This technique is suitable for transmission over AC coupled lines, as signal drooping doesn't occur here.
- ✓ clock is present.
- ✓ It is transparent.

Disadvantages:

- ✗ A single error detection capability is not present in this.
- ✗ Occupies large bandwidth.

Example: A binary data (10110001) are inserted into line coding device. Represented this data by unipolar NRZ, bipolar NRZ, unipolar RZ, bipolar RZ, AMI RZ and Manchester waveforms with time period (T_b) of each bit. Sketch all waveforms for the mentioned processes.

Solution:



Some of the important parameters to consider in selecting a signaling format are the spectral characteristics, immunity of the format to noise, bit synchronization capability, cost and complexity of implementation, and other factors that vary with the application.

Digital Communications

1. Base band signal

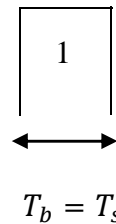
The simplest digital data signal contains a sequence of signal element (units or pulses of a data signal) where each element is binary coded. Having the choice of two possible shapes that correspond to the element values 0 or 1, each signal element has the same duration of T seconds. So that the signal element rate is $1/T$ elements per sec (or bauds). The digital signal above is clearly a 2-level or binary signal. In a sequence of M -level signal elements, where $M = 2^n$ and n is the number of bits, the M -level data symbol that determines the element value of a signal element can be represented by a sequence of n bits. For example, if $n = 3$ bits then $M = 2^3 = 8$ levels and the corresponding sequence of 3 bits are 000, 001, 010... 111.

- 2-level (binary) signal

$$T_b = T_s, n = 1 \text{ bit}, M = 2 \text{ levels}$$

$$R_b = \text{Bit rate} = \frac{1}{T_b} \dots \text{bits per second}$$

$$R_s = \text{Symbol rate} = \frac{1}{T_s} \dots \text{symbol per second (baud)}$$

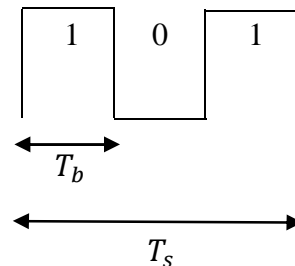


- 8-level signal

$$T_b = \frac{T_s}{3}, n = 3 \text{ bits}, M = 2^3 = 8 \text{ levels}$$

$$\text{Bit rate} = \frac{1}{T_b} \dots \text{bits per second}$$

$$\text{Symbol rate} = \frac{1}{T_s} = \frac{1}{3T_b} \dots \text{symbol/sec}$$

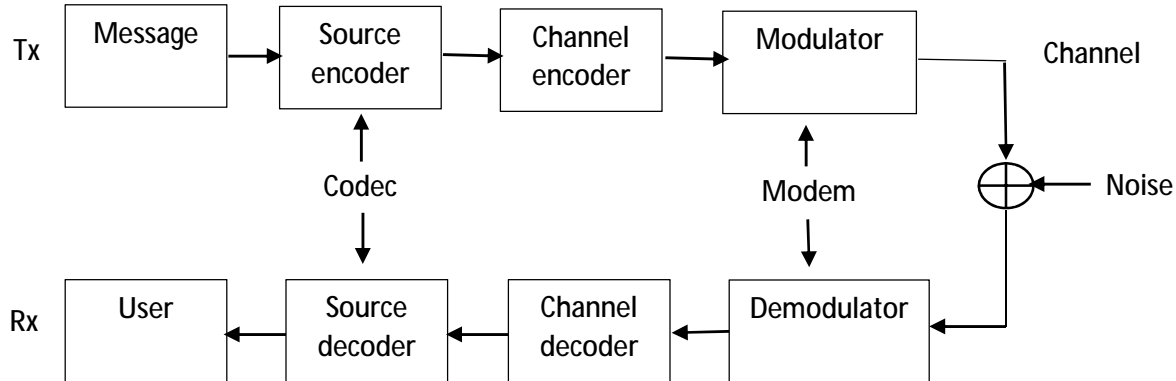


In general, $M = 2^n, n = \log_2 M, T_s = nT_b, R_b = \frac{1}{T_b}, R_s = \frac{1}{T_s}$

? Exercise.1

For 2-level and 8-level systems, what is the difference between them (bit rate? noise?) for the same T_s .

2. Model of Digital communication system



? Exercise.2

Explain briefly the function of each component in the model

3. Measure of information

Consider M-level system with symbols (x_1, x_2, \dots, x_m) . The information contents of a symbol x_i , denoted by $I(x_i)$ is defined by:

$$I(x_i) = \log_b \frac{1}{p(x_i)} = -\log_b p(x_i)$$

Where b is the radix of digits (2 for binary) and $p(x_i)$ is the probability of symbol x_i

$$I(x_1, x_2, \dots, x_m) = I(x_1) + I(x_2) + \dots + I(x_m)$$

The information measure (bits) of a message is equal to the minimum number of binary pulses required to encode that message.

Example.1

How many bits per symbol to encode 32 different symbols?

$$M = 32, P(x) = \frac{1}{M} = \frac{1}{32}$$

$$I(x) = \log_2 32 = 5 \text{ bits/symbol}$$

Example.2

The four symbols x_1, x_2, x_3, x_4 occur with probability 1/2, 1/4, 1/8, 1/8 respectively. Find the info content in the message $(x_2 x_4 x_1)$

$$I(x_2 x_4 x_1) = I(x_2) + I(x_4) + I(x_1) = \log_2 4 + \log_2 8 + \log_2 2 = 6 \text{ bits}$$

Note: $\log_n m = \frac{\ln m}{\ln n}$

4. Bandwidth and Noise

There are two factors affecting the information transfer rate on a channel

- The bandwidth of the channel will determine how quickly the signaling states on the channel can be changed.
- The level of noise in the channel will impose a limit on the number of different unique states that can be correctly decoded at Rx. In addition, the degree of distortion introduced by the channel will also limit the number and rate of change of symbol states.

So, if we had a channel with infinite BW (or no noise and distortion), it would be possible to send, say, one Mbits at the speed of light.

5. Bandwidth efficiency

It is a measure of how well a particular format (and coding scheme) is making use of the available BW. The units of BW efficiency is bits/second/Hz.

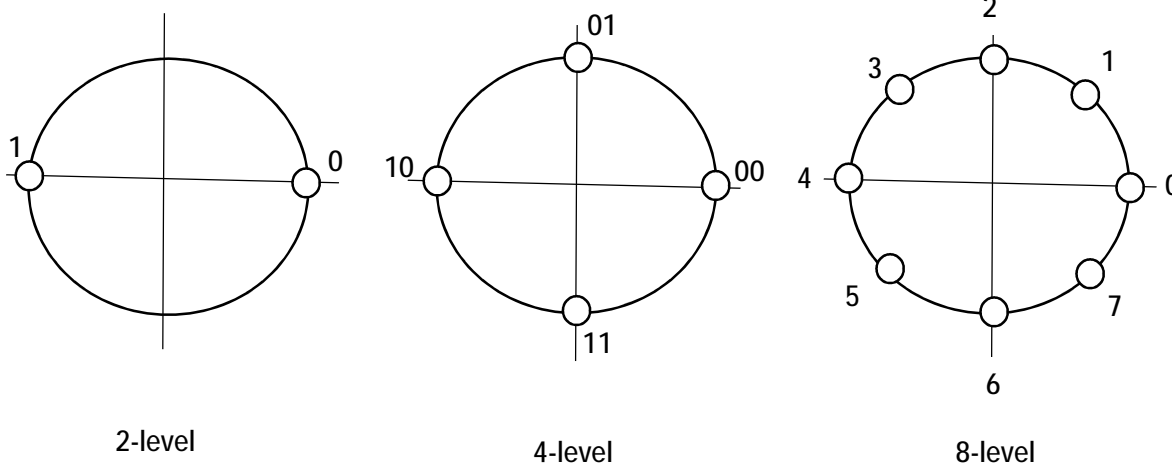
$$\text{bandwidth efficiency} = \frac{R_b}{B}$$

For example, if a system requires 4 KHz of BW to send 8000 bps of information, then

$$BW \text{ efficiency} = \text{bit rate} / BW = 8000 / 4000 = 2 \text{ bits/sec/Hz}$$

6. Multi-level signaling (M-ary)

For transmission with high data rate, it is possible to use M-level signaling: $M = 2^n$ symbol states



- **Advantage of M-ary signaling**

A higher information transfer rate is possible for a given symbol rate and channel BW. (Alternatively, reduced the required BW for a given information rate).

- **Disadvantages of M-ary signaling**

Low immunity in noise/interference compared with binary signaling, as it becomes more difficult to distinguish between symbol states. More complex symbol recovery in the Rx. required for linearity and reduced distortion in Tx/Rx hardware.

Example.3

A modem claims to operate with BW efficiency of 5 bits/sec/Hz when using 1024 symbol states.

