**Al-safwa University College**
**Department of Computer Technology Engineering**
**Fourth Class / Project Management**

# Introduction to Projects Management

**Lec. 1**

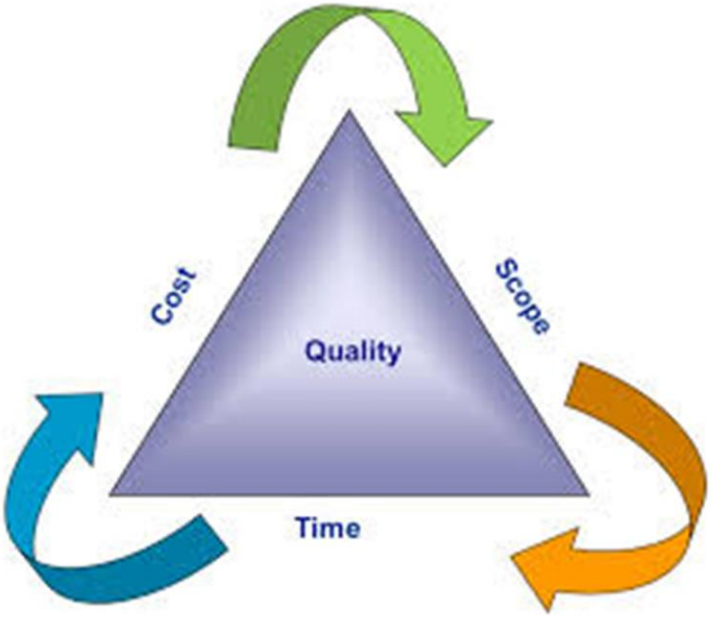
**Huda majeed**

# Introduction to Project Management

Projects are considered one of the means by which organizations and societies develop in a way that achieves balance, regardless of whether they are service or profit projects. With the aim of achieving comprehensive development and upgrading the capabilities of organizations and individuals, bearing in mind that project workers must possess basic skills; For choosing the project, managing it, in addition to having the information related to the project goal, how to plan it, its phases, organizing its elements, and evaluating the activities related to it. It is worth noting that project management is of great importance. Where it contributes to converting ideas into practical reality that would enhance the means of management, activate the optimal use of resources, and split profit and service activities.

# What is a project

- A group of milestones or phases, activities or tasks that support an effort to accomplish something.
- It has a definable goal
- Has interrelated and dependent activities tasks
- Is finite in duration
- Provides a unique product , service , or information
- Operates under scope , quality ,cost and time constraints


Cost
Scope
Quality
Time

# Definition of Project Management

- A Project management is the use of knowledge, skills, tools, and techniques to plan and implement activities to meet or exceed stakeholder needs and expectations from a project.

# Project manger

- The person assigned by the performing organization to achieve the project objectives
- Project manger should has
- (1) technical skills
- (2) project management process skills
- (3) Leadership skills

# Elements of management

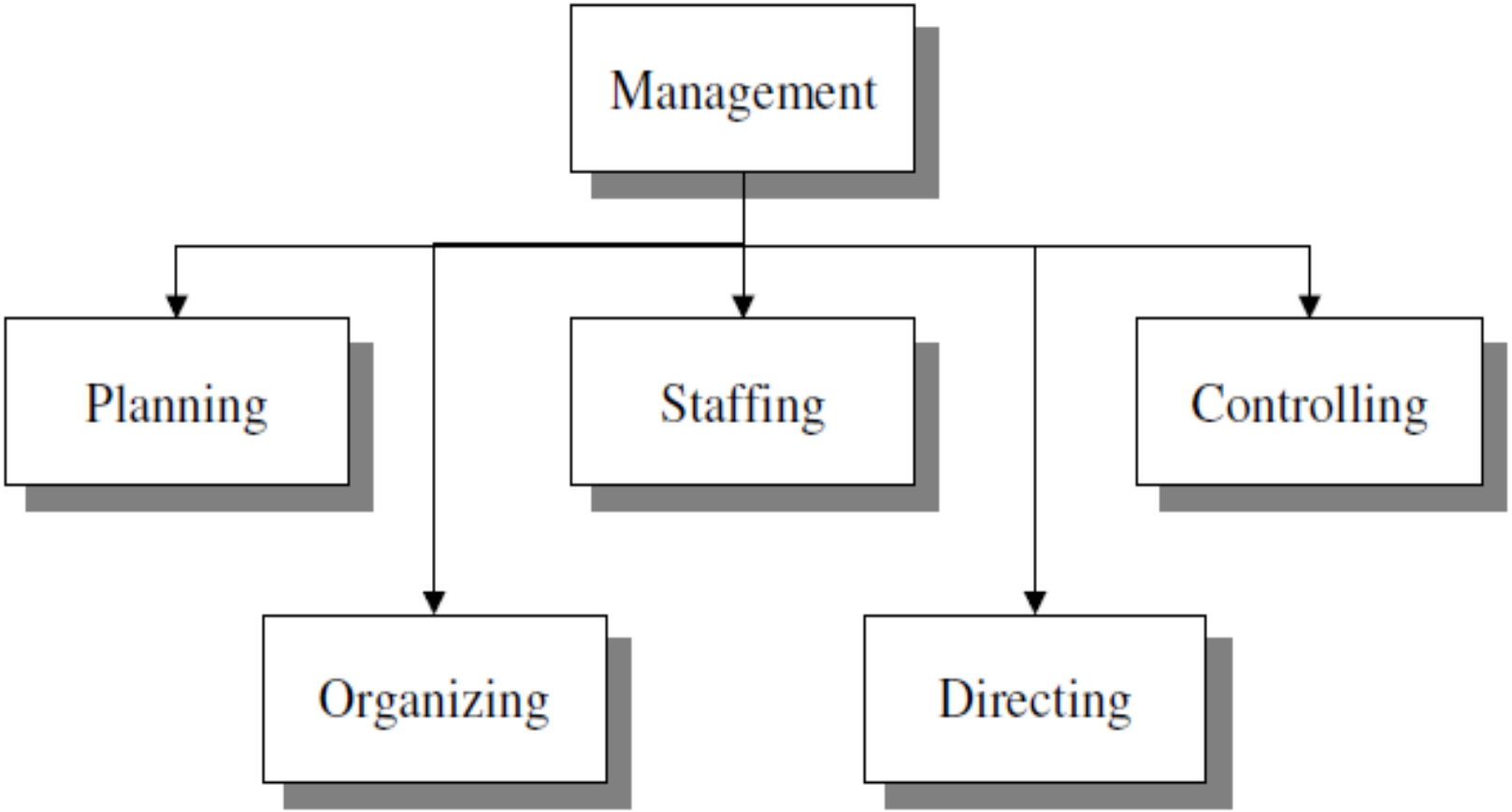
1. **Planning**
2. **Organizing**
3. **Staffing**
4. **Directing** (or **Leading**)
5. **Controlling**

- **1. Planning**
- The process of making plans for creating a product or anything the organization wants.
- Adequate planning leads to the correct completion of work. Inadequate planning leads to frustration towards the end of the project & poor project performance
- **2. Organizing involves:**
- a. Establishing a structure to be filled by people, aimed at reaching the defined goals and objectives.
- b. Defining job content, interfaces, responsibilities, authority, and resource allocation

- **3. Staffing involves:**
- a. Filling the positions in the organizational structure with suitable people.
- b. Keeping the positions filled, in order to execute the plan.
- **4. Directing (or Leading) involves:**
- a. Creating an environment in which individuals, working together in groups, can accomplish well-selected aims.
- b. Influencing people to contribute to reaching the goals and objectives.
- c. Using leadership styles, communication, conflict resolution, delegation, etc. in order to overcome the problems arising from people issues (attitudes, desires, motivations, behavior in groups, etc.) on a project.

- **5. Controlling (and co-ordination) involves:**
- a. Measuring actual performance.
- b. Comparing actual- with desired results and implementing corrective actions – e.g. by controlling the actions of the people doing the work

# Block diagram of elements of management



# Objective of project management

- Project management aims at a set of objectives that can be summarized as follows:
- 1. Reducing its administrative risks to the least.
- 2. Obtaining the required and appropriate resources for the project.
- 3. Building a team or group of work teams to complete the project.
- 4. Preparing the project plan and gathering the necessary resources to accomplish its functions.
- 5. Monitoring and control of work.
- 6. Submit periodic reports to the higher management or those responsible for the project on the progress of work on it.
- 7. Close the project upon its completion.

# Implementation of project management technique can have significant results

- 1. Cost reduction
- 2. Time reduction
- 3. Recourses allocation
- 4. Increased quality

# Steps/Phases of Project Management

1. **Project initiation.**
2. **Project planning.**
3. **Project scheduling.**
4. **Project costing.**
5. **Project control.**
6. **Project termination/evaluation.**

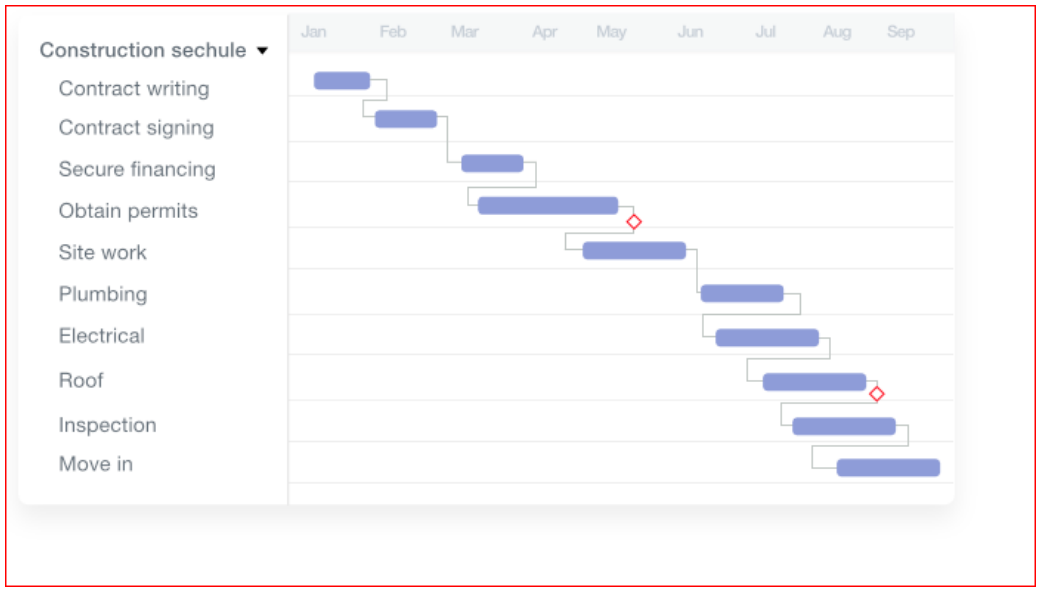
## Project Initiation

- −Concept definition, which includes identification and selection of opportunities and identification of objectives
- - Feasibility study and justification

## Project Planning

- – Scope definition
- - Project requirements - definition of deliverables
- - Project objectives - definition of major work efforts, quantifiable
- - Analysis & break down of project into smaller pieces of work
- - Development of checklist of everything that needs to be done
- - Team building
- - Selection of project manager
- - Selection of team members,
- - Use resource matrix to match skills task requirements

- **Project Scheduling**
- - Determining sequence of work
- - Building network / interdependence
- - Analysis of interdependence, estimation of total duration (CPM, PERT) and determination of Critical Path
- - Establish milestones
- - Graph on time chart (Gantt chart)
- - Determining human resource loading



- **Project Costing**
- - Estimate Costs, Capital / Operating
- - Develop Cost Spreadsheets
- **Project Control**
- - Done Periodically (At Milestones)
- - Time Control, Status, Deviations From Plan, Replanning, New Estimates
- - Cost Control, Expenditure, Deviations From Plan, New Estimates
- - Quality Control, Performance Versus Performance Criteria / Project Requirements

- **Project Termination / Evaluation**
- - Post project activity
- - Statistics from monitoring progress
- - Client feedback
- - Profitability or not of the project
- - Post implementation report

Thank you for listening

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# Network analysis

## Lec 2

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**Network Analysis** is a technique that is adopted in planning and controlling of unique and complex projects. It is a system of planning project outline by analyzing different activities associated with it. In network analysis, complex projects are broken down into smaller activities or tasks, which are then organized according to a sequence. Then, the order of tasks/activities is also decided according to a logical sequence.

**Critical Path Method** (CPM) or (Calculate Schedule) is a modeling process that defines all the project's critical activities which must be completed on time. CPM models the activities and events of a project as a network

# Steps in CPM

- 1. specify the individual activities.
- 2. Determine the sequence of those activities.
- 3. Draw a network diagram.
- 4. Estimate the completion time for each activity.
- 5. Identify the critical path (longest path through the network)
- 6. Update the CPM diagram as the project progresses.

# CPM Benefits

- Provides a graphical view of the project.
- Predicts the time required to complete the project.
- Shows which activities are critical to maintaining the schedule and which are not.

- **Critical path** is the longest-duration path through the network. The significance of the critical path is that the activities that lie on it cannot be delayed without delaying the project. Because of its impact on the entire project, critical path analysis is an important aspect of project planning.

# The critical path can be identified by determining the following four parameters for each activity

- Earliest Start time (ES): the earliest time at which the activity can start given that its. precedent activities must be completed first.
- Earliest Finish time (EF), equal to the earliest start time for the activity plus the time required completing the activity.
- Latest Finish time (LF): the latest time at which the activity can be completed without delaying the project.
- Latest Start time (LS), equal to the latest finish time minus the time required to complete the activity.

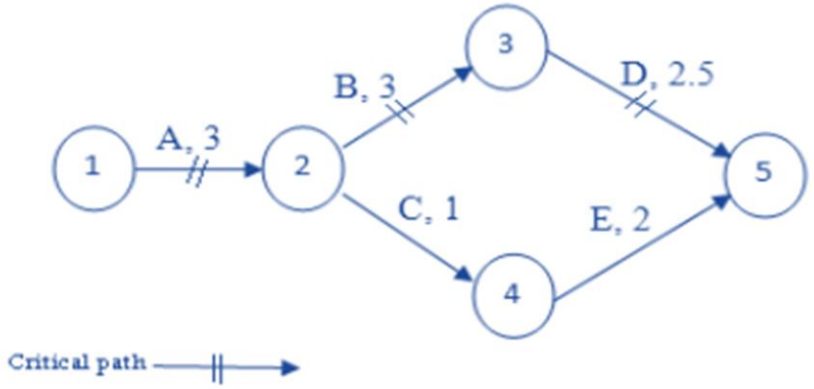
- **The slack time or Total float for an activity** is the time between its earliest and latest start time, or between its earliest and latest finish time. Slack is the amount of time that an activity can be delayed past its earliest start or earliest finish without delaying the project. The critical path is the path through the project network in which none of the activities have slack, that is, the path for which **ES=LS** and **EF=LF** for all activities in the path. A delay in the critical path delays the project. Similarly, to accelerate the project it is necessary to reduce the total time required for the activities in the critical path.

- **Activity** is an individual task needed for the completion of a project.
- **Duration** is the length of time (hours, days, weeks, months) needed to complete an activity.
- **Float** is the amount of time that an activity can slip past its duration without delaying the rest of the project.
- **Free float** is the excess time available before the start of the following activity

- **Example 1: Determine the critical path by using CPM of the following Table (project),**

| Activities | Path | Duration (day) | Description |
|---|---|---|---|
| A | 1 – 2 | 3 | وصف مختصر لكل فعالية |
| B | 2 – 3 | 3 | |
| C | 2 – 4 | 1 | |
| D | 3 – 5 | 2.5 | |
| E | 4 – 5 | 2 | |

Ans:

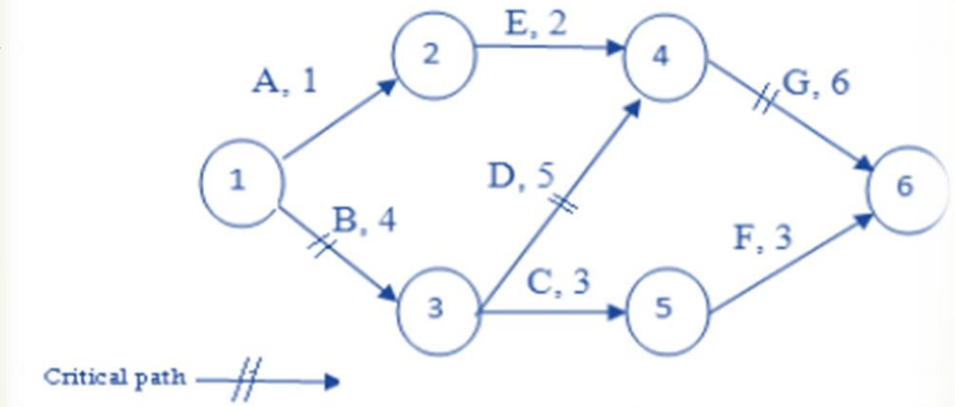



**Critical path** = 3+ 3+ 2.5= 8.5 days

- **Example 2 : Determine the critical path by using CPM of the following Table (project).**

| Activities | Path | Duration (week) | Description |
|---|---|---|---|
| A | 1 – 2 | 1 | وصف مختصر لكل فعالية |
| B | 1 – 3 | 4 | |
| C | 3 – 5 | 3 | |
| D | 3 – 4 | 5 | |
| E | 2 – 4 | 2 | |
| F | 5 – 6 | 3 | |
| G | 4 – 6 | 6 | |

**Ans:**




**Critical path=** 4+ 5+ 6= 14 weeks

**H.W.** Determine the critical path by using CPM of the following Table (project).

| Activities | Path | Duration (day) | Description |
|---|---|---|---|
| A | 1 – 2 | 2 | تحضير الموقع |
| B | 2 – 3 | 3 | تخطيط الموقع |
| C | 3 – 4 | 8 | انشاء الاعمدة |
| D | 3 – 5 | 5 | بناء الجدران الخارجية |
| E | 5 – 6 | 4 | بناء السقف |

# Thank you for Listening

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# Network analysis

## Lec 3

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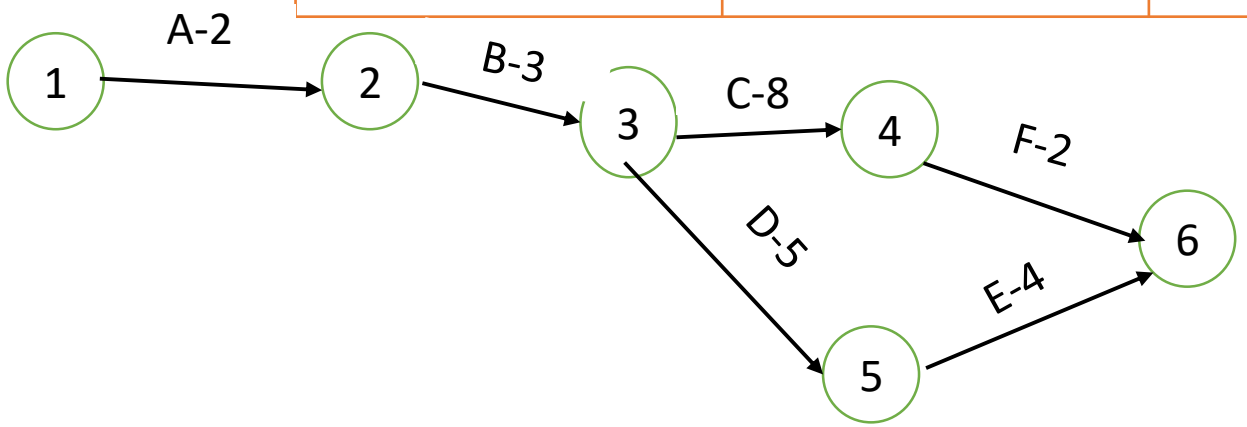
Ex: Determine the critical path by using CPM of the following table (project).

| activities | path | Duration (days) |
|---|---|---|
| A | 1-2 | 2 |
| B | 2-3 | 3 |
| C | 3-4 | 8 |
| D | 3-5 | 5 |
| E | 5-6 | 4 |
| F | 4-5 | 2 |




Path (1)=A-B-C-F=2+3+8+2=15

Path(2)=A-B-D-F=2+3+5+4=14 ,Critical path=A-B-C-F

# Six Steps PERT & CPM

- 1. Define the project and prepare the work breakdown structure.
- 2. Develop relationships among the activities - decide which activities must precede and which must follow others.
- 3. Draw the network connecting all of the activities.
- 4. Assign time and/or cost estimates to each activity.
- 5. Compute the longest time path through the network – this is called the critical. path.
- 6. Use the network to help plan, schedule, monitor, and control the project.