- How many bits are being encoded in each symbol, and what is the modem capacity if the symbol rate is 4000 symbol/sec
- How many symbol states should be employed if the user wishes to send his info in half the time?

Solution

$$a) \ n = \log_2 M = \log_2 1024 = 10 \text{ bits/symbol}$$

$$\text{Capacity} = \text{Max bit rate} = R_b = n R_s = 10 \times 4000 = 40 \text{ Kbps}$$

- To send the info in half the time, it would be necessary to send data at bit rate = 80 Kbps hence we need 20 bits in each symbol, so symbol states = $2^{20} = 1048576$

7. Channel capacity (for baseband signals)

The minimum BW required for error-free transmission

$$B_{min} = 0.5 \frac{1}{T_s}, \text{ where } T_s : \text{symbol period}$$

Since $\frac{1}{T_s}$ is the symbol rate (R_s), then

$$\text{Max } R_s = 2B$$

The channel capacity is max R_s but measured in bit/sec, so

$$\boxed{\text{channel capacity (for baseband signals)} = 2Bn}$$

8. Additive white Gaussian Noise channel (AWGN)

As M increases, the ability of the receiver to distinguish between symbols in the presence of noise/interference/distortion decreases. Hence SNR (signal to noise ratio) will be an important

factor in determining how many symbol states can be utilized and still achieve (error-free) communication.

T_s Of each symbol is also key in determining the noise tolerance of a receiver system, with longer symbols giving the receiver more time to average out the effects of noise than shorter symbols.

The capacity of AWGN channel (Shannon capacity) defined as

$$C = B \log_2(1 + \text{SNR}) \dots \text{bits/sec}$$

Where B is the channel BW, $\text{SNR} = \frac{S}{N}$, S is the signal power, N is the noise power $= N_o B$, and N_o is PSD of the noise (watt/Hz)

? Note

- The channel is error-free if $R \leq C$
- For given C, the BW can be increased for decreased signal power

Example.4

Consider AWGN channel with $B=4$ KHz and noise PSD is 2×10^{-12} W/Hz, the signal power required at the modem receiver is 0.1 mW. Calculate the capacity of this channel.

$$B = 4000 \text{ Hz}, S = (0.1)10^{-3} \text{ watt}$$

$$N = N_o B = 2(10^{-12})(4000) = (8)10^{-9} \text{ watt}$$

$$\text{SNR} = \frac{S}{N} = \frac{0.1 \times 10^{-3}}{8 \times 10^{-9}} = (1.25)10^4$$

$$C = B \log_2(1 + \text{SNR}) = 4000 \log_2[1 + 1.25(10^4)] \approx 54.44 \text{ Kb/s}$$

Example.5

The specification of two telephone links are

Link	B	SNR
Class 1	300-3400 Hz	40 dB
Class 2	600-2800 Hz	30 dB

A company has a requirement to send data over a telephone link at bit rate $R = 20$ Kbps without error. Would you advise the company to rent the more expensive class 1 service, or the cheaper class 2 service? Justify your decision.

Solution:

$$C = B \log_2(1 + \text{SNR})$$

For class one line:

$$B = 3400 - 300 = 3100 \text{ Hz}, \text{SNR} = 40 \text{ dB} = 10000$$

$$C = 3100 \log_2(1 + 10000) = 41.2 \text{ Kbps}$$

For class two line:

$$B = 2800 - 600 = 2200 \text{ Hz}, \text{ SNR} = 30 \text{ dB} = 1000$$

$$C = 2200 \log_2(1 + 1000) = 21.9 \text{ Kbps}$$

So both of links will meet the specification of $R = 20 \text{ Kbps}$ error-free. However, the performance of class 2 line is very close to Shannon bound, in practice, it is unlikely that a modem could be realized that would give the desired result on the class 2 line.

? Exercise.3

A signal with 256 symbols is transmitted by 10^4 symbol per second.

- What is the information rate R ?
- Can the output be transmitted without error over AWGN channel with $B = 10 \text{ KHz}$ and $\text{SNR} = 100$
- Find the SNR required for error-free transmission for part (b)
- Find the B required for AWGN channel for error-free transmission if $\text{SNR} = 100$

9. Power and bandwidth efficiency

For a system transmission at maximum capacity, the average signal power (S) at receiver input can be written as $S = C E_b$, where E_b is the average received energy per bit.

The average noise power (N) is $N = N_o B$, then Shannon expression can be written as:

$$C = B \log_2 \left(1 + \frac{S}{N} \right)$$

$$\frac{C}{B} = \log_2 \left(1 + \frac{C E_b}{N_o B} \right) = \log_2 \left(1 + \frac{C}{B} \frac{E_b}{N_o} \right)$$

The ratio $\left[\frac{C}{B} \right]$ represents the *bandwidth efficiency* of the system (bits/sec/Hz).

The ratio $\left[\frac{E_b}{N_o} \right]$ is the *power efficiency*. The smaller the ratio, the less energy used by each bit.

Choosing a power-efficient modem is particularly important in cellular handsets (say to maximize battery lifetime).

Example.6

A digital cellular telephone system is required to work at a BW efficiency of 4 bits/sec/Hz. What is the min E_b/N_o that must be planned for in order to ensure that users on the edge of the coverage area receive error-free communication? If the mobile telephone company wishes to double the

number of users, how much more power must the base-station and handsets radiate in order to maintain coverage and error-free communication?

Solution:

$$\frac{C}{B} = \log_2 \left(1 + \frac{C}{B} \frac{E_b}{N_o} \right), \quad \frac{C}{B} = 4$$

$$4 = \log_2 \left(1 + 4 \frac{E_b}{N_o} \right)$$

$$\frac{E_b}{N_o} = (2^4 - 1) = 3.75 = 5.74 \text{ dB}$$

In order to double the number of users for the same B, then $\frac{C}{B} = 8$

$$\frac{E_b}{N_o} = (2^8 - 1) = 31.87 = 15.03 \text{ dB}$$

Thus, the transmitted power must be increase by a factor $15.03 - 5.74 = 9.29 \text{ dB}$

? Exercise.4

Find the BW efficiency for a wireless communication system having a bit rate of 9.6 Kbps and B of 200 KHz with $\frac{E_b}{N_o}$ of 10 dB.

? Exercise.5

Data has to be transmitted which has B=3 KHz. If SNR at the receiver is 12 dB, determine $\frac{E_b}{N_o}$ for data rates: 2.4 Kbps and 4.8 Kbps. Also, determine the BW efficiency.

? Exercise.6

Sketch roughly the relationship between $\frac{C}{B}$ and $\frac{E_b}{N_o}$, then

- Find the value of $\frac{E_b}{N_o}$ when $\frac{C}{B} = 1, 0, \text{ and } \infty$
- Mark the region on the graph that been considered error-free transmission
- What is the minimum $\frac{E_b}{N_o}$ (in dB) for error-free transmission

10. Inter Symbol Interference (ISI)

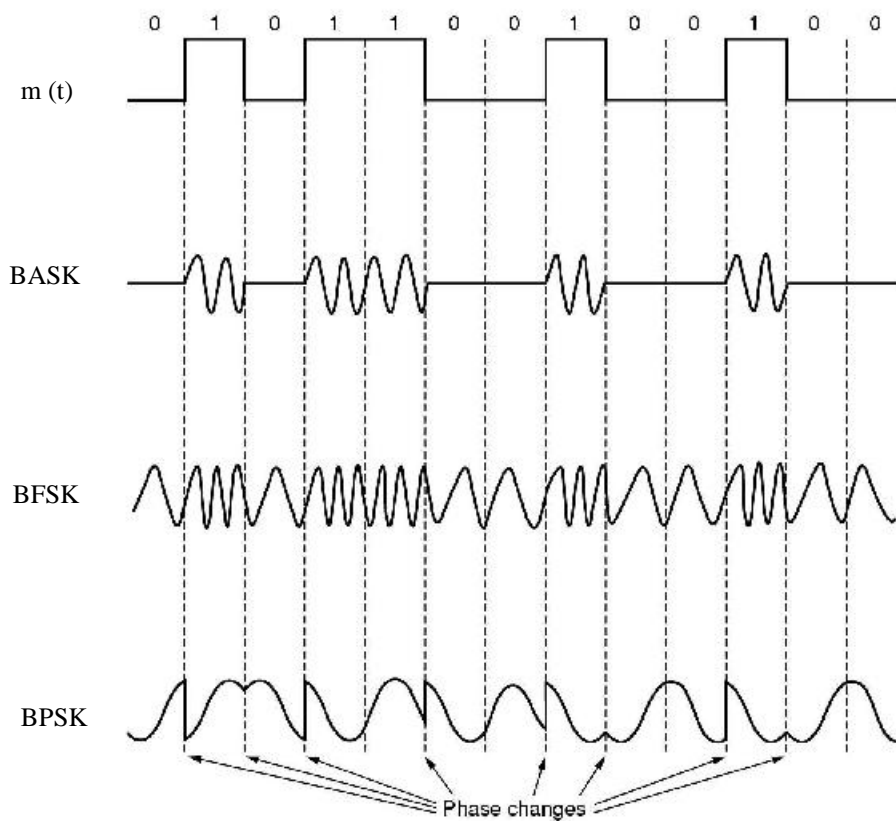
With any particle channel, the filtering effect will cause a spreading of data symbols through the channel. For consecutive symbols, this spreading causes part of the symbol energy to overlap with neighboring symbols, causing ISI. This degrade the ability of the detector to differentiate a current symbol from diffused energy of adjacent symbols. Even with no noise present in the channel, this can lead to defection errors.

It is possible to control ISI such that it does not degrade the system performance by *pulse shaping* or using *Nyquist filtering* ($f_s \geq 2f_{max}$), where f_s is the sampling frequency.

Digital Modulation Techniques

The previous chapter has been concerned with so-called baseband signaling where the channel band is assumed to extend from 0 Hz upwards. In application where bandwidth encompassing 0 Hz is not available, band pass signaling is required. Here, the task is to center the symbol energy at a given frequency of operation, for example, 900 MHz for a typical cellular telephone channel and 30 THz for an optical fiber link the process usually involves modulation the amplitude, frequency, or phase of carrier sine wave. The carrier is commonly written as $\cos(w_c t)$.

The choice of modulation method affects the *ease of implementation*; the *noise tolerance* and *occupied channel bandwidth* of the resulting band pass data modem.

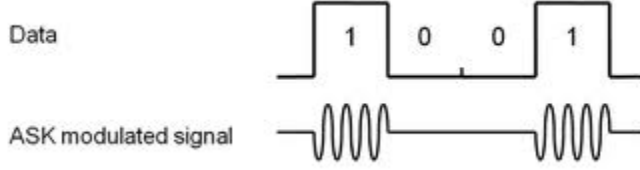


1. Amplitude Shift keying (ASK)

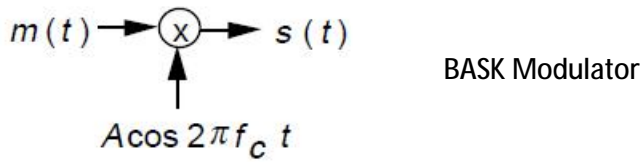
The simplest form of band-pass data modulation is ASK. Here, the symbols are represented as various discrete amplitudes of a fixed carrier frequency w_c .

- **BASK**

In binary ASK (2ASK), where only two symbol states are needed, the carrier is simply turned on or off, and this process is called ON-OFF keying (OOK)



The spectrum of ASK signal can easily be determined if the spectrum of the baseband data symbol is known, by viewing ASK modulation process as a mixing or multiplication of the baseband symbol $m(t) = \cos(w_m t)$ with carrier $A\cos(w_c t)$



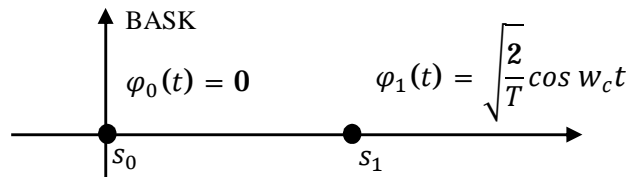
$$s(t) = A m(t) \cos w_c t, \dots 0 < t < T$$

Where A is a constant, $m(t) = 1$ or 0 , f_c is the carrier frequency, and T is the bit duration. It has a power $P = A^2/2$, so that $A = \sqrt{2P}$

$$s(t) = \sqrt{2P} \cos w_c t = \sqrt{PT} \sqrt{\frac{2}{T}} \cos w_c t = \sqrt{E} \sqrt{\frac{2}{T}} \cos w_c t$$

Where $E = P T$ is the energy contained in a bit duration.

If we take $[\varphi_1(t) = \sqrt{\frac{2}{T}} \cos w_c t]$ as the orthonormal basis function, the applicable signal space or constellation diagram of the BASK signals is shown as:

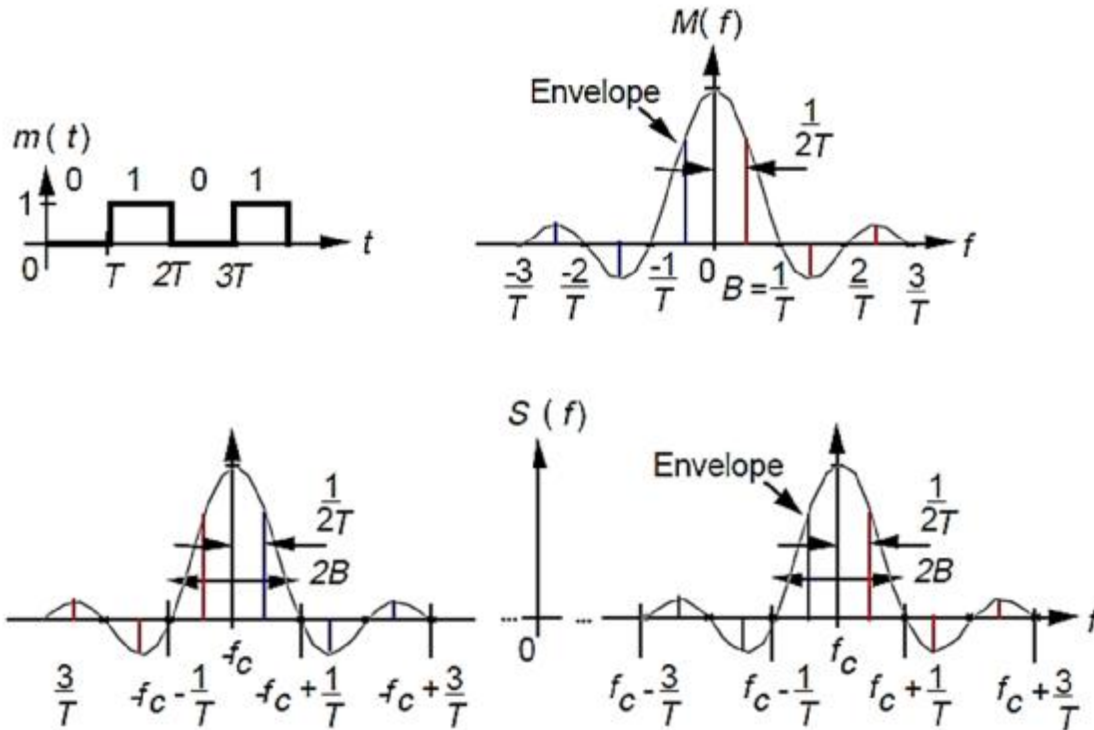


The Fourier transform of the BASK signal $s(t)$ is

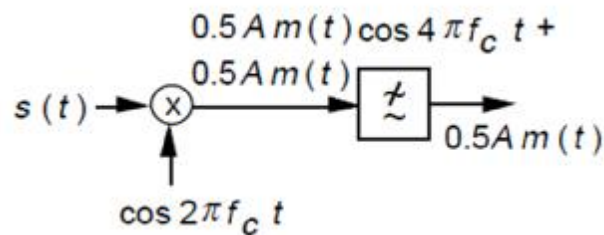
$$S(\omega) = \frac{A}{2} M(\omega - \omega_c) + \frac{A}{2} M(\omega + \omega_c)$$

The effect of multiplication by the carrier signal $A\cos(\omega_c t)$ is simply to shift the spectrum of the modulating signal $m(t)$ to ω_c

The following figure shows the amplitude spectrum of the BASK signals when $m(t)$ is a periodic pulse train.

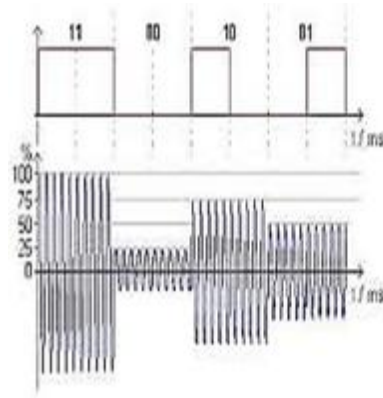


Since we define the bandwidth as the range occupied by the baseband signal $m(t)$ from 0 Hz to the first zero-crossing point, we have B Hz of bandwidth for the baseband signal and $2B$ Hz for the BASK signal. The following figure shows the coherent demodulator for BASK signals.