# • Activity-on-node (AON)

- Which type of network is better? Students often find AON networks a little easier to draw, and this can seem like a big advantage when doing it for the first time. However, some of the more advanced types of network analysis are best done using an AOA network. Every well educated student should be able to draw and analyze either, so that they are never thrown into confusion if they come across the one they haven't learnt to use. For people in employment, if your boss has a preference, then that's the one to use
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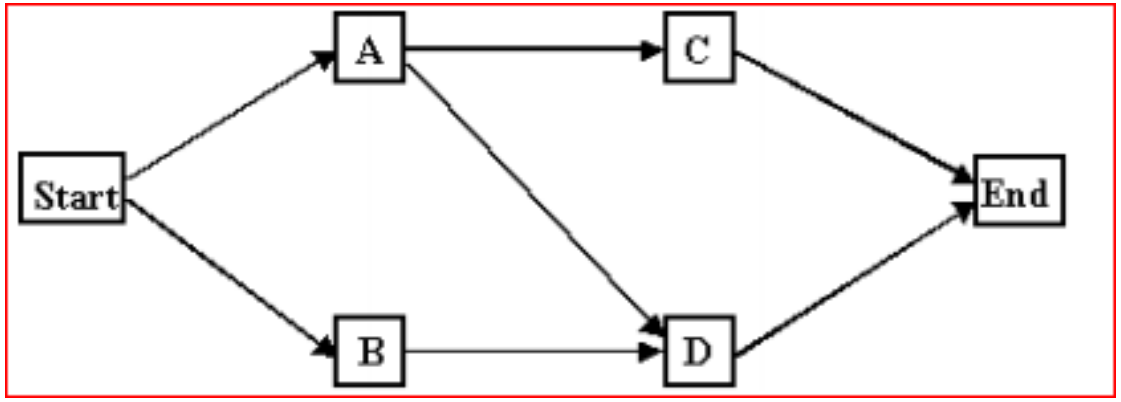
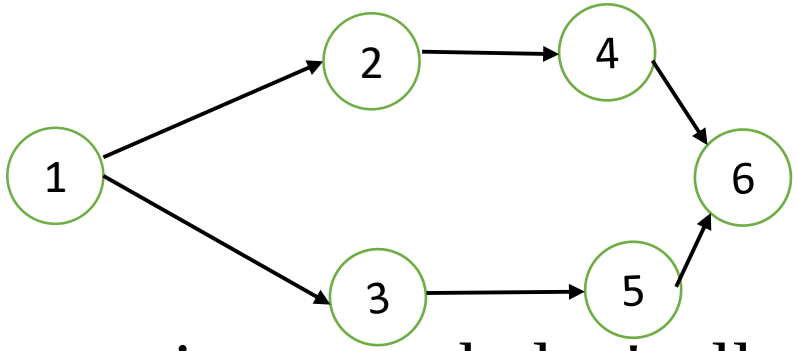
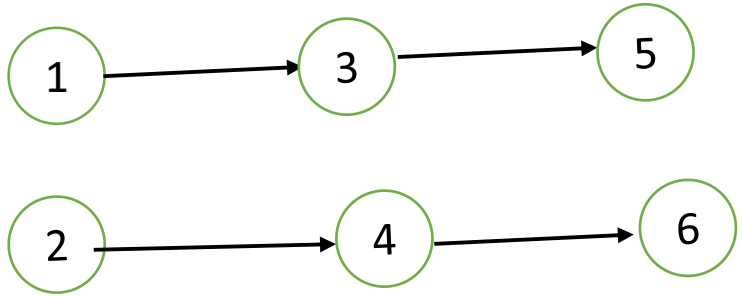
- t: Activity duration

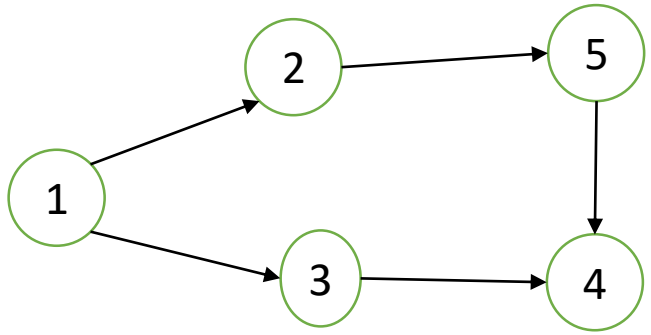


- Notes
- (1) the network should start in one activity only and end in one activity only

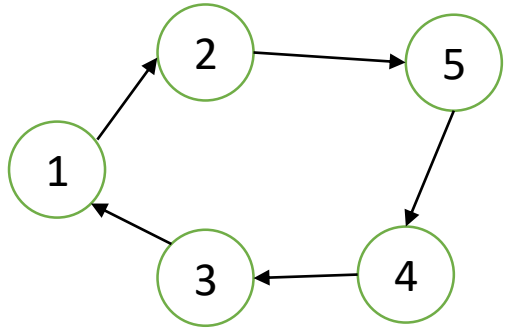



- (2) the rotate  in network don't allow

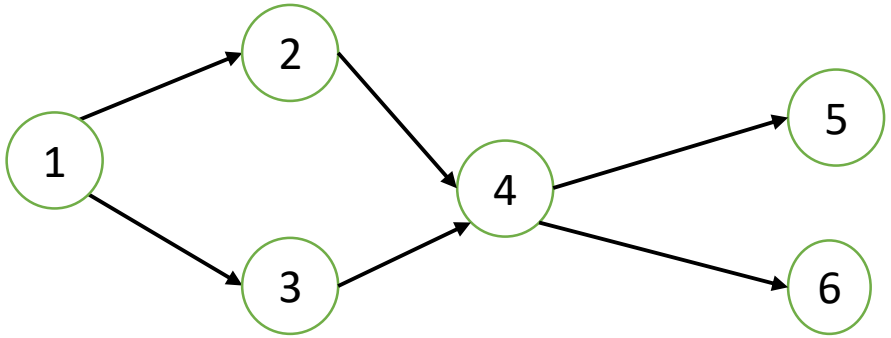


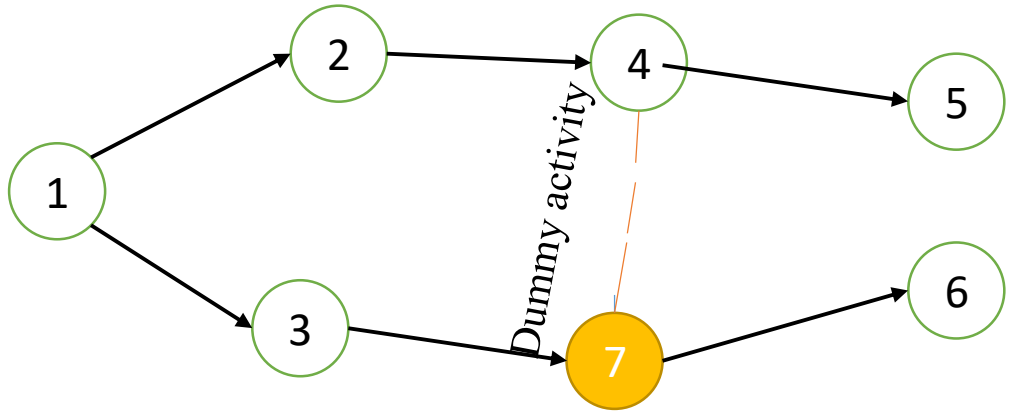

A




B

- (3) Two activity should not start and end in same time.

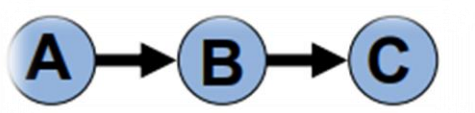


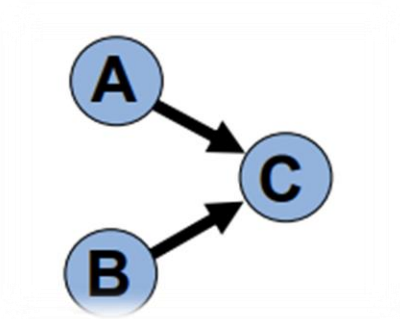



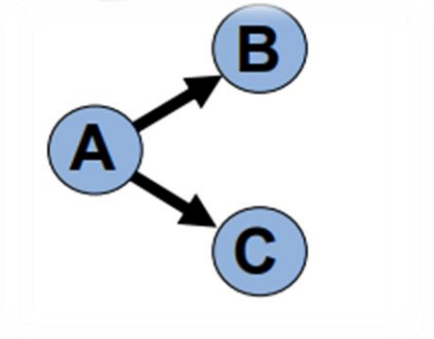
- **Dummy activity**: This is an activity, which does not consume time or
- resources. It is used to merely show clear, logical dependencies between activities so as not to violets the rules for drawing networks. It is represented in a network by dotted arrow thus.

# A Comparison of AON and AOA Network Conventions

- Activity on node



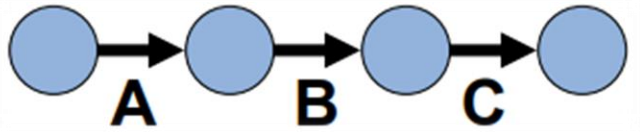

activity meaning

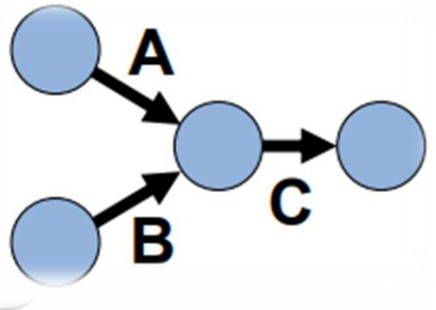
A comes before B, which comes before C.

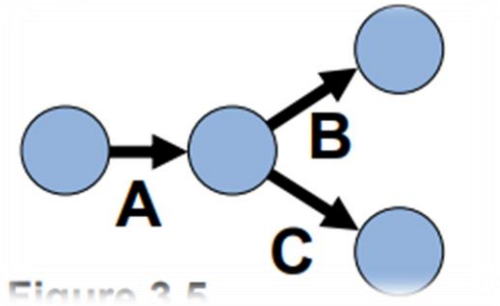
A and B must both be completed before C can start.

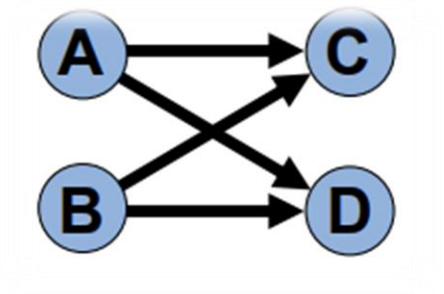
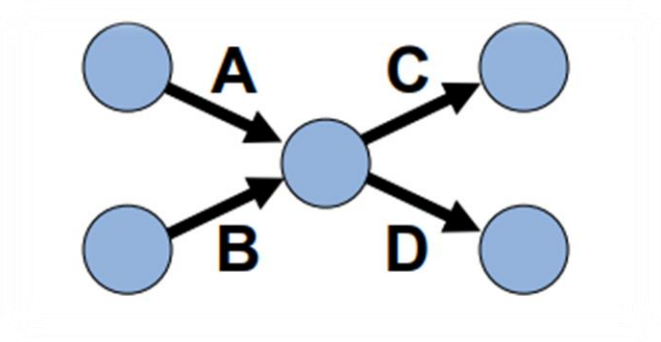
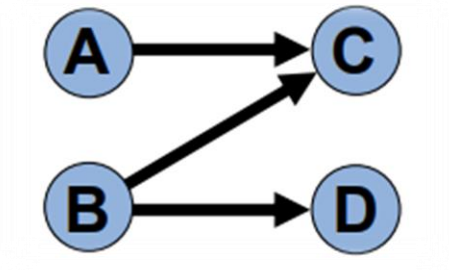
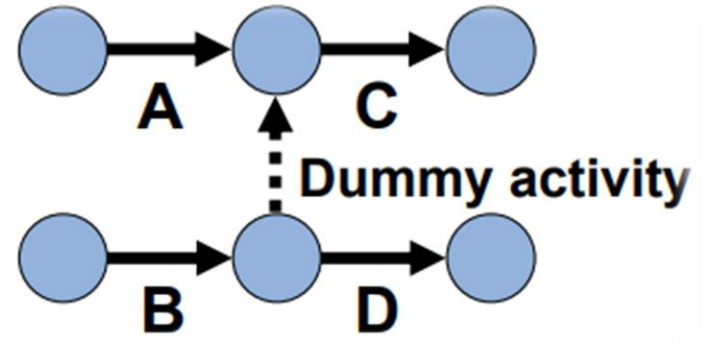
B and C cannot begin until A is completed.

activity on arrow



| Activity on node | activity meaning | activity on arrow |
| --- | --- | --- |
|  | C and D cannot begin until both A and B are completed. |  |
|  | C cannot begin until both A and B are completed; D cannot begin until B is completed. A dummy activity is introduced in AOA. |  |

| Activity On Node | Activity Meaning | Activity On Arrow |
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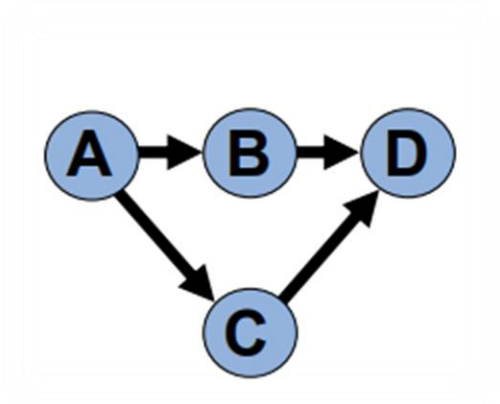
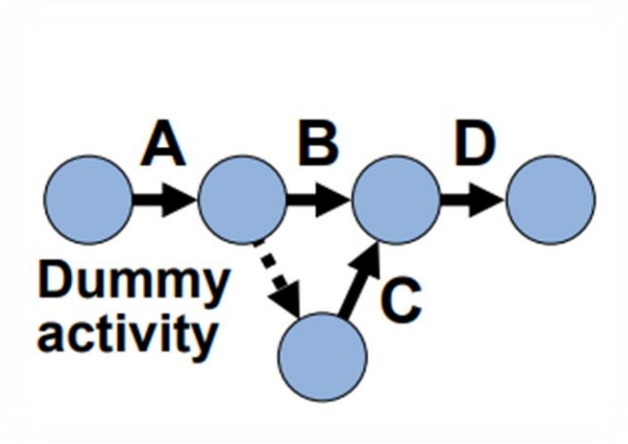
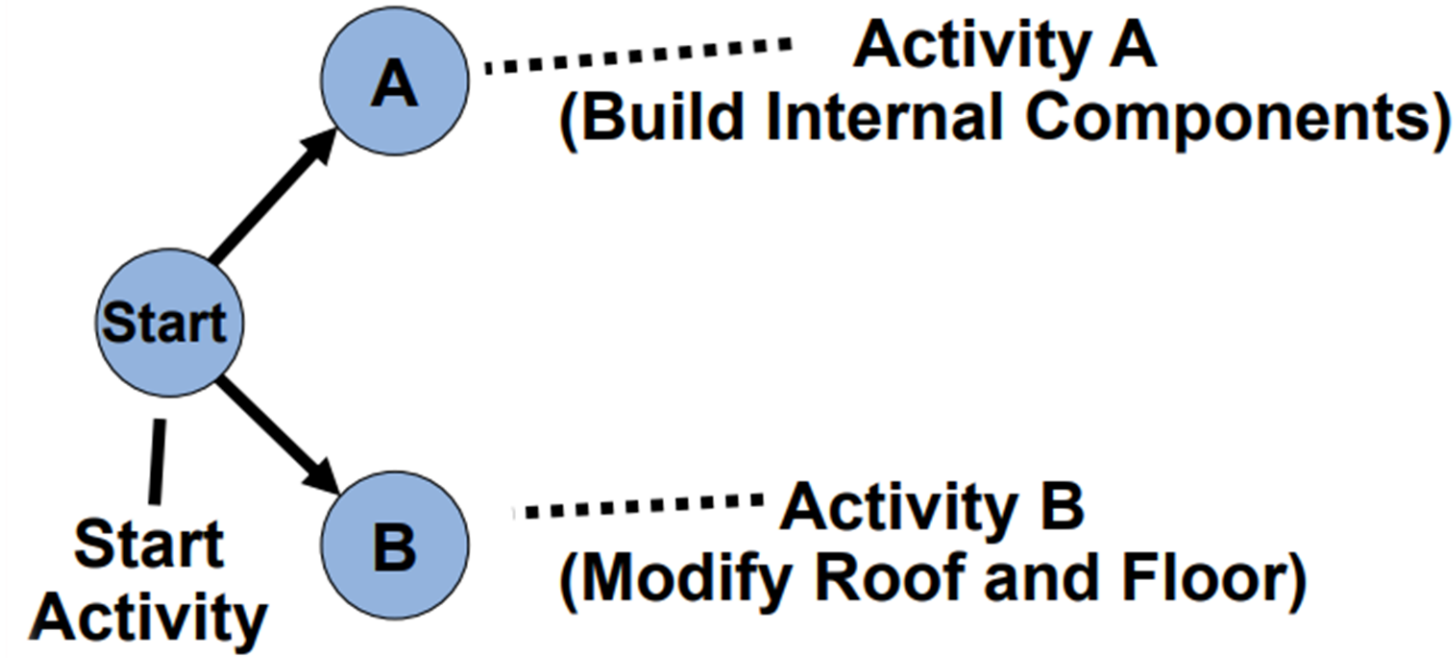
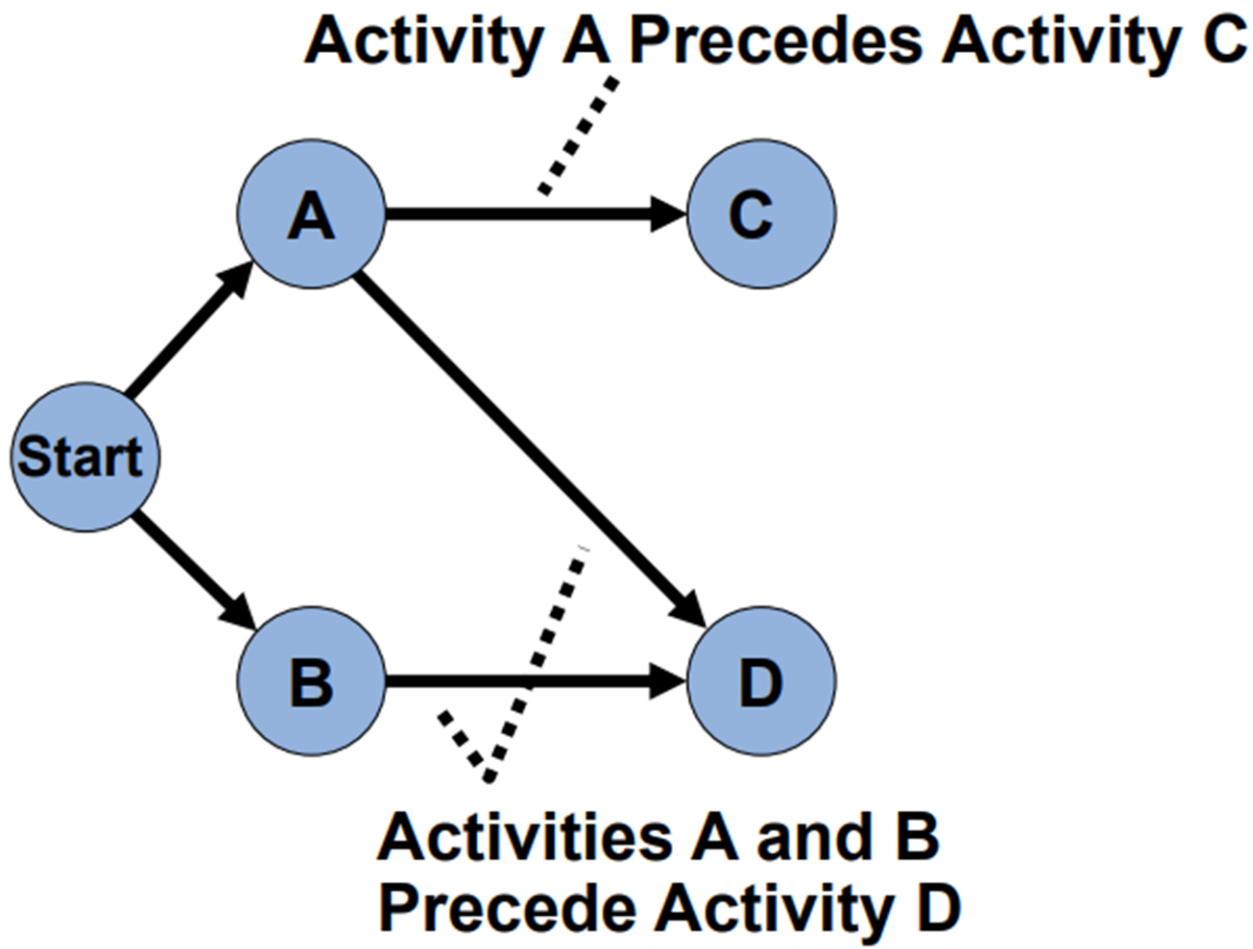
|  | B and C cannot begin until A is completed. D cannot begin until both B and C are completed. A dummy activity is again introduced in AOA. |  |