- **M-ary ASK**

If more than two levels are used, then an M-ary ASK is adopted for high bit rate (4ASK for 2bits, 8ASK for 3bits and so on). 4ASK is shown here:

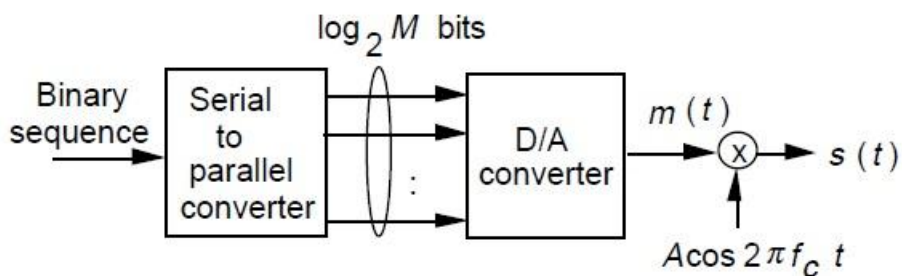


An M-ary amplitude-shift keying (M-ASK) signal can be defined by

$$s(t) = \begin{cases} A_i \cos(w_c t) , & 0 < t < T \\ 0 & , \text{ elsewhere} \end{cases}$$

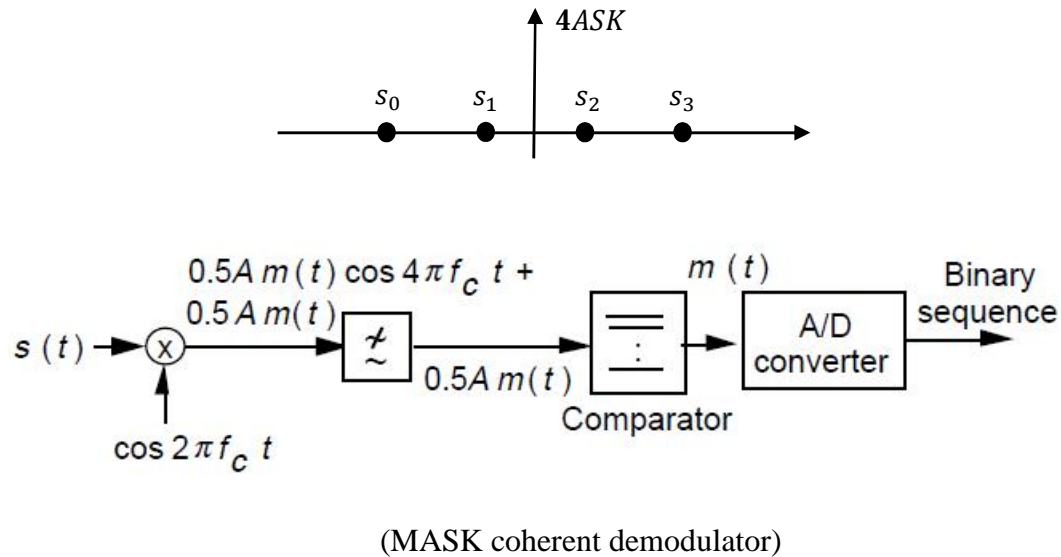
Where $A_i = A[2i - (M - 1)]$, ... for $i = 0, 1 \dots M - 1$

$$s(t) = \sqrt{2P_i} \cos w_c t = \sqrt{P_i T} \sqrt{\frac{2}{T}} \cos w_c t = \sqrt{E_i} \sqrt{\frac{2}{T}} \cos w_c t$$



MASK Modulator

Here is a 4ASK signal constellation diagram:



- *Bandwidth efficiency and Capacity of ASK*

$$\boxed{\text{Bandwidth efficiency} = n \dots \text{bit/sec/Hz}}$$

$$\boxed{C_{ASK} = nB \dots \text{bits/sec}}$$

Example.1

ASK is used for transmitted data at $R = 28.8$ Kbps over a telephone channel with bandwidth $B = 300\text{Hz to } 3400\text{Hz}$

- How many symbol states are required in order to achieve this level of performance?
- What would be the equivalent number of symbol states needed if the channel pass band extended from 0 Hz to 3100 Hz and baseband M-ary was used?
- What is the maximum capacity for the ASK if the SNR on the telephone link is 33 dB.

Solution:

- The capacity of band pass ASK is

$$C_{ASK} = B \cdot n, \quad B = 3400 - 300 = 3100, \quad n = \log_2 M$$

$$28800 = 3100 \log_2 M \rightarrow M = 626.1 \text{ or } M \approx 1024 \text{ states}$$

- The capacity of baseband M-ary system is

$$C_{baseband} = 2Bn, \quad B = 3100$$

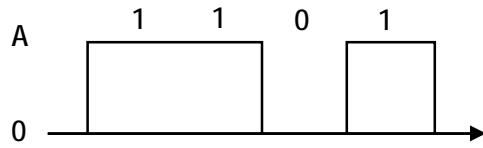
$$28800 = 2(3100)\log_2 M \rightarrow M = 25.02 \text{ or } M \approx 32 \text{ states}$$

c. Shannon capacity is

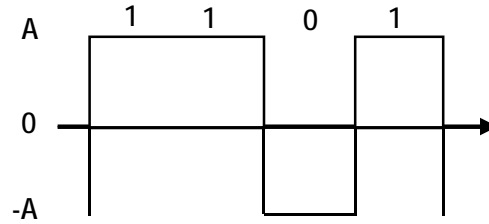
$$C = B \log_2(1 + SNR) = (3400 - 300)\log_2(1 + 10^{3.3}) = 33.996 \text{ Kbps}$$

- **Probability of Symbol error P_s**

There are two types of waveform used in digital communications: unipolar and bipolar waveforms



Unipolar



Bipolar

For unipolar waveform, the energy per symbol E_s is different depending on whether a logic 0 or 1 is sent, having a zero value for logic 0 case. The Probability of symbol error for unipolar is:

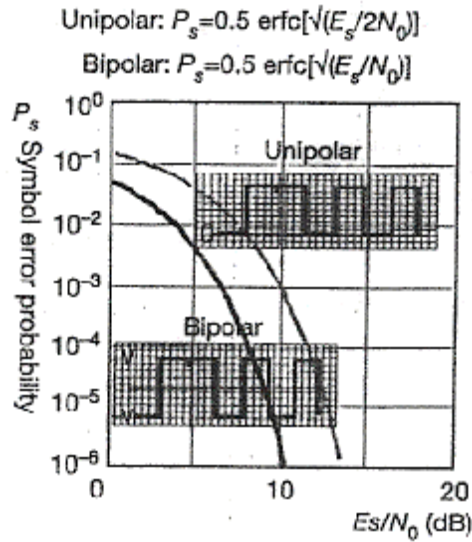
$$P_s = \frac{1}{2} \operatorname{erfc} \left[\sqrt{\frac{E_s}{2N_0}} \right] \dots \text{for unipolar}$$

Where N_0 is noise power density, and $\operatorname{erfc}(x)$ is the complementary error function:

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{-\infty}^{-x} e^{-y^2} dy$$

And the Probability of symbol error for bipolar waveform is

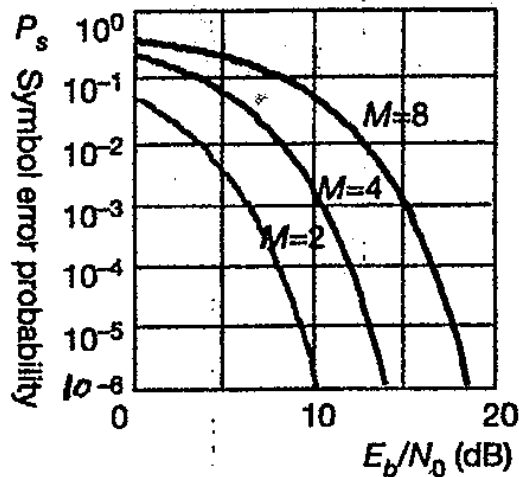
$$P_s = \frac{1}{2} \operatorname{erfc} \left[\sqrt{\frac{E_s}{N_0}} \right] \dots \text{for Bipolar}$$



It is often to draw $\frac{E_b}{N_0}$ which is called SNR rather than $\frac{E_s}{N_0}$ where $E_b = E_s/n$ and n is the number of bits per symbol.

When the number of levels is increased ($M > 2$), the ability of the receiver to distinguish between symbols in the presence of noise will decrease. The P_s for M -ary bipolar baseband signaling is:

$$P_s = \frac{M-1}{M} \operatorname{erfc} \left[\sqrt{\frac{3}{M^2-1}} \sqrt{\frac{E_s}{N_0}} \right]$$



Example.2

A company wishes to increase the through put of a telephone modem product by changing from 2-level signaling to 8-level signaling and has set a design target of maintaining a performance of no worse than one symbol error in every 10 000 symbols sent. By using the plot of symbol error

vs. E_b/N_0 for M-ary, determine the reduction in noise tolerance for the modem because of this change. What is the theoretical minimum E_b/N_0 required supporting the bandwidth efficiency achievable by the 8-level modem?

Solution:

From the plot of P_s for M-ary signaling, at $P_s=10^{-4}$, it can be seen that an increase of about 8 dB is required to maintain the same error rate. Therefore, the new modem will be approximately 8 dB less tolerant to noise.

For 8-level modem, maximum bandwidth efficiency is 6 bits/sec/Hz, so

$$\frac{C}{B} = \log_2 \left[1 + \frac{C}{B} \frac{E_b}{N_0} \right]$$

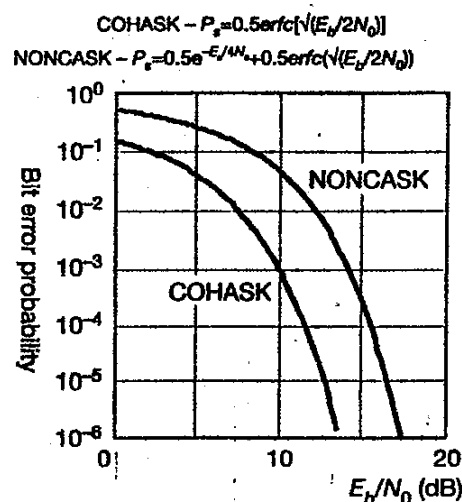
$$6 = \log_2 \left[1 + 6 \frac{E_b}{N_0} \right]$$

Therefore the minimum $\frac{E_b}{N_0}$ for error-free transmission is

$$\left(\frac{E_b}{N_0} \right)_{\min} = \frac{2^6 - 1}{6} = 10.5 \text{ or } 10.2 \text{ dB}$$

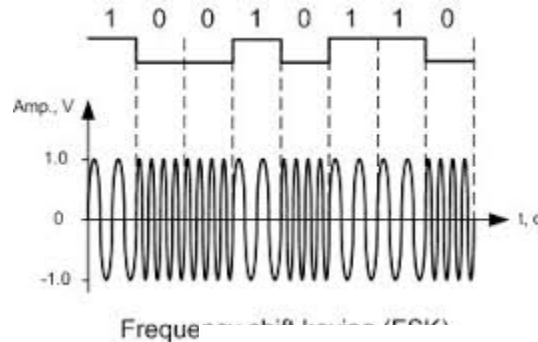
• Bit-error rate (BER) performances of ASK

The performance of digital communication systems is presented at the simplest level as a probability of bit error P_b or probability of symbol error P_s , as a function of the received E_b/N_0 ratio. Binary ASK effectively uses a unipolar baseband modulation source.



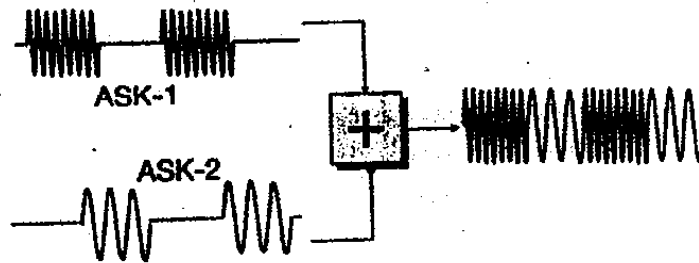
2. Frequency Shift keying (FSK)

FSK has until recent years been the most widely used form of digital modulation, being simple both to generate and to detect, and being insensitive to amplitude fluctuations in the channel. FSK conveys the data using distinct carrier frequencies to represent symbol states. ? *An important property of FSK is that the amplitude of the modulated wave is constant.*



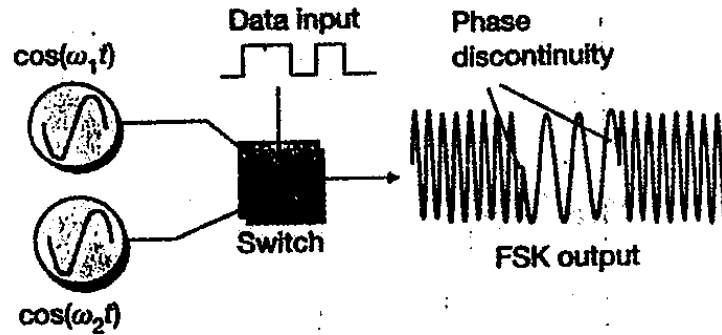
$$s(t)_{2FSK} = \begin{cases} \cos(w_1 t) , & \text{for bit 1} \\ \cos(w_2 t) , & \text{for bit 0} \end{cases}$$

Consider the case of unfiltered 2FSK. This waveform can be viewed as two separate ASK symbol streams summed prior to transmission.

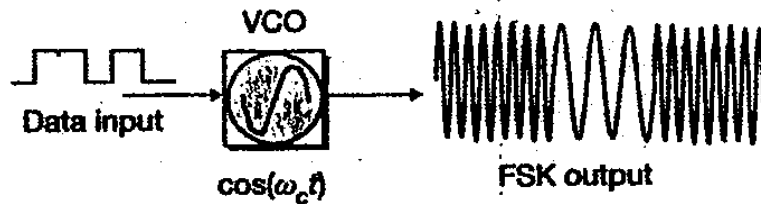


- **BFSK generation**

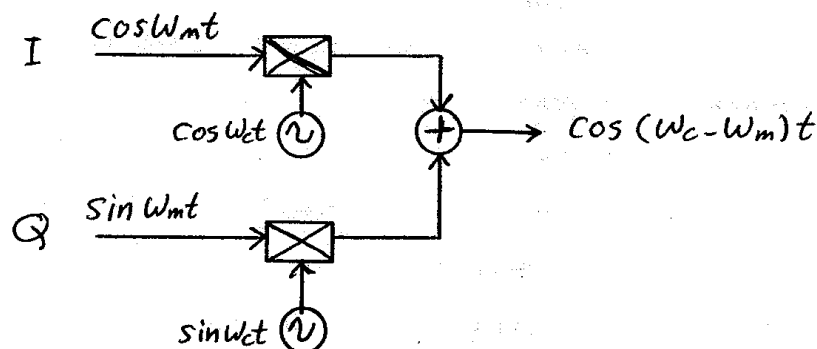
FSK can be generated by switching between distinct frequency sources; however, it is likely that there will be discrete phase jumps between the symbol states at the switching time. Any phase discontinuity at the symbol boundary will result in much greater prominence of high frequency terms in the spectrum, implying a wider bandwidth for transmission.



Alternatively, FSK can be realized by applying the data signal as a control voltage to a *voltage-controlled oscillator (VCO)*. Here the phase transition between consecutive symbols states is guaranteed to be smooth (continuous). FSK with no phase discontinuity between symbols is known as a *continuous phase (CPFSK)*.



- *The vector modulator*



The arrangement of mixer and a combiner forms an extremely useful building block in digital communication systems. It achieves a linear frequency translation of all components in the input signal (represented by its in-phase and quadrature components) by a carrier frequency component (also represented by its in-phase and quadrature components). This block is often referred to as a *vector modulator* or *quadrature modulator*, and can be used for both frequency up-conversion and down-conversion. The output of the two mixing processes is given by

$$\cos w_m t \cdot \cos w_c t = \frac{1}{2} [\cos(w_c - w_m) t + \cos(w_c + w_m) t]$$

$$\sin w_m t \cdot \sin w_c t = \frac{1}{2} [\cos(w_c - w_m) t - \cos(w_c + w_m) t]$$

When the above terms are summed, the result gives a down-converted component:

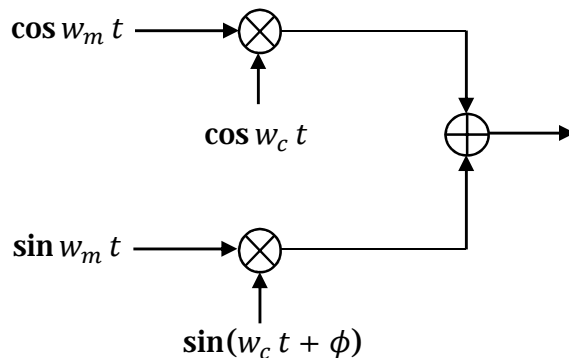
$$\cos(w_c - w_m) t$$

In addition, when subtracted from each other result in a signal up-converted component:

$$\cos(w_c + w_m) t$$

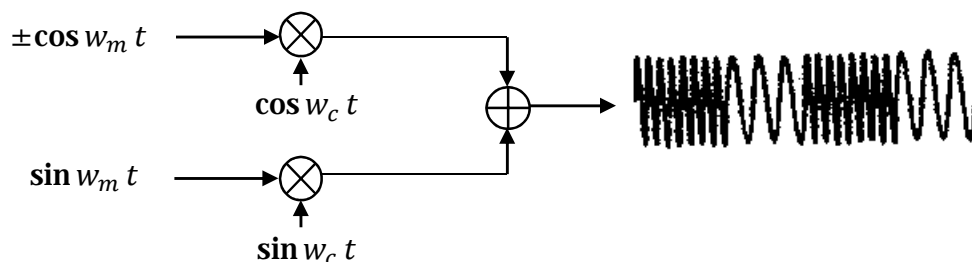
? Exercise.1

A vector modulator is fed with a perfect quadrature sine wave at the input, but there is a small phase error of 5° between the notional quadrature inputs of the carrier signal. What will be the ratio in dB between the sum and difference outputs of the vector modulator (ratio of the amplitude of the wanted to unwanted output signal)? [\pm Hint: $\sin \phi \approx \phi$ for small ϕ], [RCheck answer: 27 dB]



• **BFSK Vector modulator**

BFSK requires the generation of two symbols, one at a frequency $(w_c + w_m)$ and one at a frequency $(w_c - w_m)$. So to generate a shift of $(+w_m)$, I and Q inputs need to be fed with $(-\cos w_m t)$ and $(\sin w_m t)$ respectively. Generating a shift of $(-w_m)$ requires inputs of $\cos w_m t$ and $\sin w_m t$.