# AON and AOA Example

| Activity | Description | Immediate Predecessors |
|---|---|---|
| A | Build internal components | — |
| B | Modify roof and floor | — |
| C | Construct collection stack | A |
| D | Pour concrete and install frame | A, B |
| E | Build high-temperature burner | C |
| F | Install pollution control system | C |
| G | Install air pollution device | D, E |
| H | Inspect and test | F, G |

# (1) Solution by AON




Activity A Precedes Activity C
A
C
Start
B
D
Activities A and B
Precede Activity D

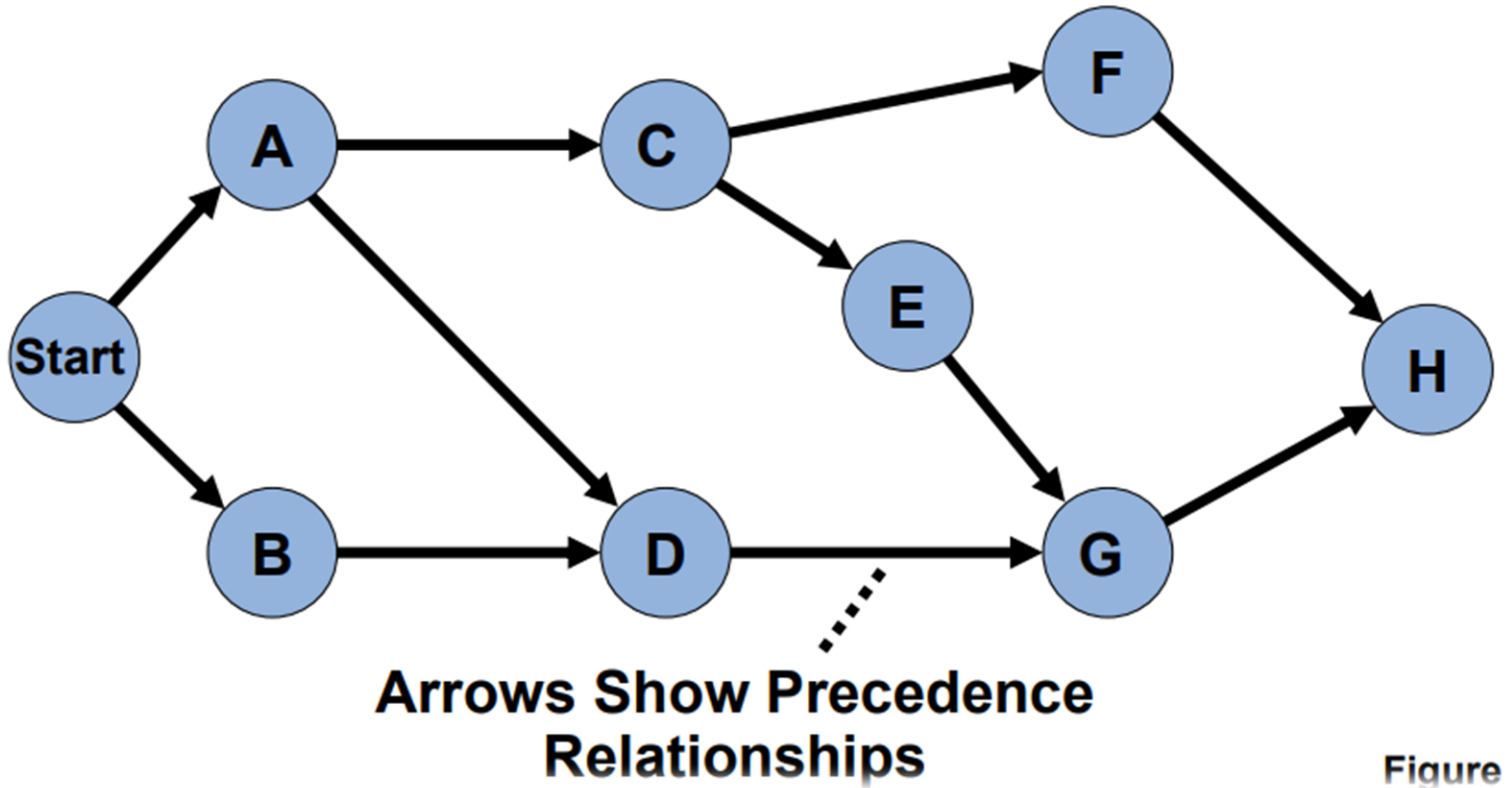



Figure 3.8

# (2)Solution by activity on arrow

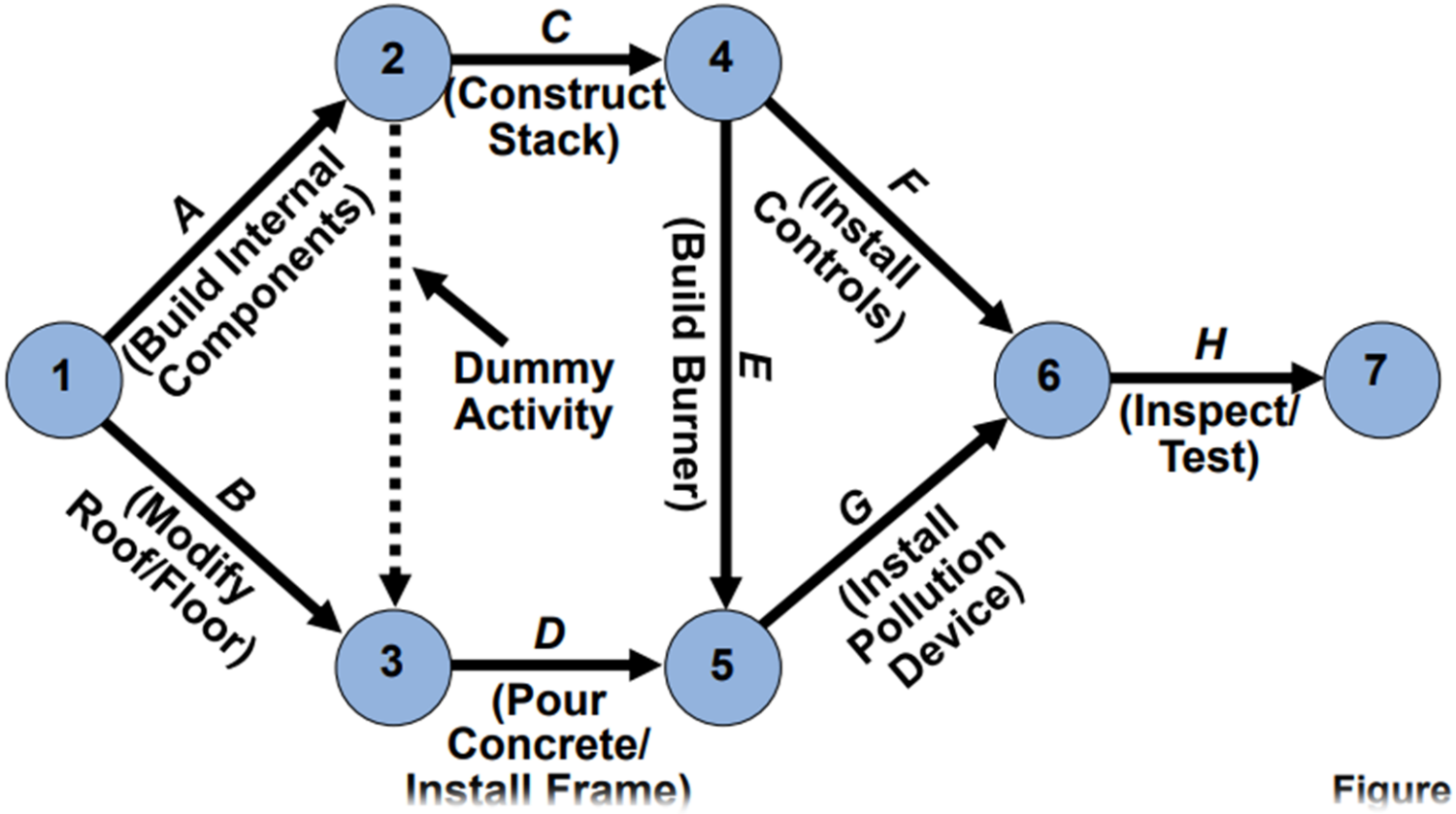



Figure 3.

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# Network analysis

## Lec 3

**Huda majeed**

Ex: Determine the critical path by using CPM of the following table (project).

| activities | path | Duration (days) |
|---|---|---|
| A | 1-2 | 2 |
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| C | 3-4 | 8 |
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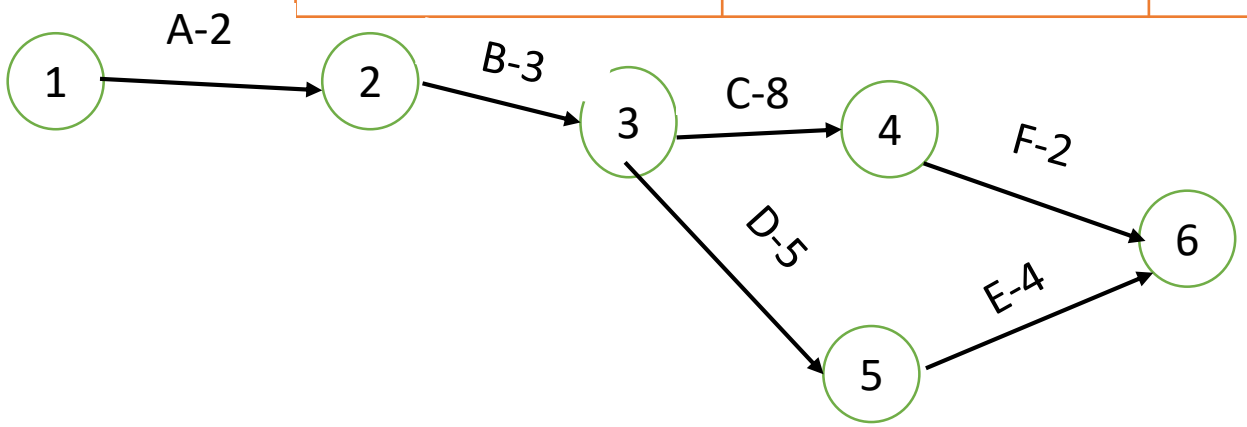
| E | 5-6 | 4 |
| F | 4-5 | 2 |

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Path(2)=A-B-D-F=2+3+5+4=14 ,Critical path=A-B-C-F

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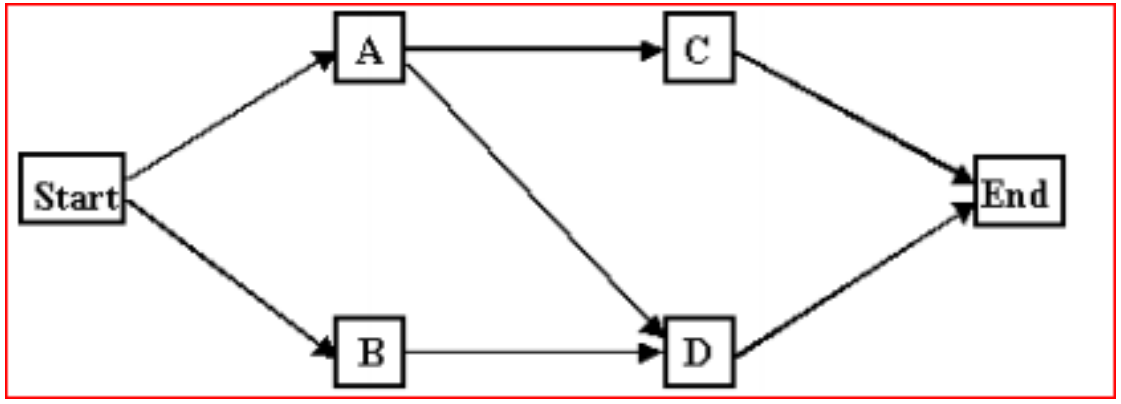
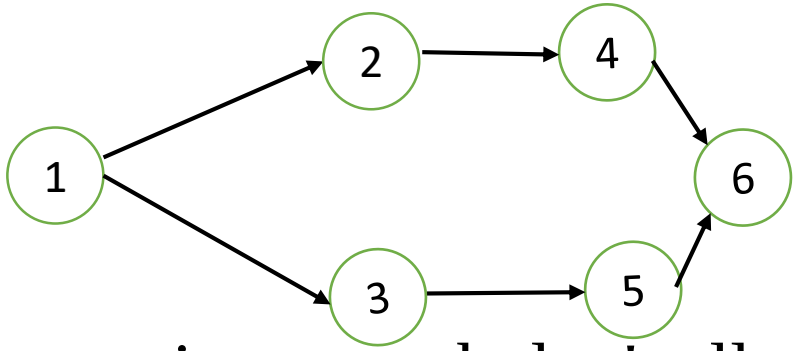
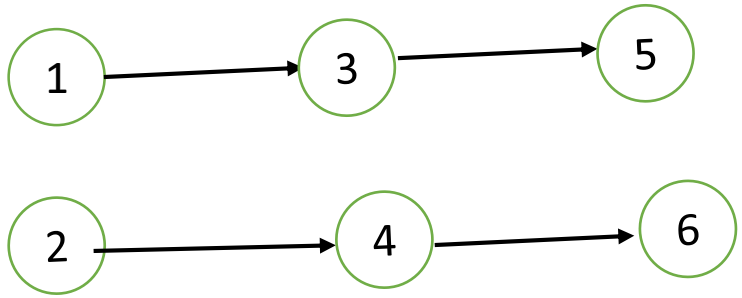
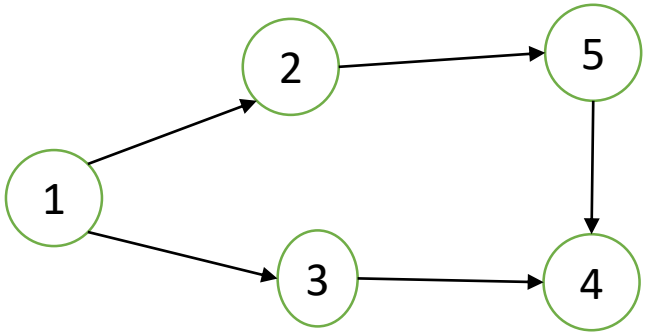


- Notes
- (1) the network should start in one activity only and end in one activity only

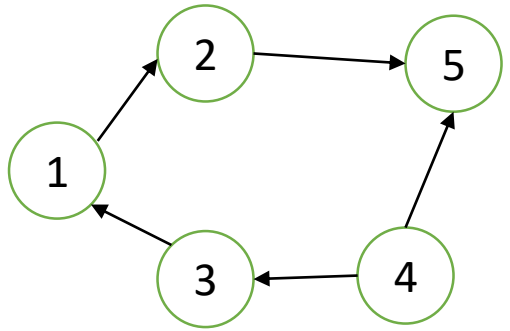



- (2) the rotate  in network don't allow

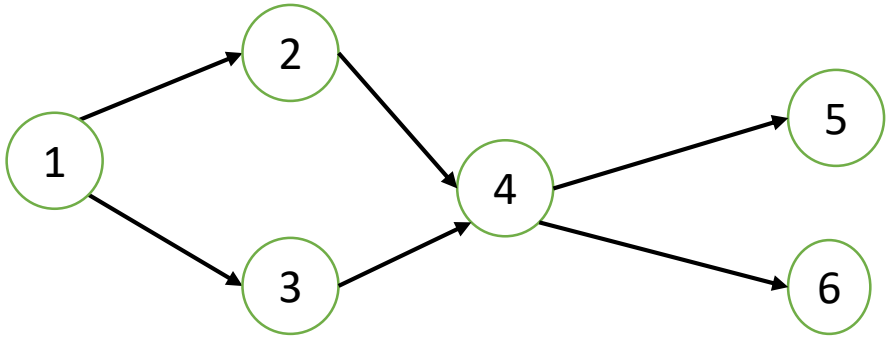


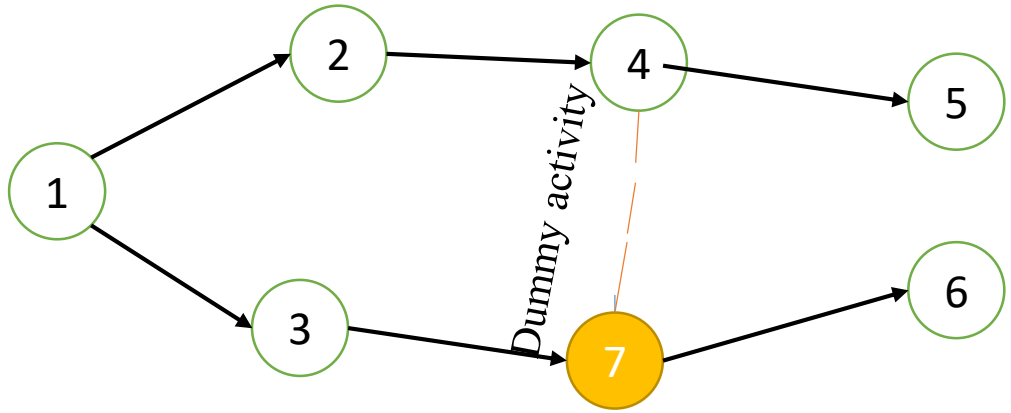

A




B

- (3) Two activity should not start and end in same time.

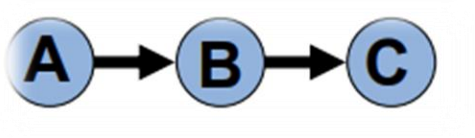


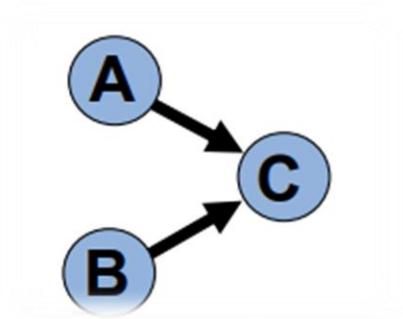



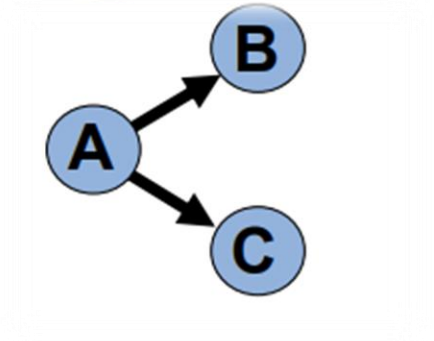
- **Dummy activity**: This is an activity, which does not consume time or
- resources. It is used to merely show clear, logical dependencies between activities so as not to violets the rules for drawing networks. It is represented in a network by dotted arrow thus.

# A Comparison of AON and AOA Network Conventions

- Activity on node



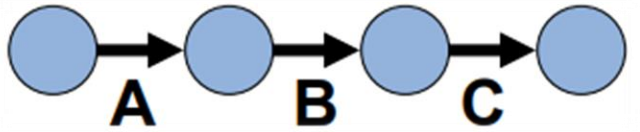

activity meaning

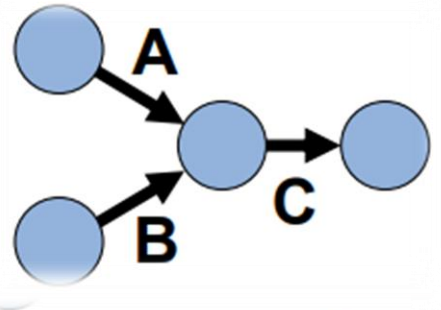
A comes before B, which comes before C.