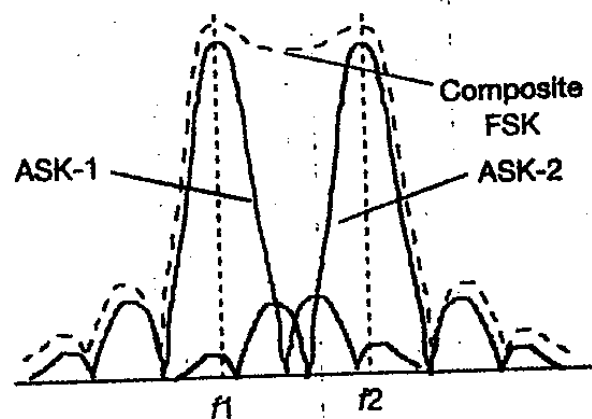
This approach is now frequently used to generate filtered CPFSK particularly in cellular handsets.

? Exercise.2

Draw the block diagram of vector modulator to generate 2ASK signal.

- *Spectrum of BFSK*

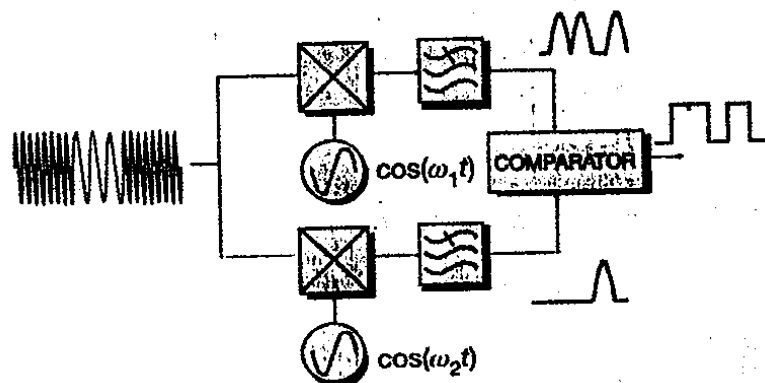
An approximation of BFSK spectrum can be obtained by plotting the spectra for two ASK streams centered on the respective carrier frequencies.



Clearly, the overall bandwidth occupied by FSK signal depends on the separation between the frequencies representing the symbol states. CPFSK system will have much lower side-lobe energy than the discontinuous case.

- *Coherent BFSK detection*

This method is very similar to that for ASK but in this case there are two detectors tuned to the two carrier frequencies.



- ***M-ary FSK system***

M-ary FSK (multi-level) is very much of interest for increasing the noise immunity of the modulation format compared with BFSK, allowing a designer to achieve reliable data transmission in the presence of high levels of noise. This is only possible by using a set of “orthogonal symbols”. Two symbol states $a_i(t)$ and $a_j(t)$ are said to be orthogonal over the symbol period T_s if:

$$\int_0^{T_s} a_i(t) \cdot a_j(t) \cdot dt \xrightarrow{i \neq j} 0$$

If the frequencies of M-FSK symbols are chosen to be of the form:

$$a(t) = \cos \left[\omega_c t + \frac{2\pi k t}{2T_s} \right] \text{ for } k = 1, 2 \dots (M - 1)$$

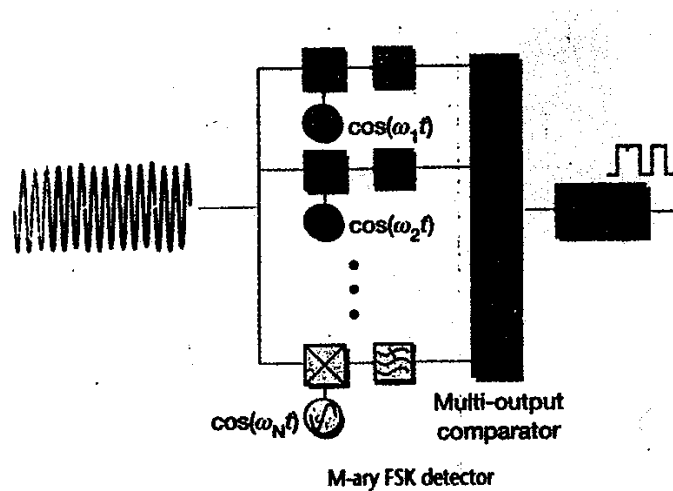
Then these frequencies are orthogonal over a symbol period.

Example.3

For 8-FSK and $R_s=1200$, the required frequencies are 1000,1600,2200,2800,3400,4000,4600 and 5200 Hz.

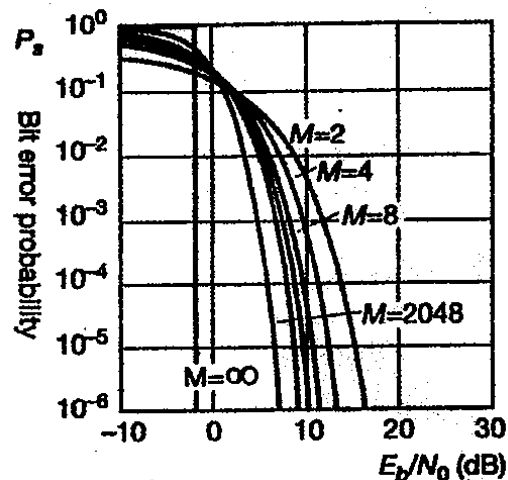
? Orthogonal system gives *better SNR at detector output, improving the probability of correct symbol detection* but *required high bandwidth*.

- ***M-ary FSK detection***



A typical M-ary FSK detector consists of a bank of “correlators” (mixers with coherent carrier reference), followed by a decision circuit at the output determining which correlator has the largest output and hence which symbol was sent.

- **BER performance for M-ary FSK**



? As the number of symbol states is increased, the BER improves but at the expense of BW.

- **Advantage of FSK**

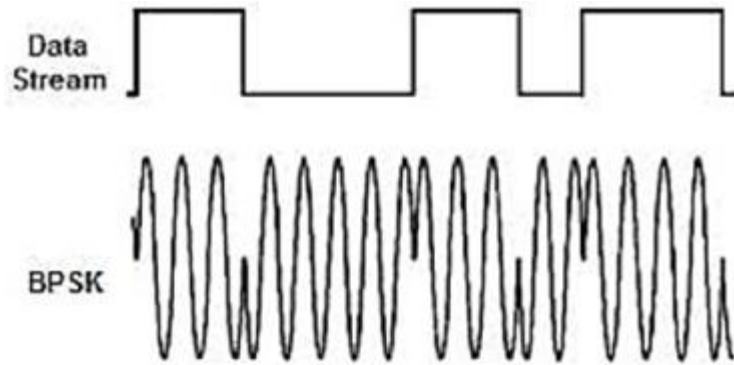
- ✓ FSK is constant envelope modulation and hence insensitive to amplitude variations in the channel.
- ✓ The detection of FSK is based on relative frequency changes between symbol states and thus does not require absolute frequency accuracy in the channel.
- ✓ In deep space missions where the path loss is so great, M-ary FSK is very effective modulation.

- **Disadvantage of FSK**

- ✓ FSK is less bandwidth efficient than ASK or PSK
- ✓ The bit/symbol error rate performance of FSK is worse than for PSK.

3. Phase shift keying (PSK)

With PSK, the information is contained in the instantaneous phase of the modulated carrier. Usually this phase is imposed and measured with respect to fixed carrier of known phase-coherent PSK. For binary PSK (2PSK), phase states of 0° and 180° are used. It is also possible to transmit data encoded as the phase change (phase difference) between consecutive symbols (Differentially coherent PSK). There is no non-coherent detection for PSK.