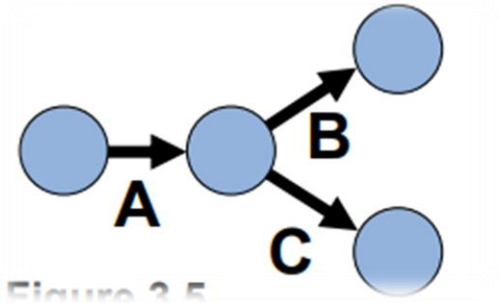
A and B must both be completed before C can start.

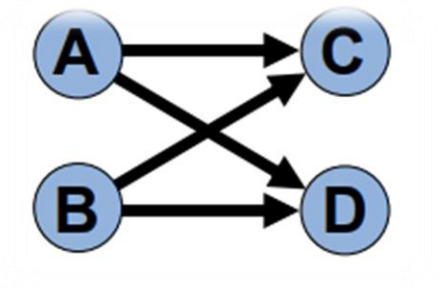
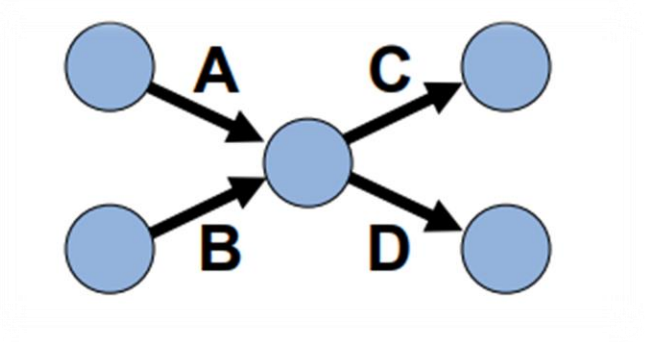
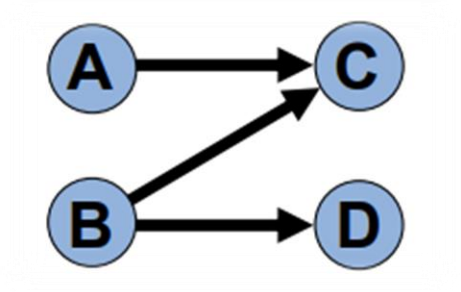
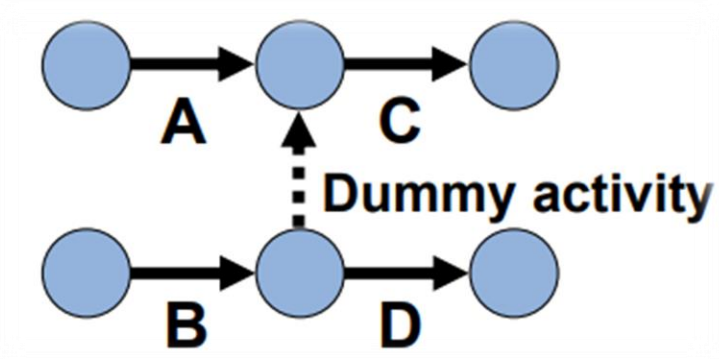
B and C cannot begin until A is completed.

activity on arrow

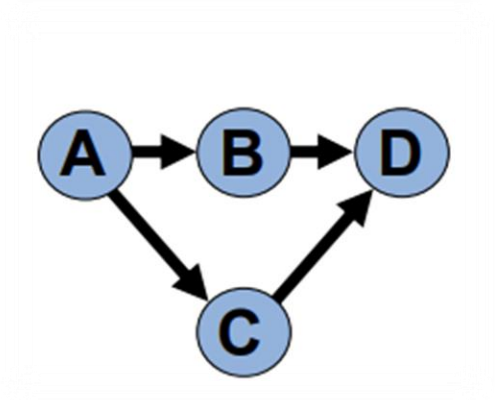


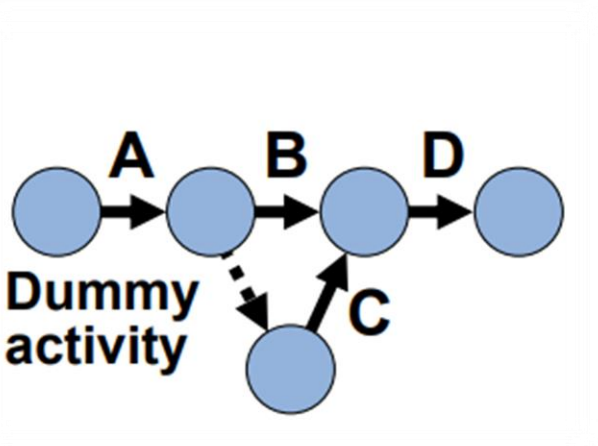



| Activity on node | activity meaning | activity on arrow |
| --- | --- | --- |
|  | C and D cannot begin until both A and B are completed. |  |
|  | C cannot begin until both A and B are completed; D cannot begin until B is completed. A dummy activity is introduced in AOA. |  |

Activity On Node  Activity Meaning  Activity On Arrow



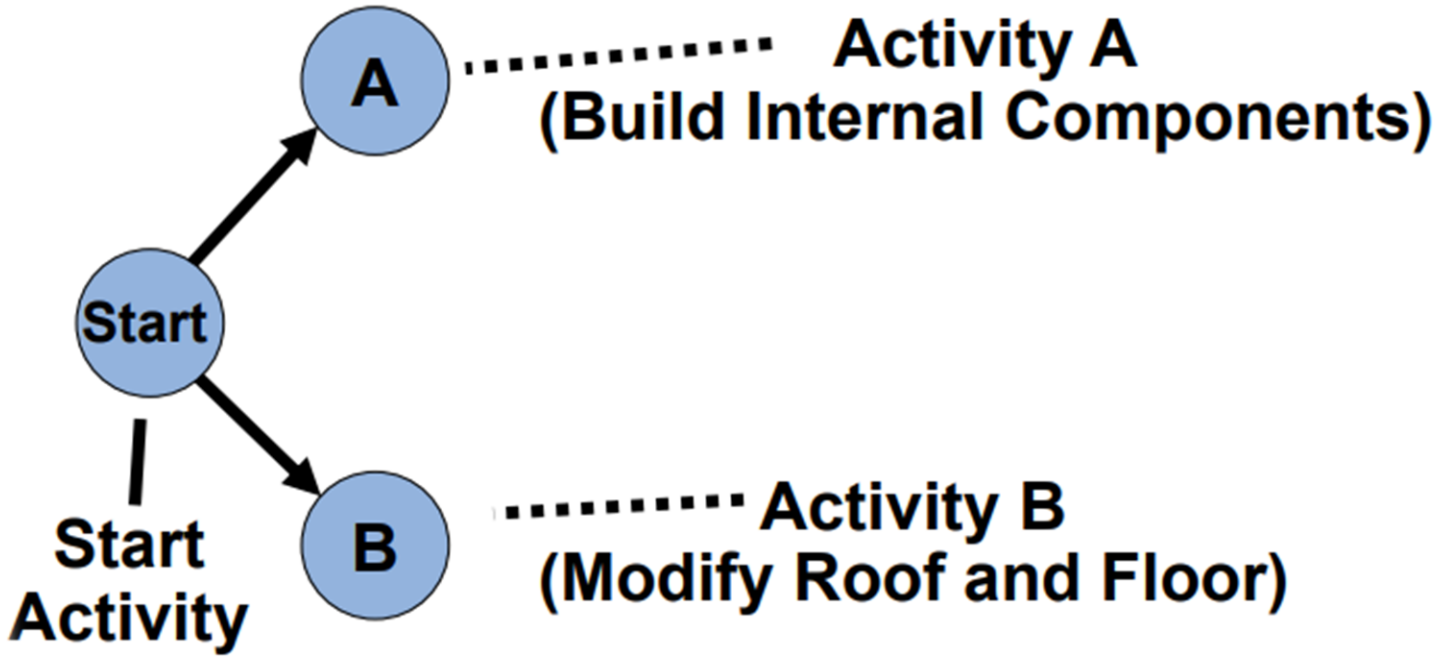

B and C cannot begin until A is completed. D cannot begin until both B and C are completed. A dummy activity is again introduced in AOA.

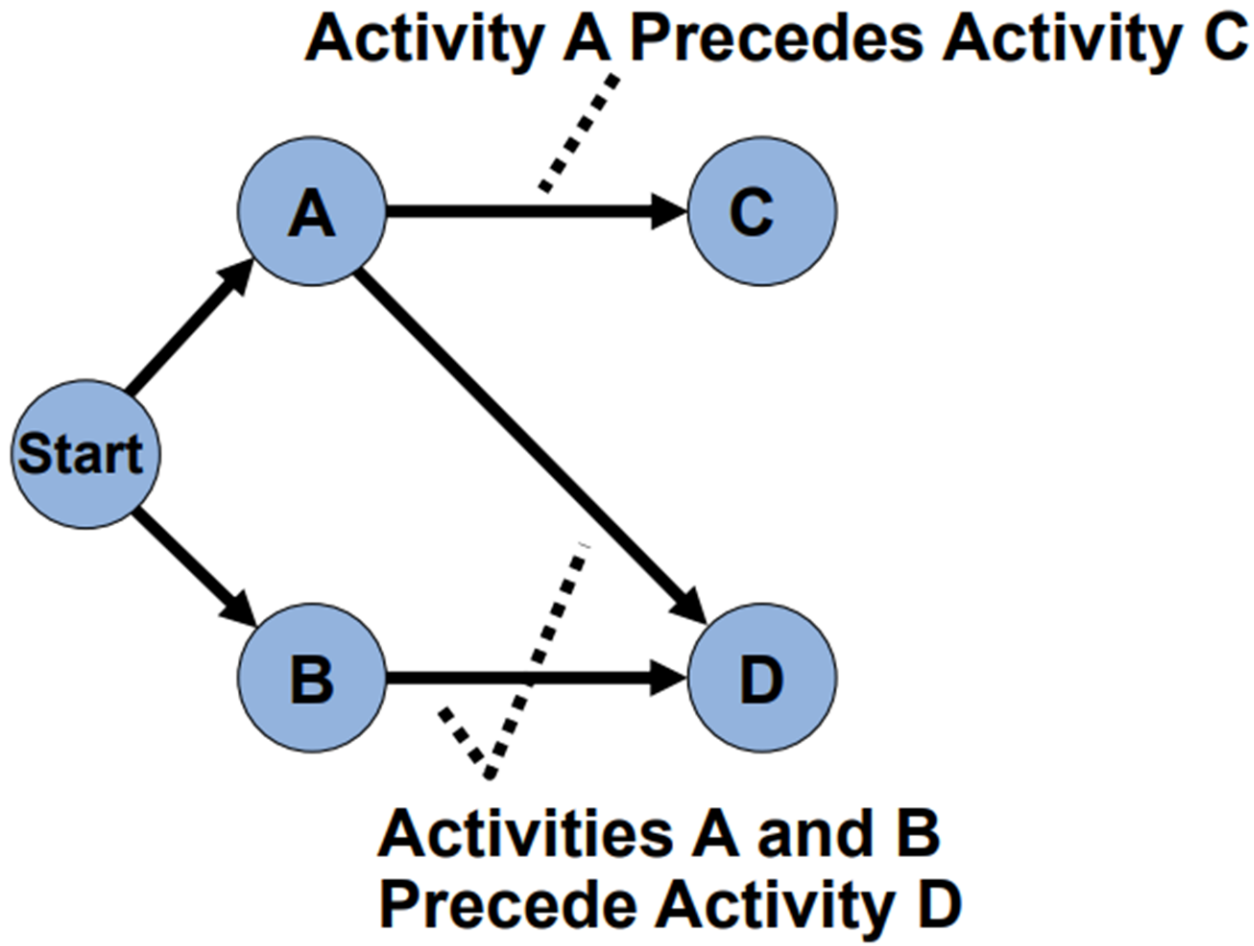


# AON and AOA Example

| Activity | Description | Immediate Predecessors |
|---|---|---|
| A | Build internal components | — |
| B | Modify roof and floor | — |
| C | Construct collection stack | A |
| D | Pour concrete and install frame | A, B |
| E | Build high-temperature burner | C |
| F | Install pollution control system | C |
| G | Install air pollution device | D, E |
| H | Inspect and test | F, G |

# (1) Solution by AON




Activity A Precedes Activity C
A
C
Start
B
D
Activities A and B
Precede Activity D

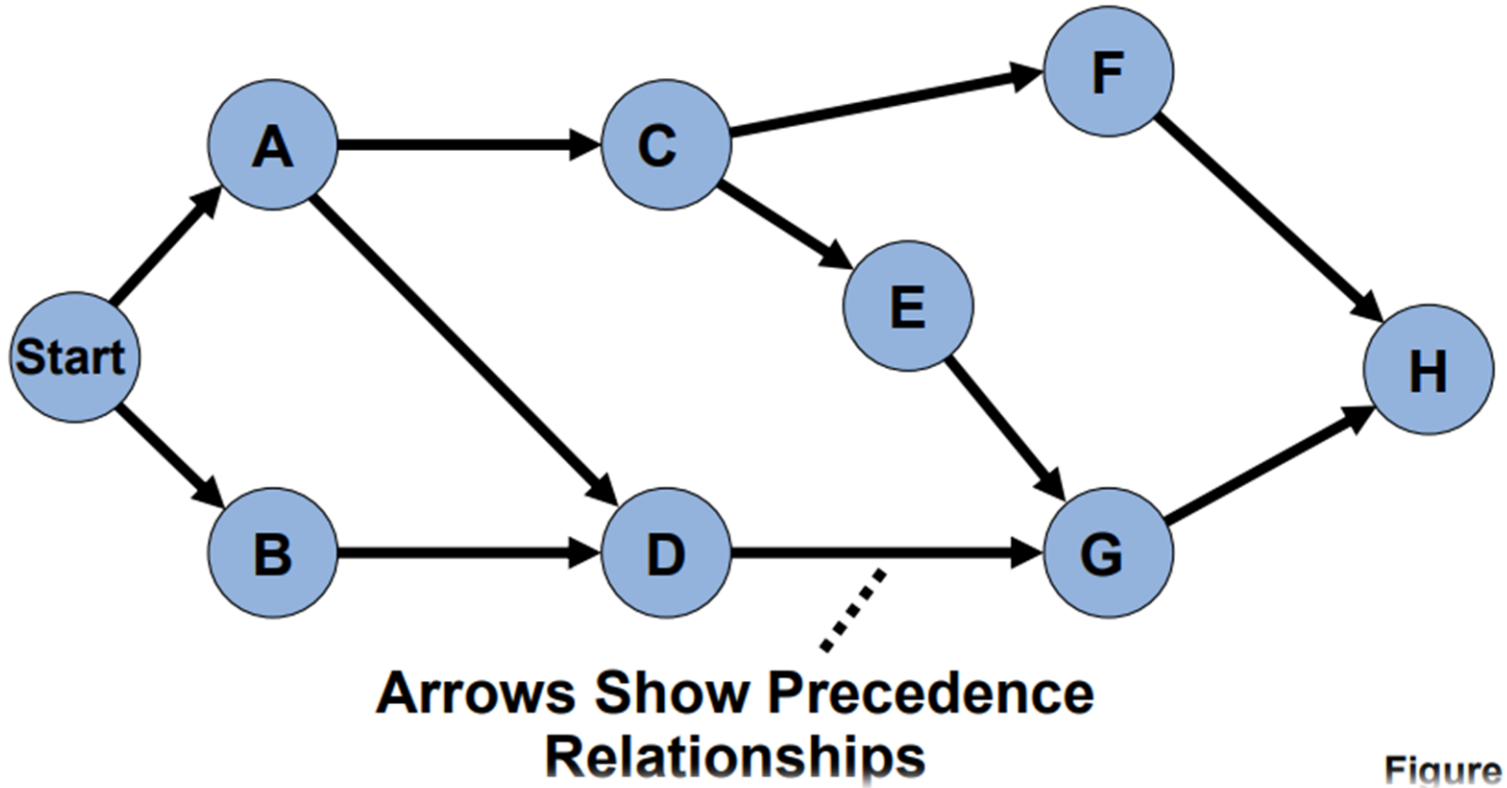



Figure 3.8

# (2)Solution by activity on arrow

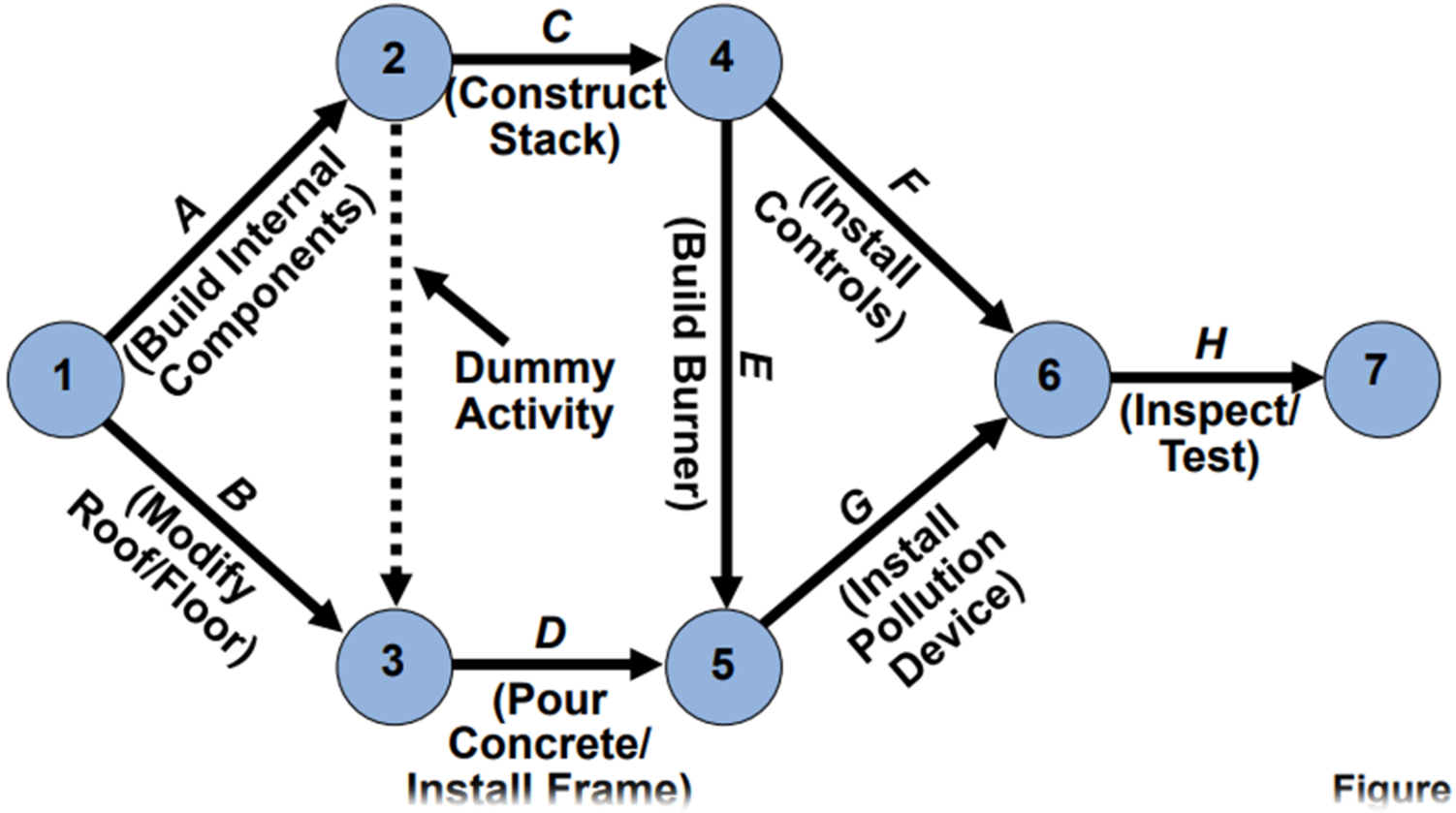



Figure 3.

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# Network analysis

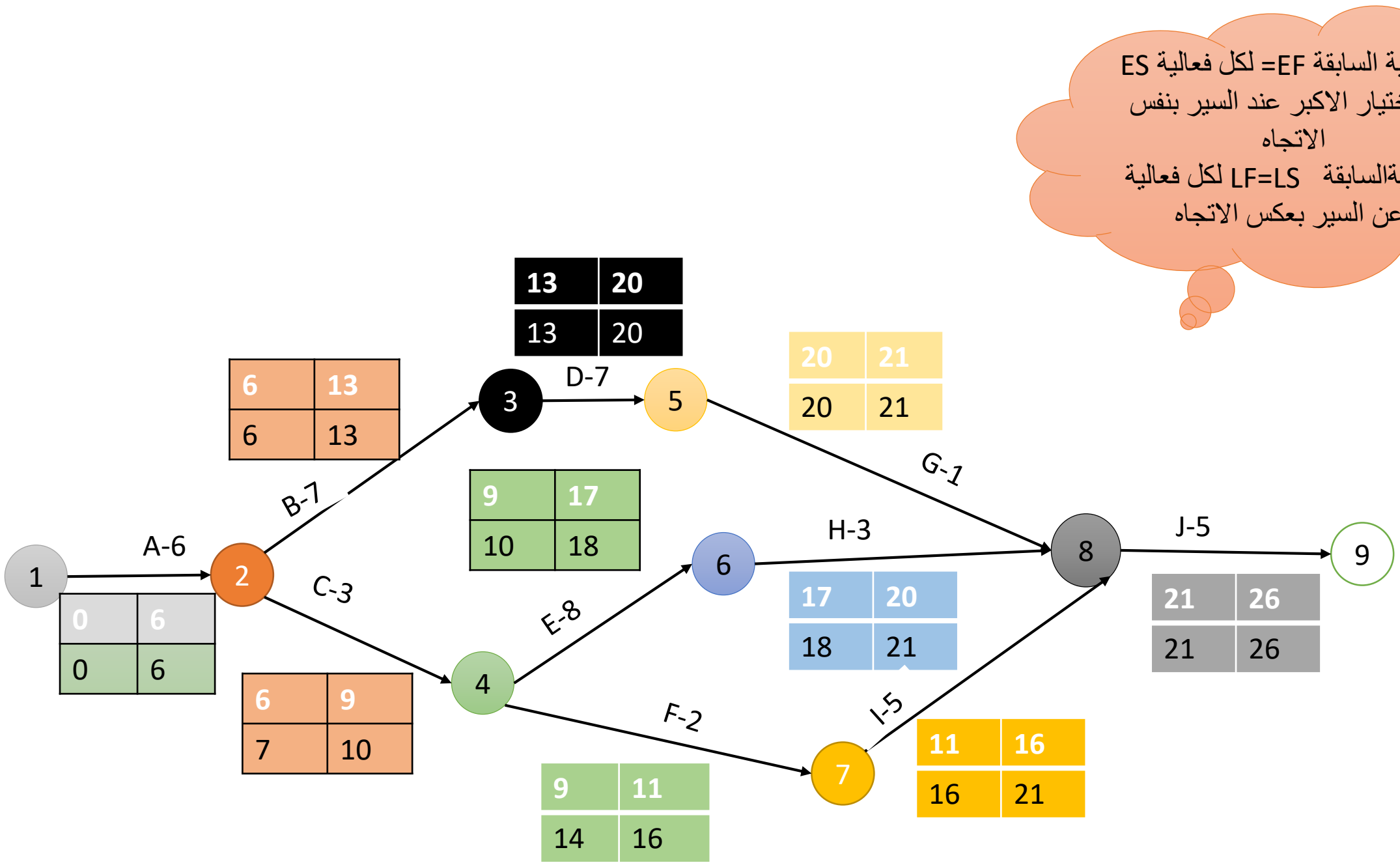
## Lec 5

**Huda majeed**

Example : Data for a small project are given, draw the net work and determine the ES,LS, LF,T.F, and C.P.

| Act | path | Dur |
|---|---|---|
| A | 1-2 | 6 |
| B | 2-3 | 7 |
| C | 2-4 | 3 |
| D | 3-5 | 7 |
| E | 4-6 | 8 |
| F | 4-7 | 2 |
| G | 5-8 | 1 |
| H | 6-8 | 3 |
| I | 7-8 | 5 |
| J | 8-9 | 5 |

للفعالية السابقة EF= لكل فعالية ES
مع اختيار الاكبر عند السير بنفس الاتجاه
للفعاليةالسابقة LF=LS لكل فعالية
عن السير بعكس الاتجاه

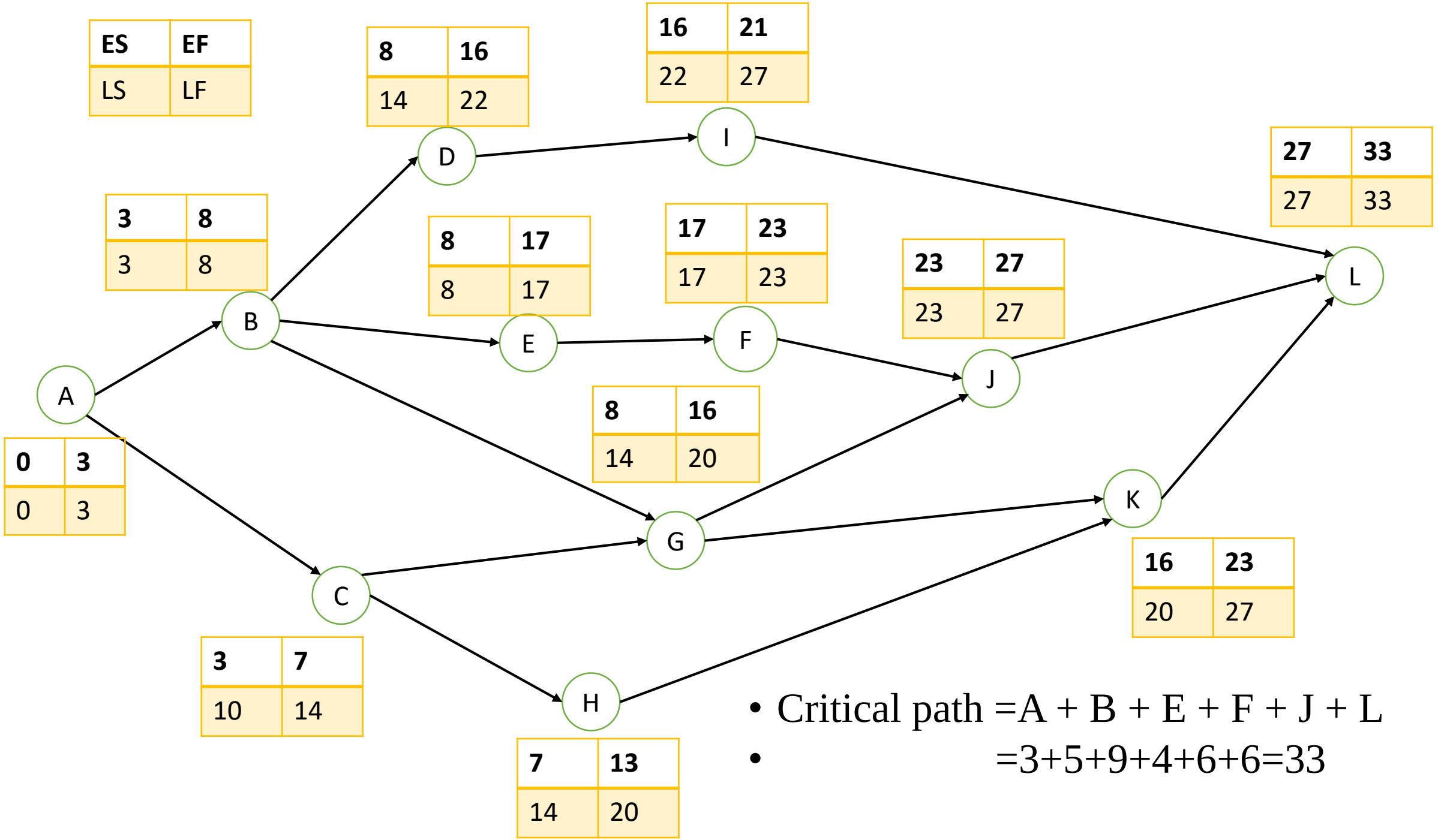


| Act | Dur | ES | EF | LS | LF | T.F |
|---|---|---|---|---|---|---|
| 1-2 | 6 | 0 | 6 | 0 | 6 | 0 |
| 2-3 | 7 | 6 | 13 | 6 | 13 | 0 |
| 2-4 | 3 | 6 | 9 | 4 | 10 | 1 |
| 3-5 | 7 | 13 | 20 | 13 | 20 | 0 |
| 4-6 | 8 | 9 | 17 | 10 | 18 | 1 |
| 4-7 | 2 | 9 | 11 | 14 | 16 | 5 |
| 5-8 | 1 | 20 | 21 | 20 | 21 | 0 |
| 6-8 | 3 | 17 | 20 | 18 | 21 | 1 |
| 7-8 | 5 | 11 | 16 | 16 | 21 | 5 |
| 8-9 | 5 | 21 | 26 | 21 | 26 | 0 |

Critical path=A-B-D-G-J=6+7+7+1+5=26

Example : Data for a small project are given, draw the net work and determine the ES,LS, LF,T.F, and C.P.

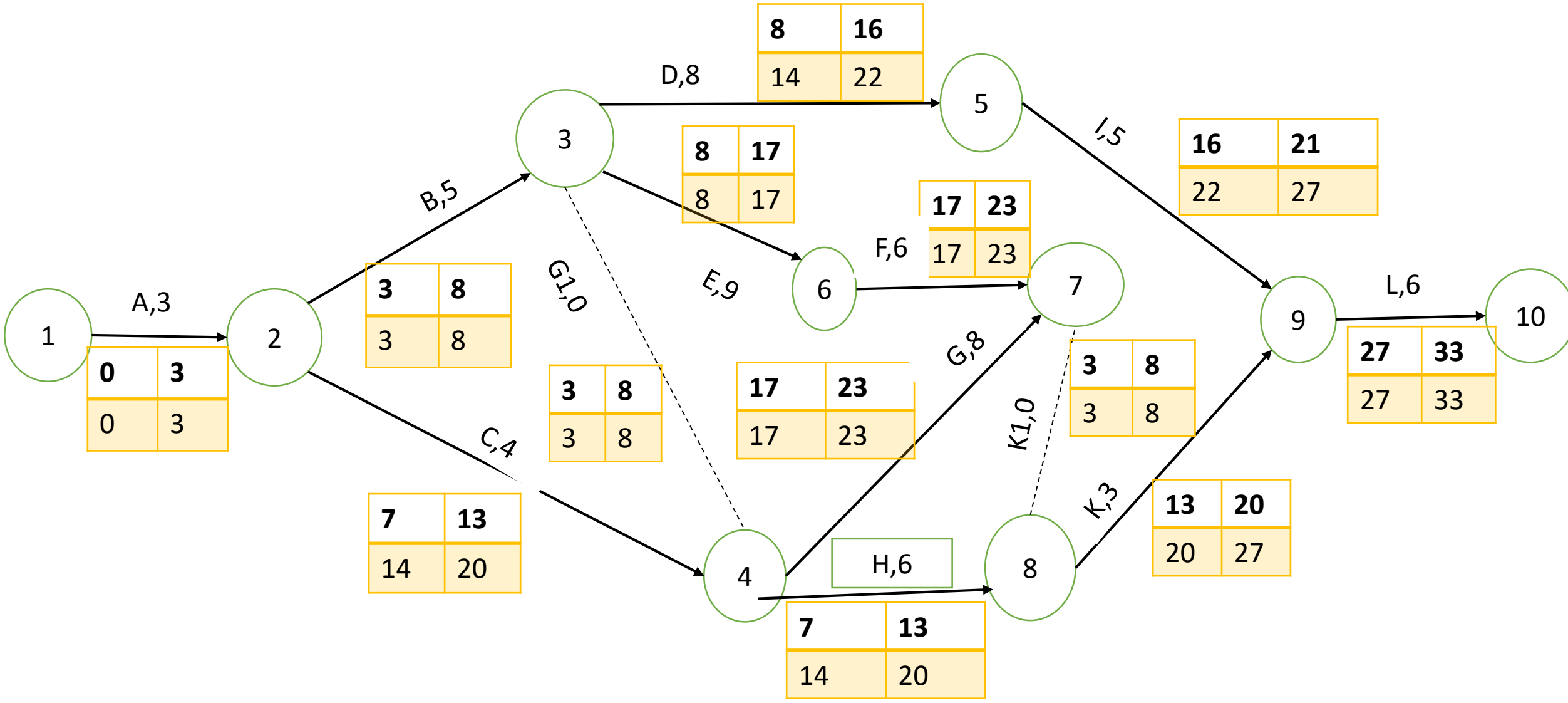
| Act | Dur | Follo | ES | EF | LS | LF | T.F |
|---|---|---|---|---|---|---|---|
| A | 3 | B,C | 0 | 3 | 0 | 3 | 0 |
| B | 5 | D,E,G | 3 | 8 | 3 | 8 | 0 |
| C | 4 | H,G | 3 | 7 | 10 | 14 | 7 |
| D | 8 | I | 8 | 16 | 14 | 22 | 6 |
| E | 9 | F | 8 | 17 | 8 | 17 | 0 |
| F | 6 | J | 17 | 23 | 17 | 23 | 0 |
| G | 8 | J,K | 8 | 16 | 14 | 20 | 8 |
| H | 6 | K | 7 | 13 | 14 | 20 | 7 |
| I | 5 | L | 16 | 21 | 22 | 27 | 6 |
| J | 4 | L | 23 | 27 | 23 | 27 | 0 |
| K | 7 | L | 16 | 23 | 20 | 27 | 4 |
| L | 6 | ----- | 27 | 33 | 27 | 33 | 0 |




- Critical path =A + B + E + F + J + L
- =3+5+9+4+6+6=33

Example : Data for a small project are given, draw the net work and determine the ES,LS, LF,T.F, and C.P.

| Act | Dur | Followed | ES | EF | LS | LF | TF |
|---|---|---|---|---|---|---|---|
| A | 3 | B,C | 0 | 3 | 0 | 3 | 0 |
| B | 5 | D,E,G | 3 | 8 | 3 | 8 | 0 |
| C | 4 | H,G | 3 | 7 | 10 | 14 | 7 |
| D | 8 | I | 8 | 16 | 14 | 22 | 6 |
| E | 9 | F | 8 | 17 | 8 | 17 | 9 |
| F | 6 | J | 17 | 23 | 17 | 23 | 6 |
| G | 8 | J,K | 7 | 15 | 15 | 23 | 8 |
| H | 6 | K | 7 | 13 | 14 | 20 | 7 |
| I | 5 | L | 16 | 21 | 22 | 27 | 6 |
| J | 4 | L | 23 | 27 | 23 | 27 | 0 |
| K | 7 | L | 13 | 20 | 20 | 27 | 7 |
| L | 6 | ------ | 27 | 33 | 27 | 33 | 0 |


1
A,3
2
B,5
3
C,4
4
D,8
5
E,9
6
F,6
7
G,8
H,6
8
I,5
9
K,3
K1,0
L,6
10
G1,0
0 3
0 3
3 8
3 8
8 16
14 22
8 17
8 17
3 8
3 8
17 23
17 23
7 13
14 20
17 23
17 23
3 8
3 8
16 21
22 27
13 20
20 27
27 33
27 33
7 13
14 20

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# Network analysis

## Lec 6

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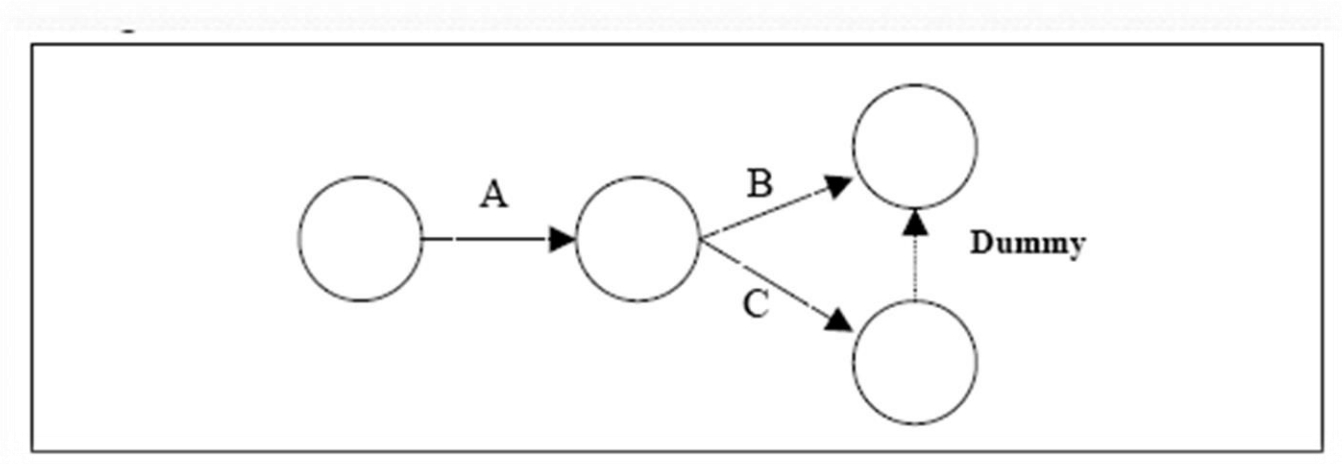
## Dummy activity

An imaginary activity which does not consume any resource and time is called dummy activity .dummy activities are simply used to represent aconnection between events in order to contain a logic in the network .it is represented by a dotted line in a network.

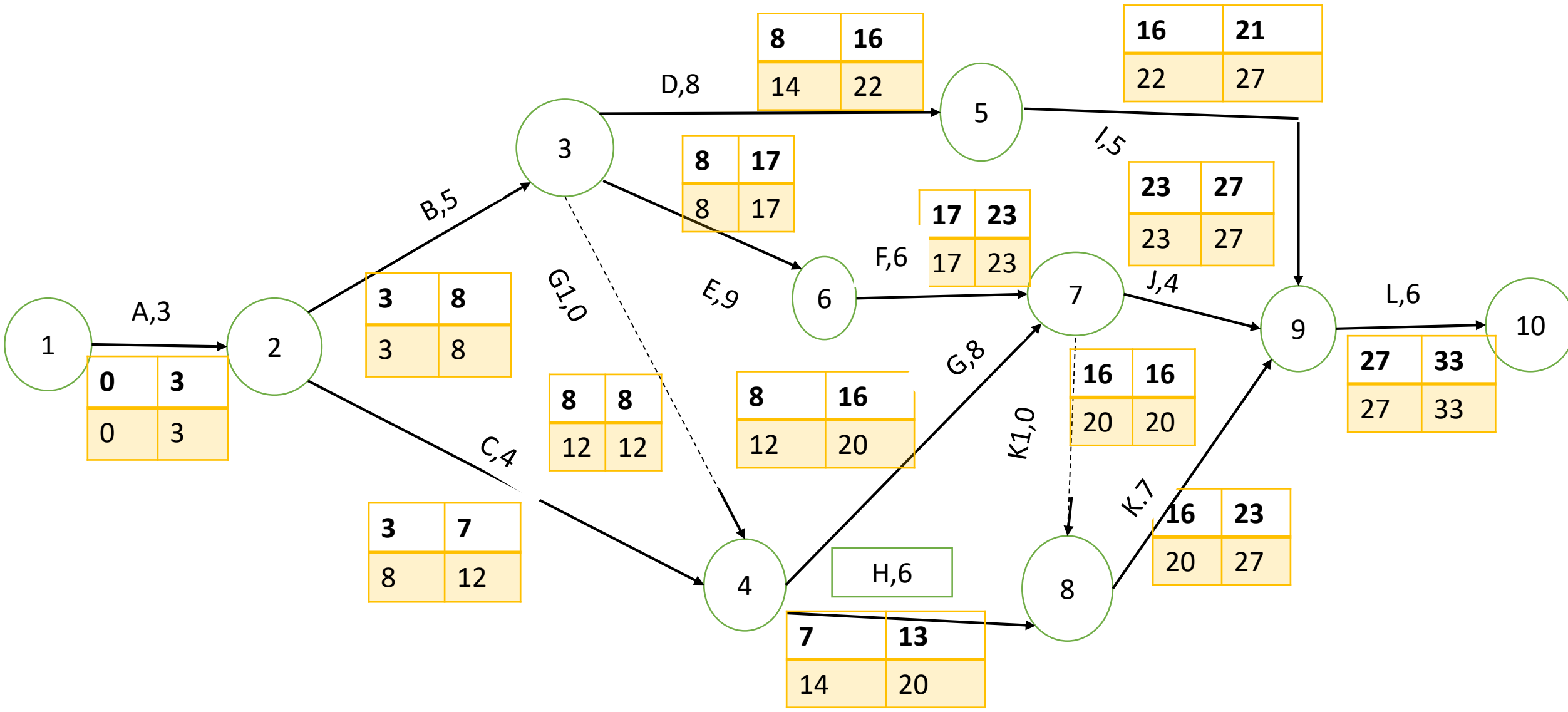


# Errors to be avoided in constructing a network

- A. two activities starting from a tail event must not have a same end event .to ensure this, it is absolutely necessary to introduce a dummy activity, as shown in figure.
- B.looping error should not be formed in network, as it represents performance of activities repeatedly in acyclic manner ,as shown below in figure.