For BPSK:

$$S_1 = \sqrt{\frac{2E_s}{T}} \cos(w_c t + 0) \quad \dots \text{for bit 0}$$

$$S_2 = \sqrt{\frac{2E_s}{T}} \cos(w_c t + \pi) \quad \dots \text{for bit 1}$$

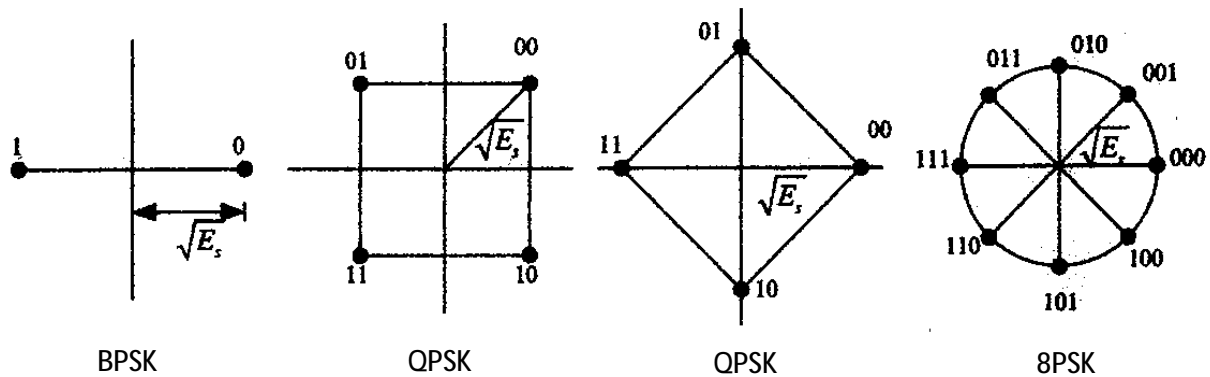
Where E_s is energy per symbol, T is symbol time and $\sqrt{\frac{2E_s}{T}}$ is the amplitude (A) of the signal.

In general, for MPSK:

$$S_i = \sqrt{\frac{2E_s}{T}} \cos\left(w_c t + \frac{2\pi i}{M}\right) \quad \dots i = 0, 1, \dots, (M - 1)$$

Where $\frac{2\pi i}{M} = \theta_i$ is the modulation angle.

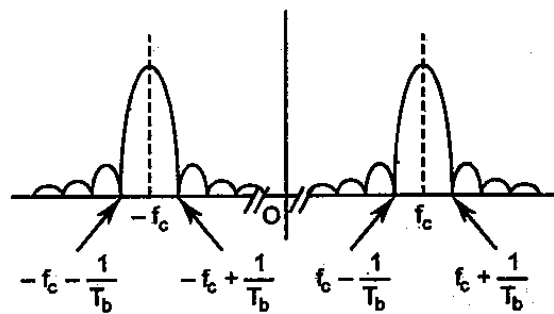
The constellation mapping for MPSK can be shown below



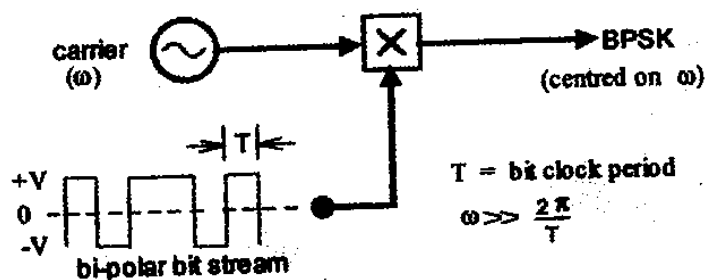
? QPSK is quadrature PSK (4PSK).

- BPSK spectrum**

The bandwidth of BPSK signal is identical to that of BASK. In fact, BPSK can be viewed as ASK signal with the carrier amplitudes as $+A$ and $-A$ (rather than $+A$ and 0 for ASK).

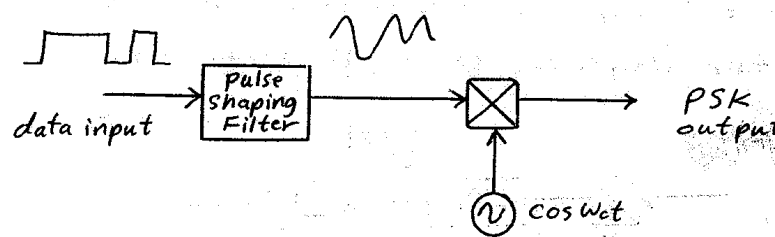


- PSK generation**



The simplest means of realizing unfiltered BPSK is to switch the sign of the carrier using the data signal, causing 0° or 180° phase shift.

? The square pulses for data signal are not practical to send. *They are hard to create and required a lot of bandwidth.* The solution here is to send shaped pulses that convey the same information but use smaller bandwidth and have other good properties such as ISI rejection.

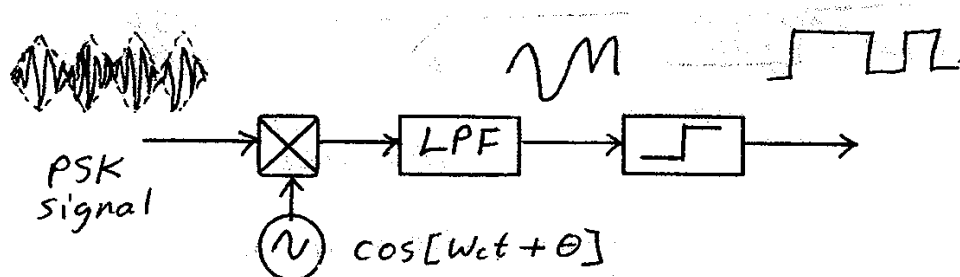


There are some common pulse shaping methods that control the shape and the bandwidth of the signal:

- ✓ Root raised cosine (used with QPSK)
- ✓ Half sinusoid (used with MSK (minimum shift keying))
- ✓ Gaussian (used with GMSK. This system is used in several mobile systems around the world such as in GSM (global special mobile))
- ✓ Quadrature partial response (QPR)

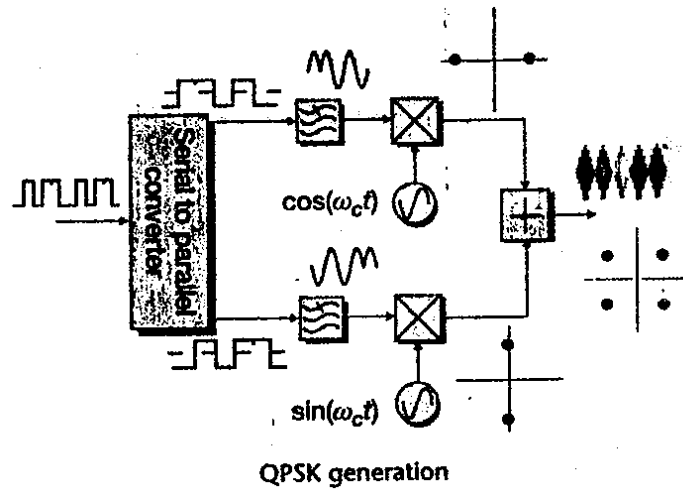
• Detection of BPSK

There is no non-coherent detection for PSK, and various forms of coherent detection must be employed. The ideal detector thus requires perfect knowledge of the unmodulated carrier phase at the receiver (carrier recovery). As with ASK, any phase error θ of the locally generated carrier reference reduces the signal level at the output of the detector by $\cos \theta$. This in turn degrades the E_s/N_0 performance.



Thus, we need zero phase error for optimum detection. Note that if the phase error θ reaches 90° , the output falls to zero.

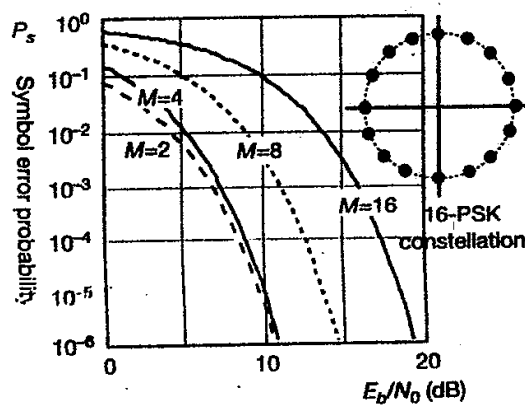
- **Quadrature phase-shift keying (QPSK)**



QPSK uses the orthogonality between cosine and sine carrier. This would imply that if we send BPSK on the cosine of a carrier, and simultaneously send a second BPSK using the sine of a carrier, then it would be possible to detect each one independently of the other. *Orthogonality property of QPSK means that it can be used to send information at twice the speed of BPSK in the same bandwidth.* The block diagram of QPSK modulator is simply two BPSK using quadrature carriers summed in parallel. The source data is first split into two data streams, with each data stream running at half the rate of input data.

- **Performance of MPSK**

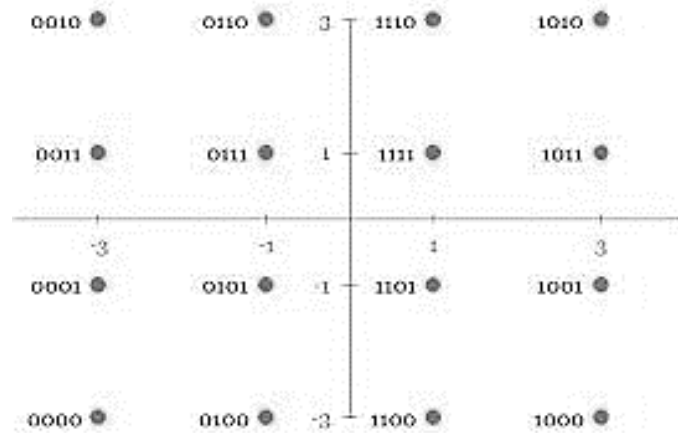
Increasing M allows further improvements in bandwidth efficiency but requires more $\frac{E_b}{N_0}$ for same P_s .



$$\text{Max bandwidth efficiency}_{MPSK} = \log_2 M \dots \text{bit/sec/Hz}$$

4. Quadrature Amplitude Modulation (QAM)

So far, we have considered only signal property modulators using amplitude, frequency, or phase symbols to conveying the data. We can combine two or more symbols types, which gives improved performance (trade-off between bandwidth efficiency and noise performance).