Example : Data for a small project are given, draw the net work and determine the ES,LS, LF,T.F, and C.P.

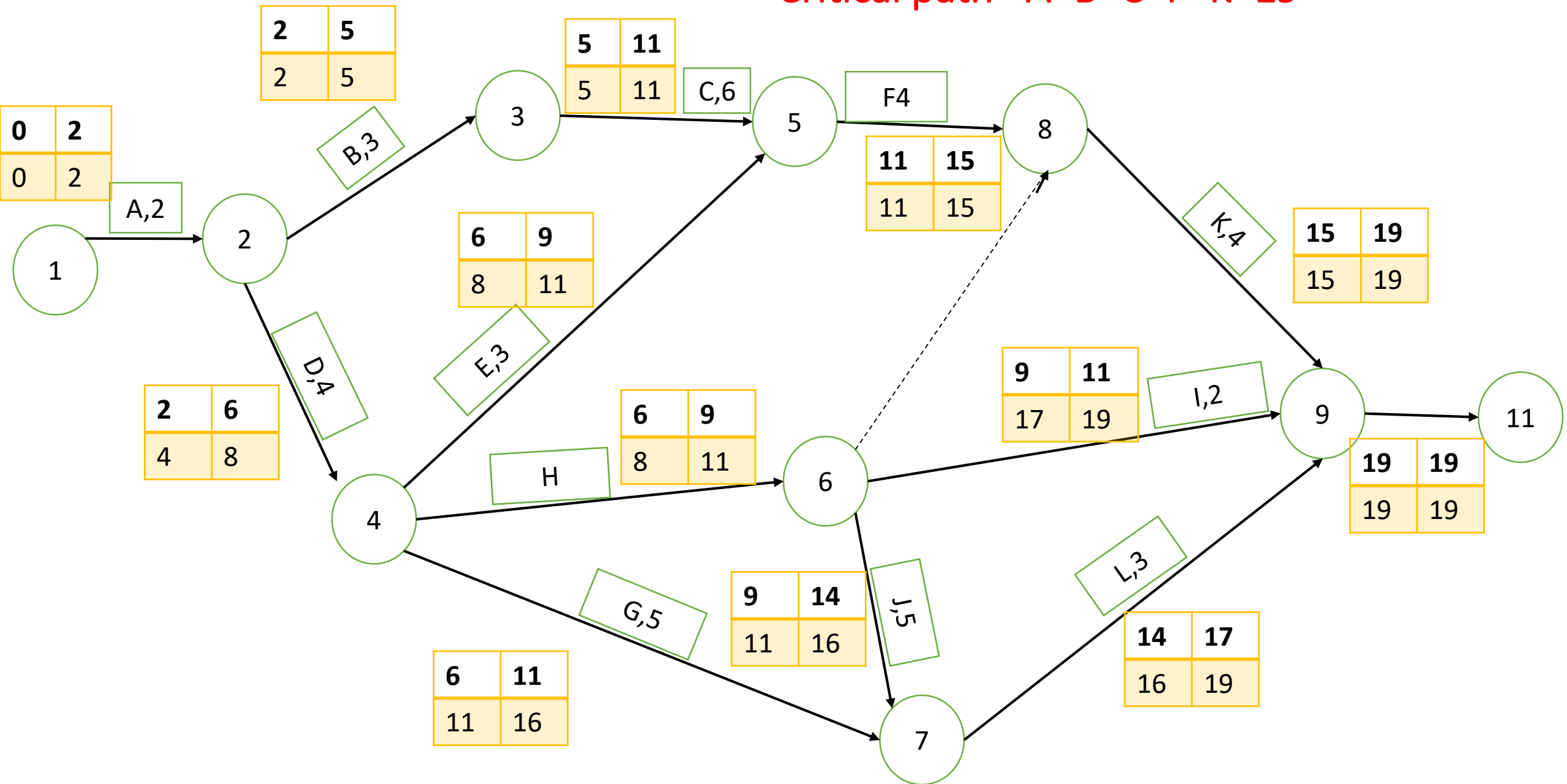
| Act | Dur | Followed | ES | EF | LS | LF | TF |
|---|---|---|---|---|---|---|---|
| A | 3 | B,C | 0 | 3 | 0 | 3 | 0 |
| B | 5 | D,E,G | 3 | 8 | 3 | 8 | 0 |
| C | 4 | H,G | 3 | 7 | 8 | 12 | 4 |
| D | 8 | I | 8 | 16 | 14 | 22 | 6 |
| E | 9 | F | 8 | 17 | 8 | 17 | 9 |
| F | 6 | J | 17 | 23 | 17 | 23 | 6 |
| G | 8 | J,K | 8 | 16 | 12 | 20 | 8 |
| H | 6 | K | 7 | 13 | 14 | 20 | 7 |
| I | 5 | L | 16 | 21 | 22 | 27 | 6 |
| J | 4 | L | 23 | 27 | 23 | 27 | 0 |
| K | 7 | L | 16 | 23 | 20 | 27 | 7 |
| L | 6 | ------ | 27 | 33 | 27 | 33 | 0 |


1
A,3
2
B,5
3
C,4
4
D,8
5
E,9
6
F,6
7
G,8
8
G1,0
H,6
I,5
J,4
K1,0
K,7
9
L,6
10
0 3
0 3
3 8
3 8
3 7
8 12
8 8
12 12
8 16
14 22
8 17
8 17
17 23
17 23
8 16
12 20
16 21
22 27
23 27
23 27
16 16
20 20
16 23
20 27
7 13
14 20
27 33
27 33

# Example : Data for a small project are given, draw the net work and determine the ES,LS, LF,T.F, and C.P.

| Activity | Dur | Immediate predecessor |
|---|---|---|
| A | 2 | ----- |
| B | 3 | A |
| C | 6 | B |
| D | 4 | A |
| E | 3 | D |
| F | 4 | C,E |
| G | 5 | D |
| H | 3 | D |
| I | 2 | H |
| J | 5 | H |
| K | 4 | F,H |
| L | 3 | G,J |

# Critical path =A+B+C+F+K=19



Example : Data for a small project are given, draw the net work and determine the ES,LS, LF,T.F, and C.P.

| Activity | Dur | Immediate predecessor | ES | EF | LS | LF | T.F |
|---|---|---|---|---|---|---|---|
| A | 2 | ----- | 0 | 2 | 0 | 2 | 0 |
| B | 3 | A | 2 | 5 | 2 | 5 | 0 |
| C | 6 | B | 5 | 11 | 5 | 11 | 0 |
| D | 4 | A | 2 | 6 | 4 | 8 | 2 |
| E | 3 | D | 6 | 9 | 8 | 11 | 2 |
| F | 4 | C,E | 11 | 15 | 11 | 15 | 0 |
| G | 5 | D | 6 | 11 | 11 | 16 | 5 |
| H | 3 | D | 6 | 9 | 8 | 11 | 2 |
| I | 2 | H | 9 | 11 | 17 | 19 | 8 |
| J | 5 | H | 9 | 14 | 11 | 16 | 2 |
| K | 4 | F,H | 15 | 19 | 15 | 19 | 0 |
| L | 3 | G,J | 14 | 17 | 16 | 19 | 2 |

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# Program Evaluation and Review Technique (PERT )

## Lec 7

**Huda majeed**

# Program Evaluation and Review Technique (PERT)

The Program Evaluation and Review Technique (PERT) is a network model that allows for randomness in activity completion times. PERT uses three time estimates optimistic, pessimistic and most likely, which help in establishing the probability of completing a project within a specified time and take calculated risk before commencing a project. It has the potential to reduce both the time and cost required to complete a project.

## What are the different steps involved in PERT planning?

PERT planning involves the following steps:

1. Identify the specific activities and milestones.
2. Determine the interdependencies and proper sequence of the activities.
3. Construct a network diagram.
4. Estimate the time (three time estimates, if probabilities are to be computed) required for each activity.
5. Determine the critical path.
6. Update the PERT chart as the project progresses.

# What are the benefits of PERT?

PERT is useful because it provides the following information:

* Expected project completion time.
* Probability of completion before a specified date.
* The critical path activities that directly impact the completion time.
* The activities that have slack time and that can lend resources to critical path activities.
* Activity starts and end dates.

## Estimate activity times

For each activity, the model usually includes three time estimates:

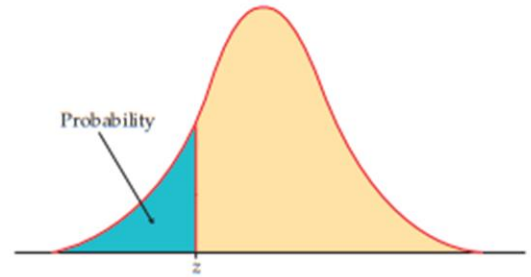
Optimistic time (**To**) - generally the shortest time in which the activity can be completed.

Most likely time (**Tm**) - the completion time having the highest probability. This is different from expected time. Seasoned managers have an amazing way of estimating very close to actual data from prior estimation errors.

Pessimistic time (**Tp**) - the longest time that an activity might require.

- The expected time for each activity can be approximated using the following weighted average:
- $$\text{Expected time } (Te) = \frac{(To+4Tm+Tp)}{6}$$
- This expected time might be displayed on the network diagram. The standard deviation and Variance for each activity are given by:
- $$variance\ (V) = (Tp - To)/6)^2 = \boldsymbol{\sigma\ 2}$$
- The standard deviation $=\sqrt{\boldsymbol{\sigma 2}}$ OR $\boldsymbol{\sigma} = (\boldsymbol{Tp} - \boldsymbol{To})\ /\ 6$
- $Z=(X\text{-}Tcp)/\ \boldsymbol{\sigma}$

Table entry for $z$ is the area under the standard Normal curve to the left of $z$.




## TABLE A

**Standard Normal probabilities**

| $z$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| −3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| −3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| −3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| −3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| −2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| −2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| −2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| −2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| −2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| −2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| −2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| −2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| −2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| −2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| −1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| −1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| −1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| −1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| −1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| −1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| −1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| −1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| −1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| −1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| −0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| −0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| −0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| −0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| −0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| −0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| −0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| −0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| −0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| −0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

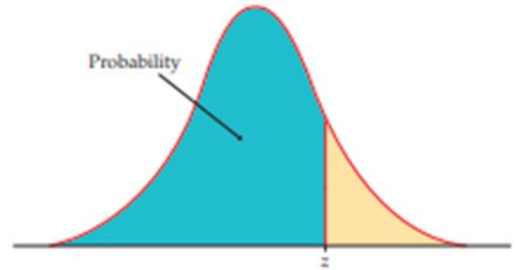



Table entry for $z$ is the area under the standard Normal curve to the left of $z$.

**TABLE A**

**Standard Normal probabilities (continued)**

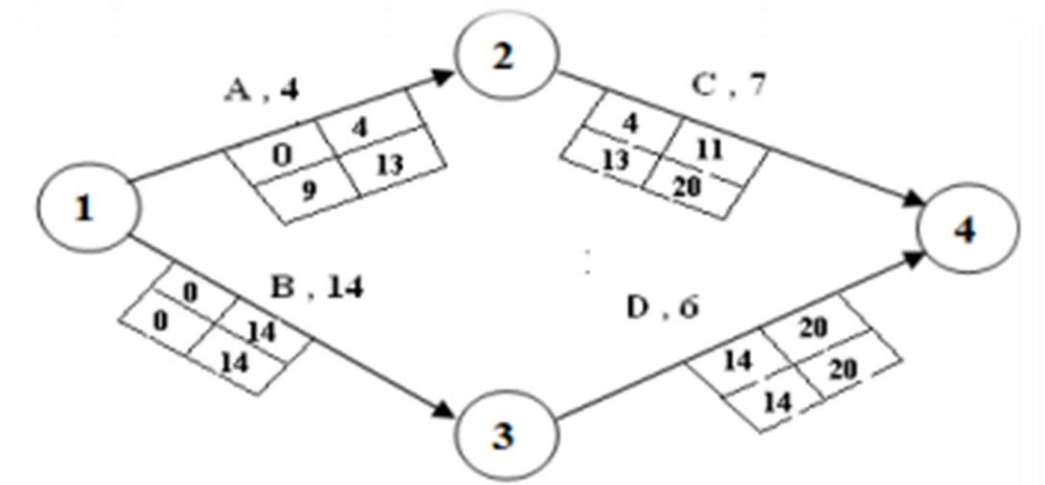
| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

Ex: By using PERT draw the network diagram and determine $Te$ , $S$, $V$, $ES$, $EF$, $LS$, $LF$ $and$ $C$. $P$for the following Table. Find the probability for D=23 weeks.

| Activities | Path | To | Tm | Tp | Te | S | V | Es | Ef | Ls | Lf | C.P |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 1-2 | 2 | 4 | 6 | 4 | 0.6 | 0.36 | 0 | 4 | 9 | 13 | |
| B | 1-3 | 3 | 16 | 17 | 14 | 2.3 | 5.29 | 0 | 14 | 0 | 14 | * |
| C | 2-4 | 3 | 7 | 7 | 6.3 | 0.6 | 0.36 | 4 | 11 | 13 | 20 | |
| D | 3-4 | 4 | 6 | 8 | 6 | 0.6 | 0.36 | 14 | 20 | 14 | 29 | * |

- Ans:
- $Expected\ time\ (Te) = (To+4Tm+Tp)\ /6$
- $V = [(Tp - To)\ /\ 6]\ \text{^}2 = \sigma^2$
- $\sigma = (Tp - To)\ /\ 6$ or $\sigma = \sqrt{V}$ (For each activity)
- $cp = B + D = 20$
- $V = 5.29 + 0.36 = 5.65$
- $\sqrt{v} = 2.37$
- $\boldsymbol{Z} = (\boldsymbol{X}-\boldsymbol{T})/\ \sqrt{\boldsymbol{v}} = (23-20)\ /\sqrt{5.65} = 1.26$ (Probability Formula)

The probability for 23 week is 89.62%



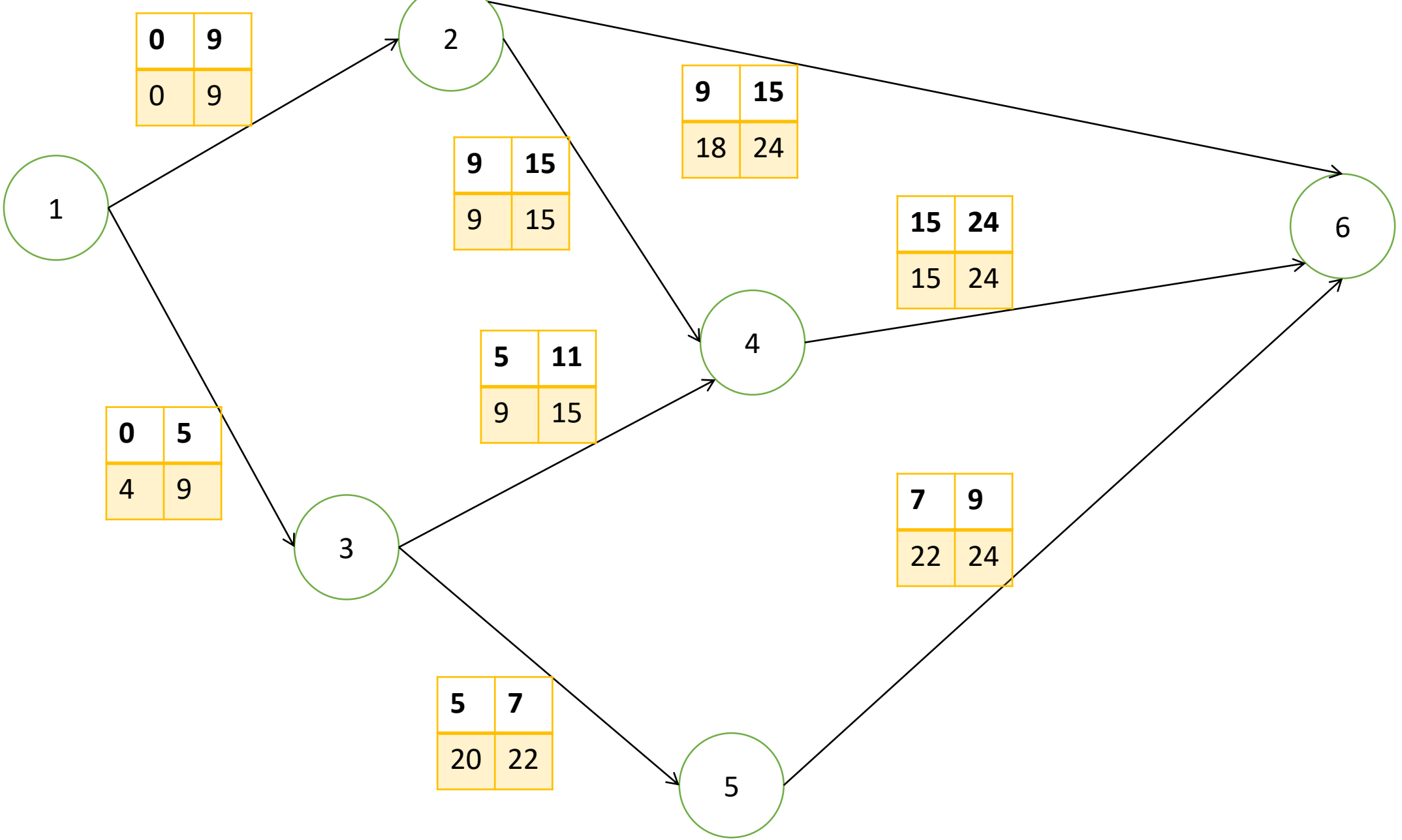
EX2: For a given activities determine

A-critical path using PERT

B-calculate the variance and standard devation

C-calculate probability of completing of completing the project in26 days

| activties | Path | to | tm | tp |
|---|---|---|---|---|
| A | 1-2 | 6 | 9 | 12 |
| B | 1-3 | 3 | 4 | 11 |
| C | 2-4 | 2 | 5 | 14 |
| D | 3-4 | 4 | 6 | 8 |
| E | 3-5 | 1 | 1.5 | 5 |
| F | 2-6 | 5 | 6 | 7 |
| G | 4-6 | 7 | 8 | 15 |
| H | 5-6 | 1 | 2 | 3 |


0 9
0 9
2
9 15
9 15
9 15
18 24
1
15 24
15 24
6
4
5 11
9 15
0 5
4 9
3
7 9
22 24
5 7
20 22
5

| Activties | Path | to | tm | tp | Te | ES | EF | LS | LF | T.F | Variance |
|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 1-2 | 6 | 9 | 12 | 9 | 0 | 9 | 0 | 9 | 0 | 1 |
| B | 1-3 | 3 | 4 | 11 | 5 | 0 | 5 | 4 | 9 | 4 | 1.778 |
| C | 2-4 | 2 | 5 | 14 | 6 | 9 | 15 | 9 | 15 | 0 | 4 |
| D | 3-4 | 4 | 6 | 8 | 6 | 5 | 11 | 9 | 15 | 4 | 0.444 |
| E | 3-5 | 1 | 1.5 | 5 | 2 | 5 | 7 | 20 | 22 | 2 | 0.444 |
| F | 2-6 | 5 | 6 | 7 | 6 | 9 | 15 | 18 | 24 | 9 | 0.11 |
| G | 4-6 | 7 | 8 | 15 | 9 | 15 | 24 | 15 | 24 | 0 | 1.778 |
| H | 5-6 | 1 | 2 | 3 | 2 | 7 | 9 | 22 | 24 | 2 | 0.111 |

Critical path=A-C-G=9+6+9=24

$$V = [(Tp - To) / 6] \text{ ^2} = \sigma^2 =$$

$$\sigma = \sqrt{v} = \sqrt{(1 + 4 + 1.77)}$$

$$= 2.604$$

X=26

Tcp=24

Z=(X-Tcp)/$\sqrt{v}$

=(26-24)/2.606= 0.768

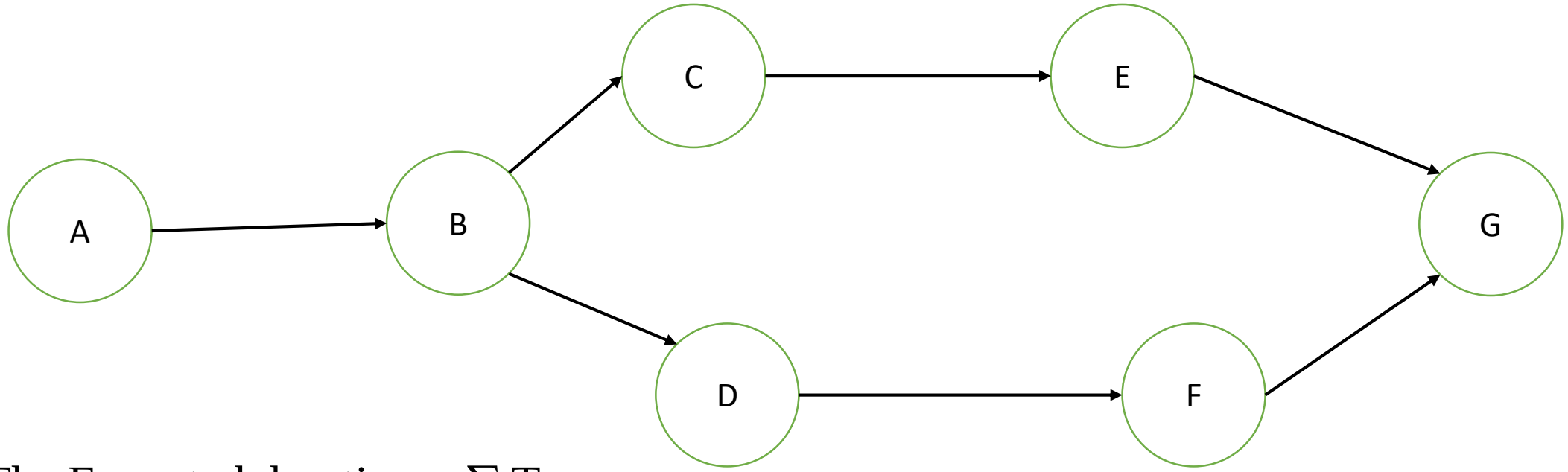
From z score table

The probability is 77%

- Example 2: - By using PERT draw the network diagram and determine Te, $\sigma$, V, and C.P for the following table

| Act | predecessor | to | tm | tp |
|---|---|---|---|---|
| A | --- | 2 | 4 | 9 |
| B | A | 5 | 8 | 14 |
| C | B | 4 | 10 | 13 |
| D | B | 4 | 7 | 10 |
| E | C | 11 | 14 | 20 |
| F | D | 9 | 13 | 16 |
| G | E,F | 2 | 4 | 6 |

| Act | predecessor | to | tm | tp | Te | varience | Standard deviation |
|---|---|---|---|---|---|---|---|
| A | --- | 2 | 4 | 9 | 4.5 | 1.36 | 1.17 |
| B | A | 5 | 8 | 14 | 8.5 | 2.25 | 1.5 |
| C | B | 4 | 10 | 13 | 9.5 | 2.25 | 1.5 |
| D | B | 4 | 7 | 10 | 7 | 1 | 1 |
| E | C | 11 | 14 | 20 | 14.5 | 2.25 | 1.5 |
| F | D | 9 | 13 | 16 | 12.8 | 1.36 | 1.17 |
| G | E,F | 2 | 4 | 6 | 4 | 0.44 | 0.67 |




The Expected duration = ∑ Te $_{CP}$ =

Variance = ∑ $\boldsymbol{v}$ $_{CP}$ =

Standard Deviation = $\sum\sqrt{\boldsymbol{vcp}}$

What is the probability that the project finishes in less than 38 days ?

$$Z = \frac{x - T}{\sqrt{\sigma_T{}^2}} \qquad \frac{38-41}{2.92} = -1.03$$

More than 43 days ? $\qquad \frac{43-41}{2.92} = 0.68$

What is the probability that activity G finishes in less than 3 days ?

$(\mathbf{3} - \mathbf{4}) / \mathbf{0.67} = -\mathbf{1.49}$

More than 1 day late ?

$(\mathbf{5} - \mathbf{4}) / \mathbf{0.67} = \mathbf{1.49}$

If P (Z) = 95% (0.95) what is the estimated project completion day ?

Z = 1.645 ( From Z Score Table )

$1.645 = (X - 41) / 2.92 \longrightarrow (1.645)(2.92) + 41 = X$

X = 45.803

Al-safwa University College
Department of Computer Technology Engineering
Fourth Class / Project Management

# Program Evaluation and Review Technique (PERT )

## Lec 8

**Huda majeed**

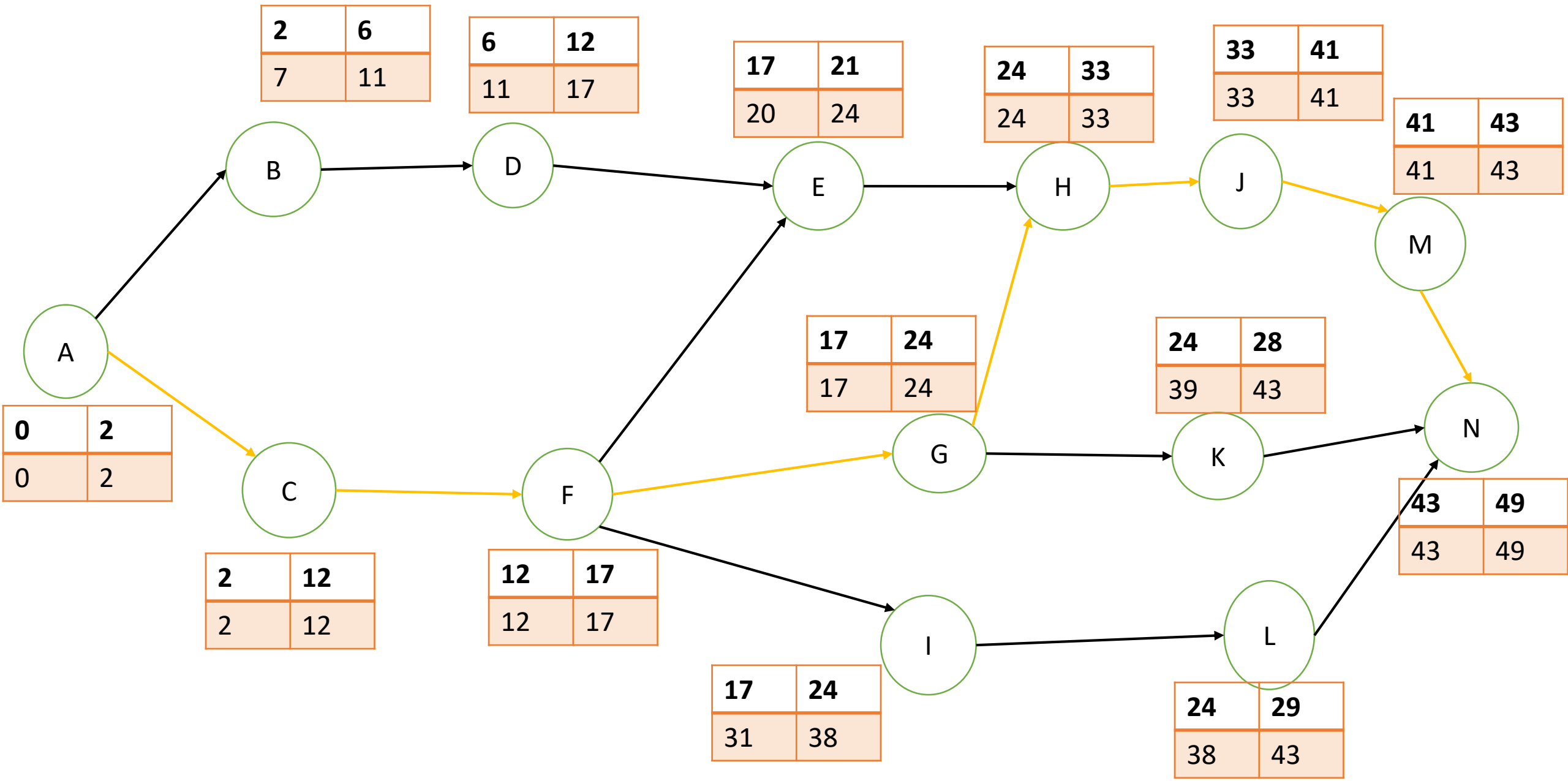
EX3: For a given activities determine

A-critical path using PERT

B-calculate the variance and standard devation

C-calculate probability of completing of completing the project in44 days

| Activity | Preceded by | to | tm | tp | te | v | ES | EF | LS | LF | T.F |
|---|---|---|---|---|---|---|---|---|---|---|---|
| A | ------ | 1 | 2 | 3 | 2 | 1/9 | 0 | 2 | 0 | 2 | 0 |
| B | A | 2 | 3.5 | 8 | 4 | 1 | 2 | 6 | 7 | 11 | 4 |
| C | A | 6 | 9 | 18 | 10 | 4 | 2 | 12 | 2 | 12 | 0 |
| D | B | 4 | 5.5 | 10 | 6 | 1 | 6 | 12 | 11 | 17 | 5 |
| E | D,F | 1 | 4.5 | 5 | 4 | 4/9 | 17 | 22 | 17 | 22 | 0 |
| F | C | 4 | 4 | 10 | 5 | 1 | 12 | 17 | 12 | 17 | 0 |
| G | F | 5 | 6.5 | 11 | 7 | 1 | 17 | 24 | 17 | 24 | 0 |
| H | E,G | 5 | 8 | 17 | 9 | 4 | 24 | 33 | 24 | 33 | 0 |
| I | F | 3 | 7.5 | 9 | 7 | 1 | 17 | 24 | 31 | 38 | 7 |
| J | H | 3 | 9 | 9 | 8 | 1 | 33 | 41 | 41 | 39 | 0 |
| K | G | 4 | 4 | 4 | 4 | 0 | 24 | 28 | 39 | 43 | 13 |
| L | I | 1 | 5.5 | 7 | 5 | 1 | 24 | 29 | 38 | 43 | 12 |
| M | J | 1 | 2 | 3 | 2 | 1/9 | 41 | 43 | 41 | 43 | 0 |
| N | M,K,L | 5 | 5.5 | 9 | 6 | 4/9 | 43 | 49 | 43 | 49 | 0 |


A
B
C
D
E
F
G
H
I
J
K
L
M
N
0 2
0 2
2 6
7 11
2 12
2 12
6 12
11 17
12 17
12 17
17 21
20 24
17 24
17 24
17 24
31 38
24 33
24 33
24 28
39 43
24 29
38 43
33 41
33 41
41 43
41 43
43 49
43 49

Z=(X-Te)/ σ

$$V = [(Tp - To) / 6] \text{ ^2} = \sigma^2 =$$

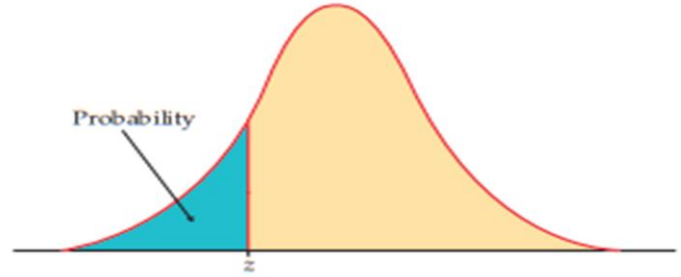
$$\sigma = \sqrt{v} = \sqrt{\left(\frac{1}{9}\right) + 4 + (1) + 1 + 4 + 1 + \left(\frac{1}{9}\right) + (\frac{4}{9})} = 3.4$$

X=44

Z=(44-49)/3.4=-1.47 from Z scor table

The probability is 7.08%

Table entry for $z$ is the area under the standard Normal curve to the left of $z$.




## TABLE A

**Standard Normal probabilities**

| $z$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| −3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| −3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| −3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| −3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| −2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| −2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| −2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| −2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| −2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| −2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| −2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| −2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| −2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| −2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| −1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| −1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| −1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| −1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| −1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| −1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| −1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| −1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| −1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| −1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| −0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| −0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| −0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| −0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| −0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| −0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| −0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| −0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| −0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| −0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

# Linear Programming

Linear Programming is the branch of applied mathematics that deals with solving optimization problems of a particular functional form. A linear programming problem consists of a linear objective function (of decision variables) which is to be minimized or maximized, subject to a certain set of linear constraints on decision variables. The constraints are usually in the form of linear inequalities of decision variables used in the objective function. Linear programming is a relatively recent mathematical discipline.

Graphical Method

• Graphical solution is limited to linear programming models containing only two decision variables (can be used with three variables but only with great difficulty).

• Graphical methods provide visualization of how a solution for a linear programming problem is obtained.

## Procedure to Solution in Graphical Method

Step I: Convert each inequality as equation

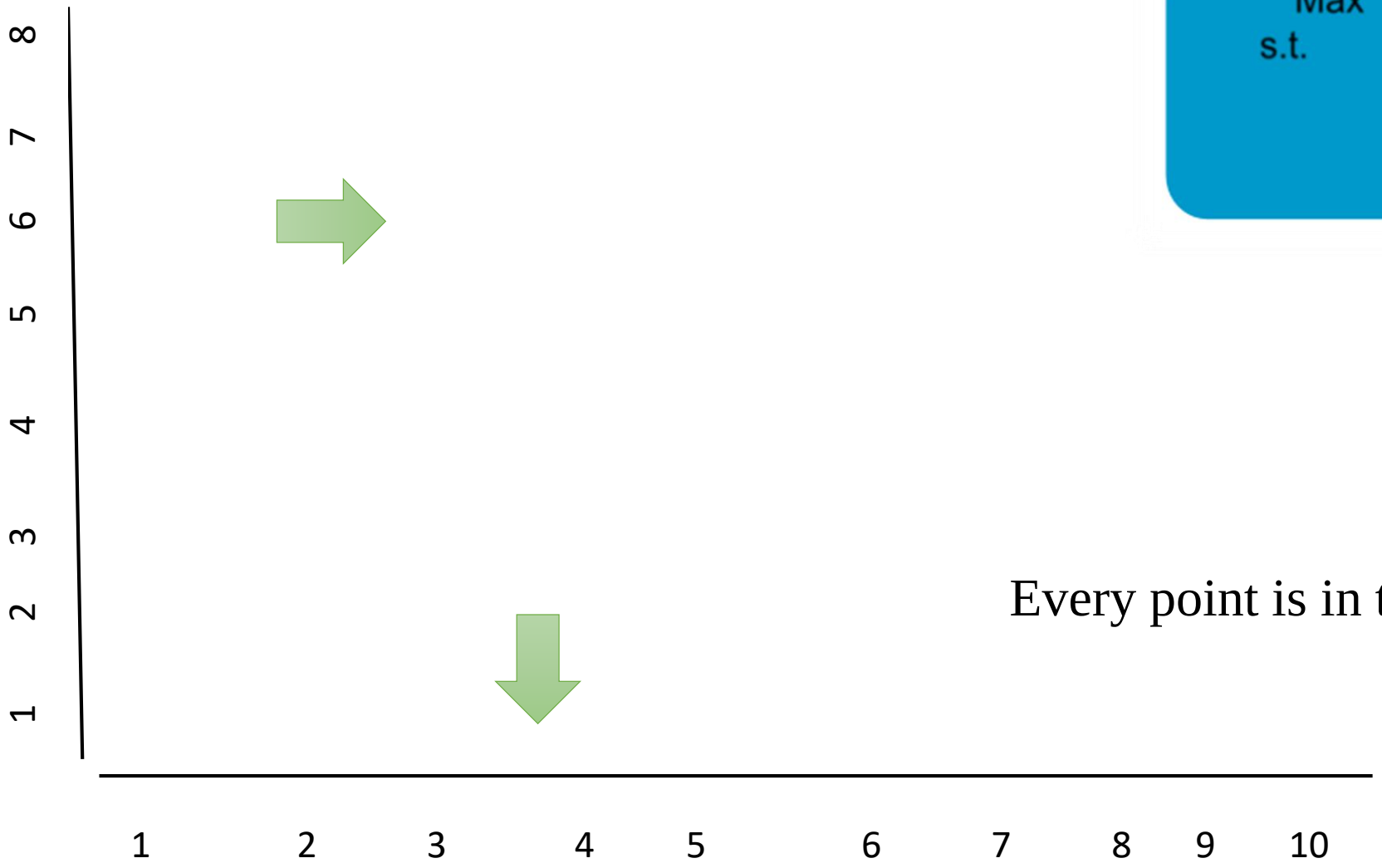
Step II: Plot each equation on the graph

Step III: Shade the ‘Feasible Region’. Highlight the common Feasible region. Feasible Region: Set of all possible solutions.

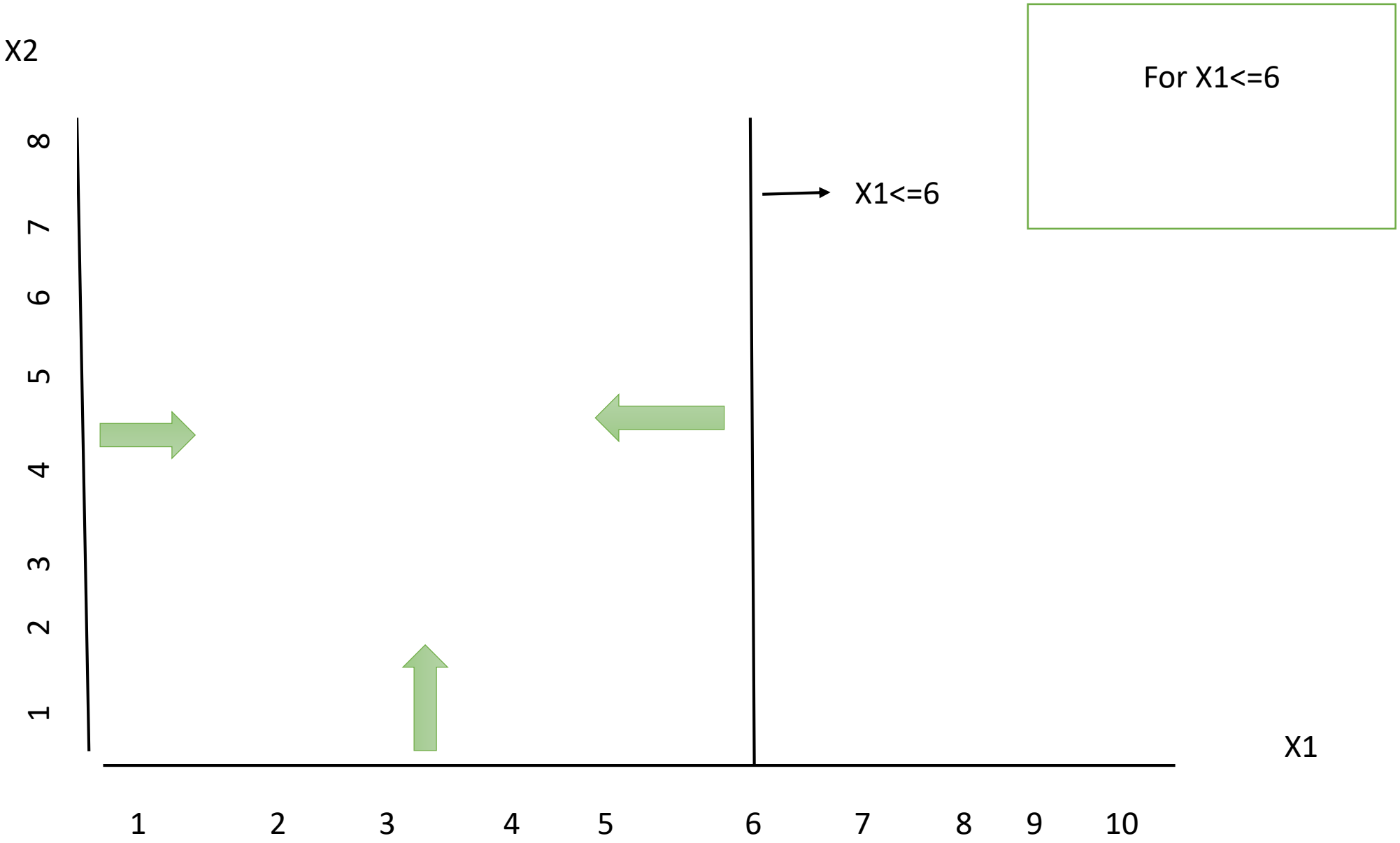
- Step IV: Compute the coordinates of the corner points (of the feasible region). These corner points will represent the ‘Feasible Solution’. Feasible Solution: If it satisfies all the constraints and non-negativity restrictions.
- Step V: Substitute the coordinates of the corner points into the objective function to see which gives the
- Optimal Value. That will be the ‘Optimal Solution’.
- Optimal Solution: If it optimizes (maximizes or minimizes) the objective function.
- Unbounded Solution: If the value of the objective function can be increased or decreased indefinitely, such solutions are called Unbounded solution.
- Infeasible (Inconsistent) Solution: It means the solution of problem does not exist. This is possible when there is no common feasible region.