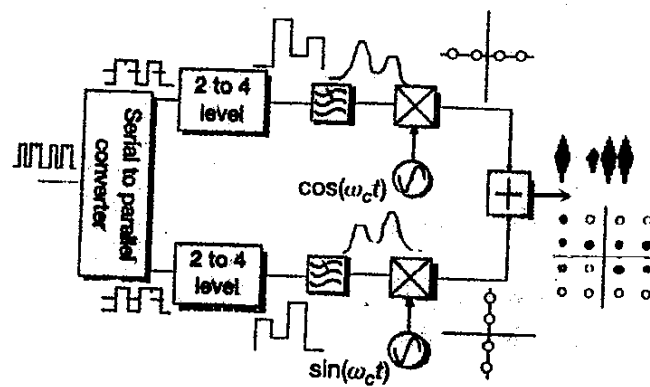
16-QAM constellation

The simplest form of QAM is in fact the QPSK symbol set, which Can be viewed as two quadrature amplitude modulated carriers, with amplitude levels of $+A$ and $-A$.

Increasing the number of amplitude levels on each carrier to 4 (for example $\pm A, \pm 3A$) gives 16 possible combinations of symbols at the output, each equally spaced on the constellation diagram, and each represented by a unique amplitude and phase.

- ***QAM generation***

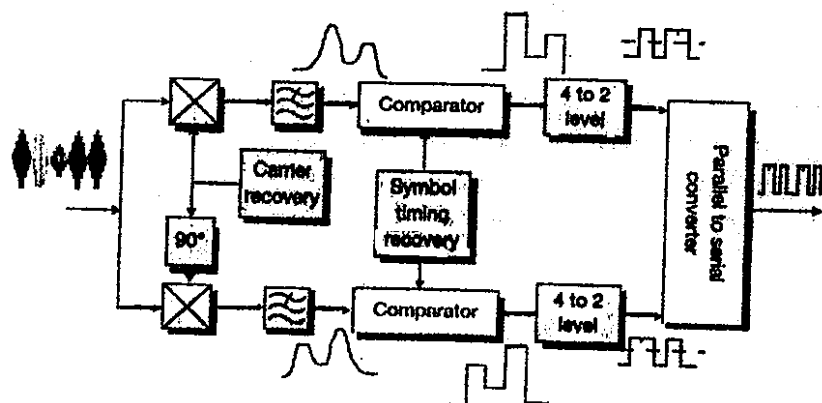
The modulator is making use of orthogonality of the sine and cosine carriers to allow independent detection of the two ASK data.



16-QAM generation

Pulse shaping is performed by filtering the multi-level baseband input symbol streams as in ASK.

- **QAM detection**

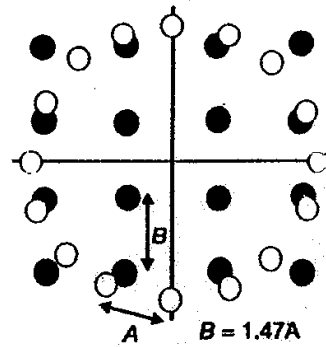


16-QAM detection

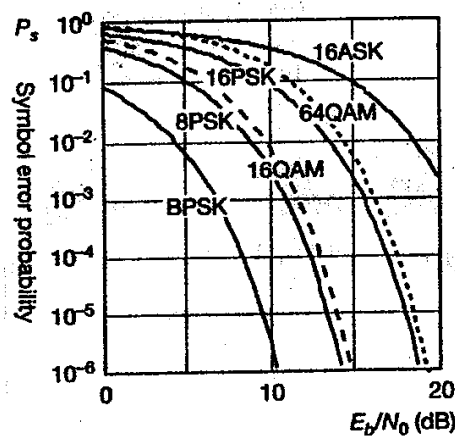
QAM can be decoded using coherent detection just as for PSK (requires carrier recovery). The output of each demodulator is a baseband multi-level symbol set; this should undergo matched filtering for optimum performance in noise. The aim of comparator is to determine the level at the sampling instant, and hence decode the corresponding bit pattern.

- **M-ary QAM vs. M-ary PSK**

Comparing the constellation diagrams of 16 QAM with 16 PSK, we can see that the spacing between symbol states for QAM is greater than that for PSK, which means that the detection process in QAM should be less susceptible to noise. However, the power for QAM is greater than that for PSK and this must be taken into account if the transmission process is power limited.



Comparison of 16-PSK and 16-QAM for equal average symbol power



Comparison of M-ary data systems

Example.4

A digital TV has a source analogue video signal with BW from 0 Hz to 2 MHz. This signal is sampled at four times the highest frequency using 16-bit ADC. The resulting data signal is sent over the air using 16QAM modulation. Assume ideal pulse-shaping filter, what is the bandwidth occupied by the transmitted digital video signal?

Solution:

Sampling rate at ADC = $4f_h = 4(2) \text{ MHz} = 8 \text{ Msamples/sec}$

Bit rate at ADC output = 16 bits (8M) = 128 Mbps

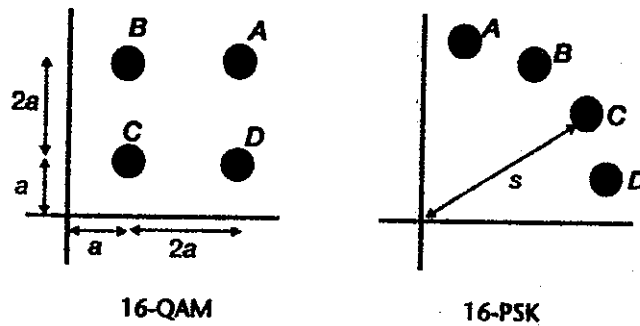
16 QAM uses $n = 4 \text{ bits per sample}$, so $\frac{C}{B} = 4 \text{ bit/sec/Hz}$

Hence $B = \frac{128}{4} = 32 \text{ MHz}$

Example.5

A transmitter for digital radio system is peak power limited to 150 W with $50\ \Omega$ antenna. Determine the average power that can be supported for both 16 PSK and 16 QAM transmission if each point in the constellation has an equal probability of transmission.

Solution:



With reference to one quadrant of the 16 QAM constellation, the average power developed by each of the vectors A, B, C, D is as follows:

$$A^2 = (3a)^2 + (3a)^2 = 18a^2, \quad B^2 = (3a)^2 + (a)^2 = 10a^2$$

$$C^2 = (a)^2 + (a)^2 = 2a^2, \quad D^2 = (3a)^2 + (a)^2 = 10a^2$$

$$\text{Average power} = \frac{(18a^2 + 10a^2 + 10a^2 + 2a^2)/4}{R} = \frac{10a^2}{R}$$

$$\text{The maximum vector power is 150 w, so } \frac{A^2}{R} = 150 = \frac{18a^2}{R} = \frac{18a^2}{50} \rightarrow a = \sqrt{\frac{150 \times 50}{18}} = 20.4$$

$$\text{The average power for all symbol states is: } P_{av}(\text{QAM}) = \frac{10a^2}{R} = 83.33 \text{ w}$$

$$P_{av} \text{ for 16 PSK is the same for all symbol states } P_{av}(\text{PSK}) = \frac{S^2}{R} = 150 \text{ w}$$

? Exercise.3

If the maximum vector length in 16 QAM is 100 v rms, determine the average power that would be delivered into $R=50\ \Omega$ antenna load if each point in the constellation has an equal probability of transmission. [**R** Check answer: $P_{av} = 111 \text{ w}$]

? Exercise.4

If the peak symbol power for 16QAM is 200 w, measured in $R=50\ \Omega$ antenna load. What are the amplitudes of the different symbol vectors in the transmitted waveforms?

? Exercise.5

Orthogonal 4FSK modem has $R_s = 2400 \text{ symbols/sec}$. If the lowest symbol frequency is 8 kHz, what will be the other three symbol frequencies?

? Exercise.6

64QAM data link operates at 256 kbps. What is the symbol rate on the channel, and what is the occupied bandwidth?

? Exercise.7

What is the minimum bandwidth required to support 256 kbps data stream using BPSK, QPAK, and 64QAM?

? Exercise.8

A customer requires a microwave radio link to provide a bit rate of 2 Mbps in a bandwidth of 400 kHz. The minimum SNR on the channel is 30 dB. Can the channel support the required capacity? Moreover, how many symbol states would be required?