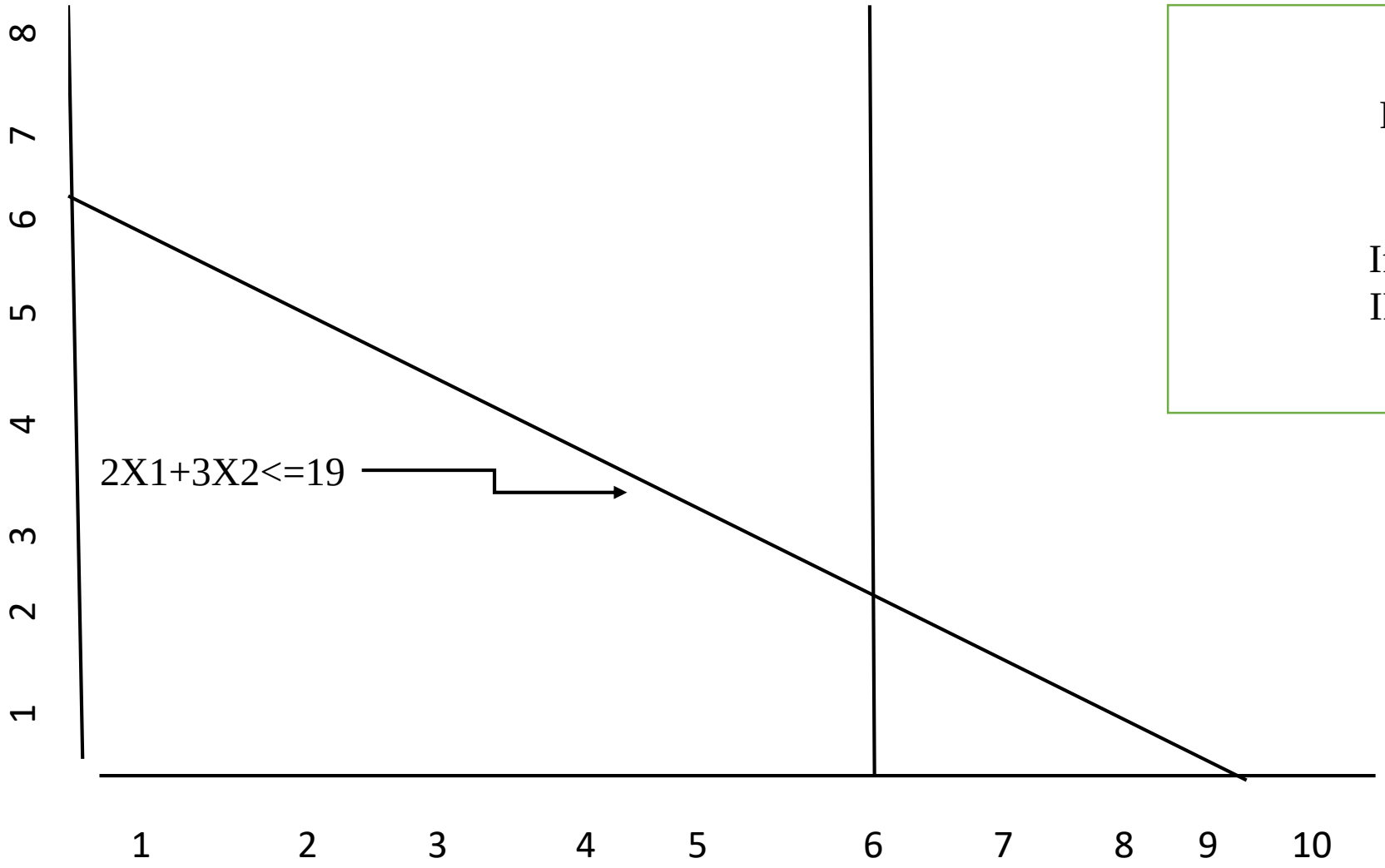
# Example1 :Solve by graphical method

$$\begin{aligned} \text{Max} \quad & z = 5x_1 + 7x_2 \\ \text{s.t.} \quad & x_1 \le 6 \\ & 2x_1 + 3x_2 \le 19 \\ & x_1 + x_2 \le 8 \\ & x_1, x_2 \ge 0 \end{aligned}$$



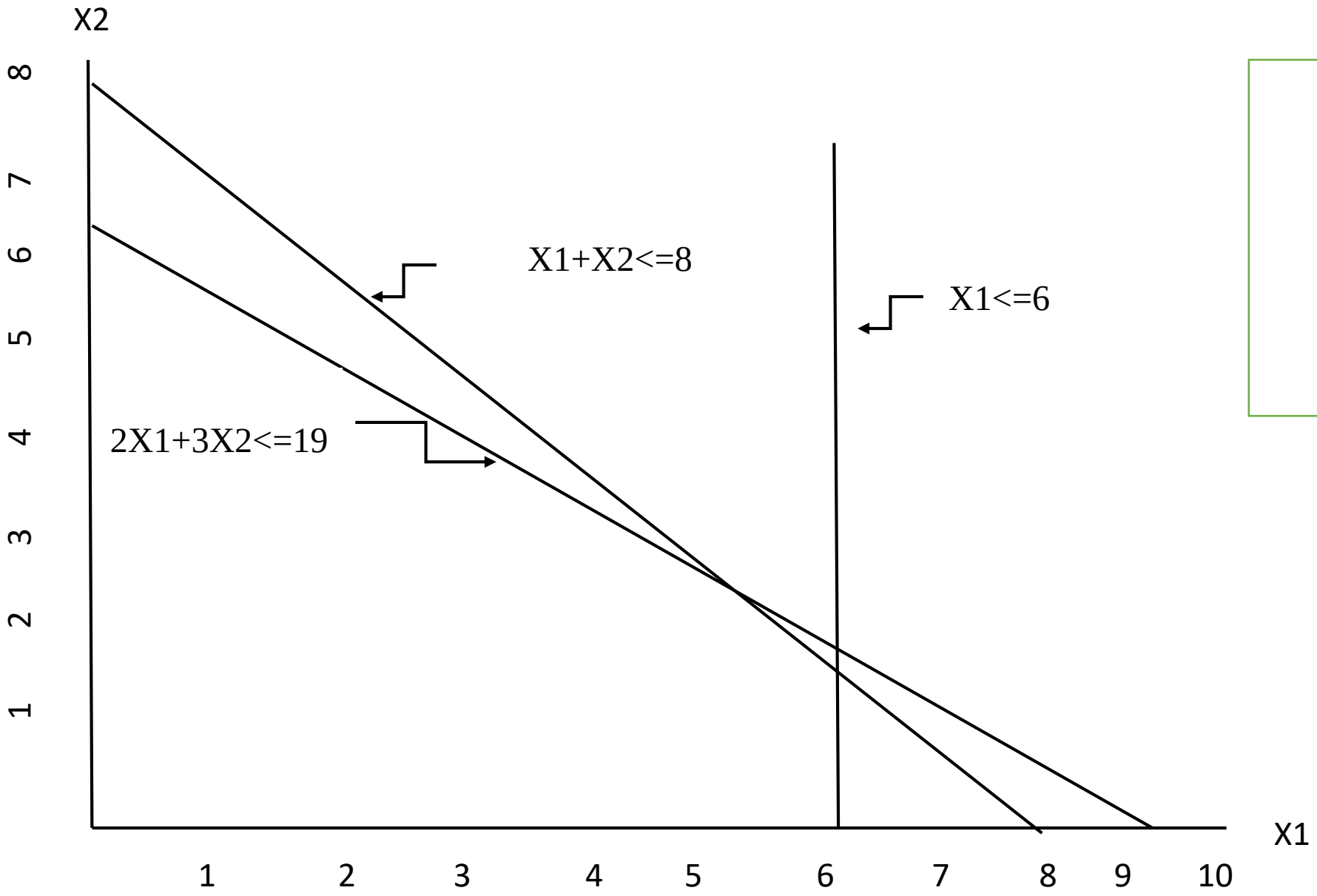

Every point is in this non-negative


X2
8
7
6
5
4
3
2
1
X1<=6
For X1<=6
X1
1
2
3
4
5
6
7
8
9
10



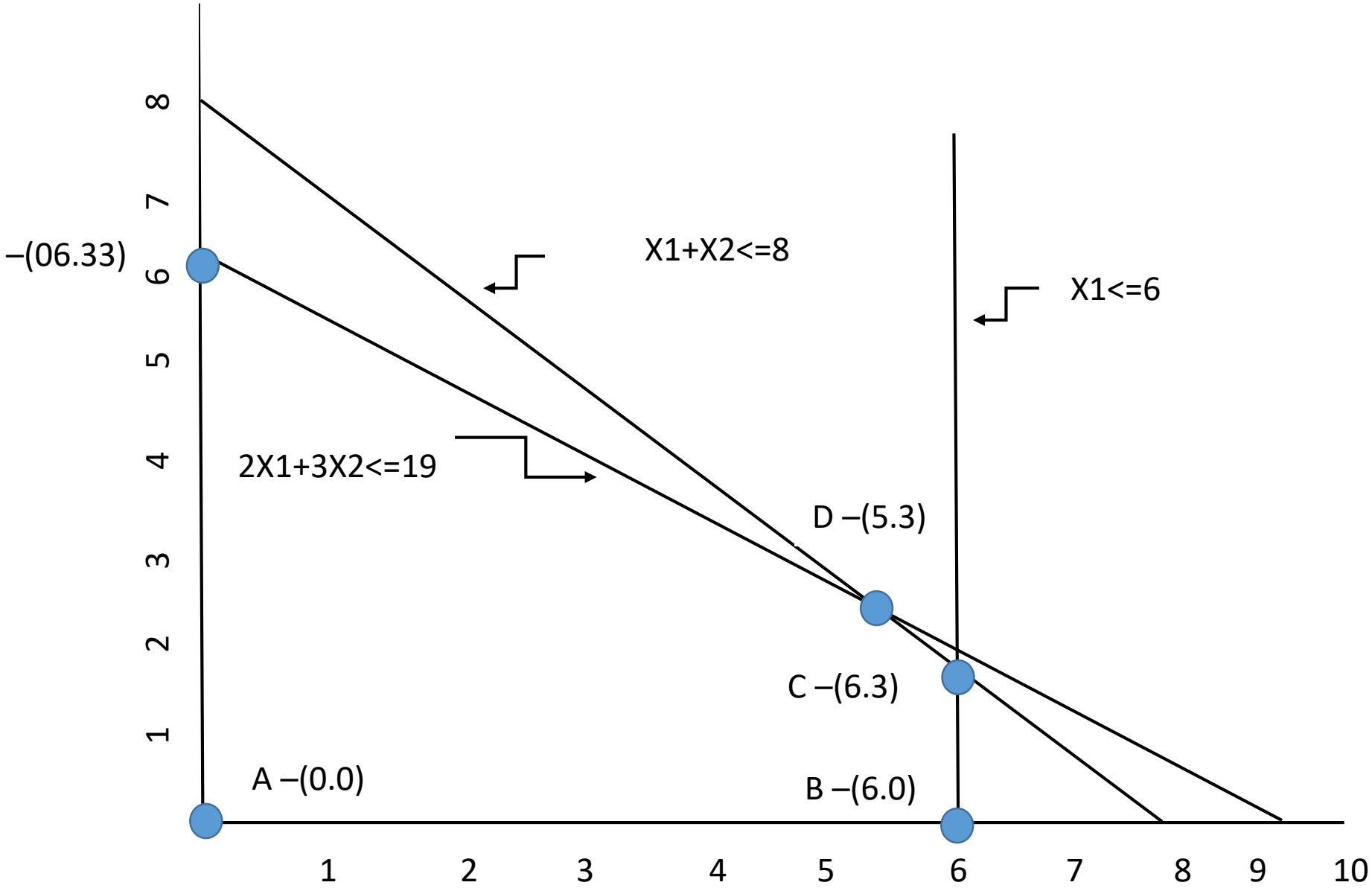

For 2X1+3X2=19

If X1=0 X2=6.33
IF X2=0 X1=9.5




For X1+X2=8

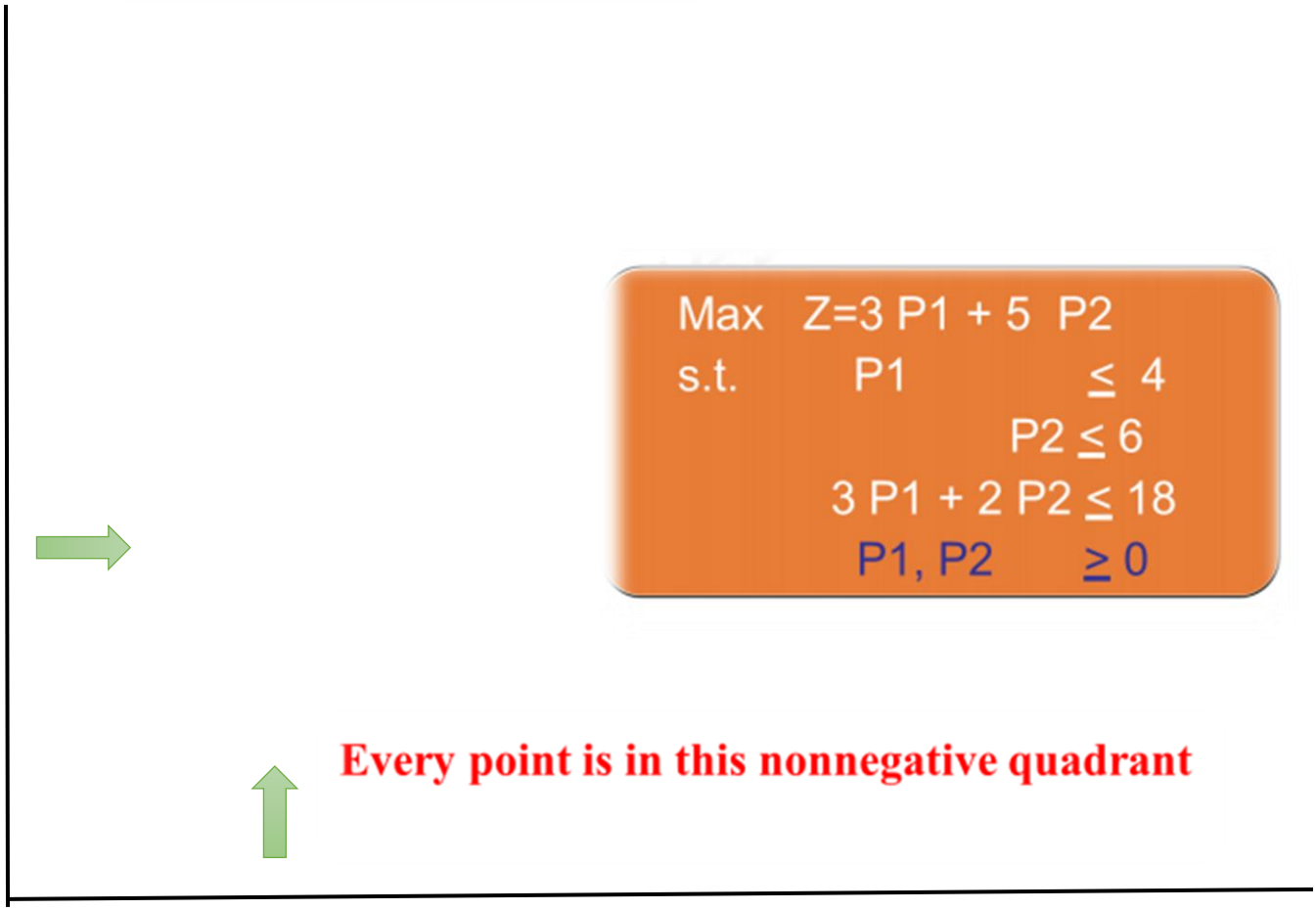
If X1=0 X2=8
IF X2=0 X1=8

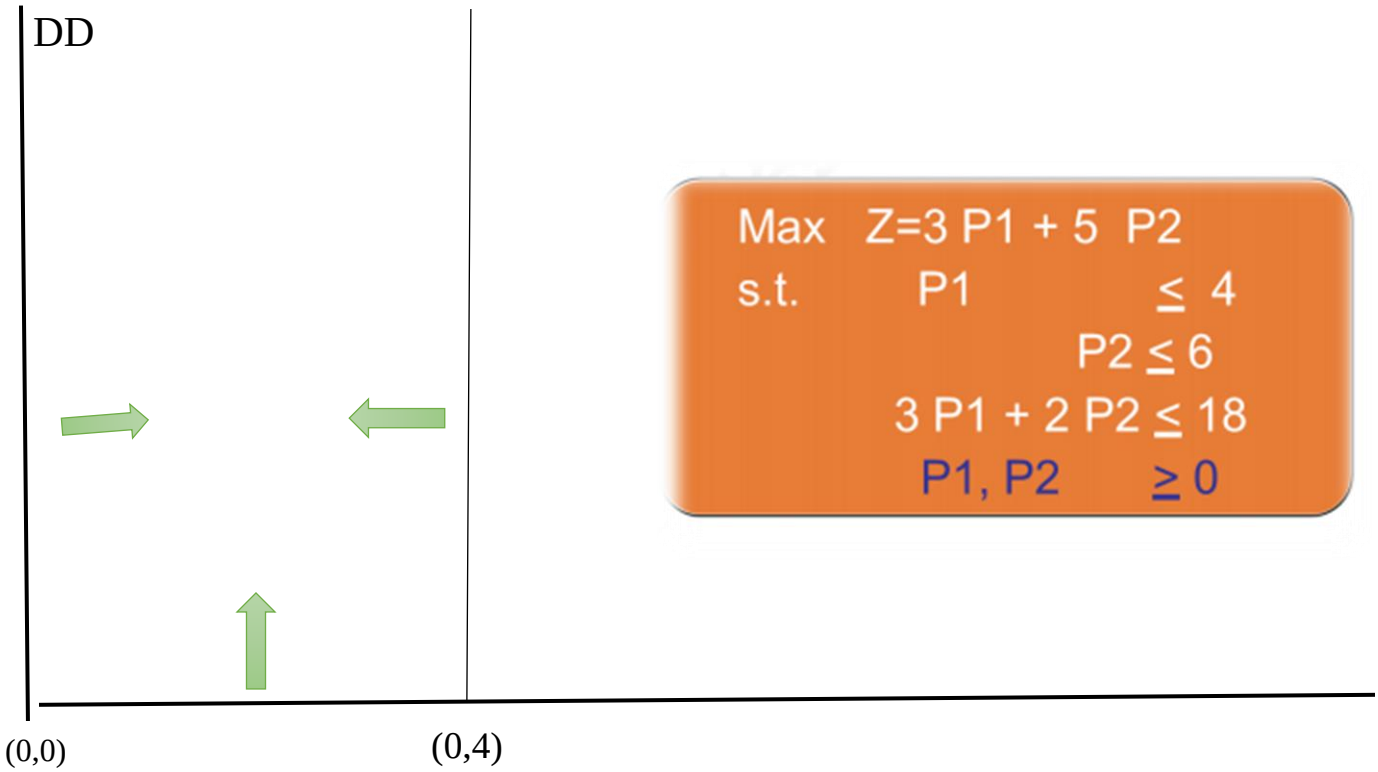

–(06.33)
X1+X2<=8
X1<=6
2X1+3X2<=19
D –(5.3)
C –(6.3)
A –(0.0)
B –(6.0)
1
2
3
4
5
6
7
8
9
10

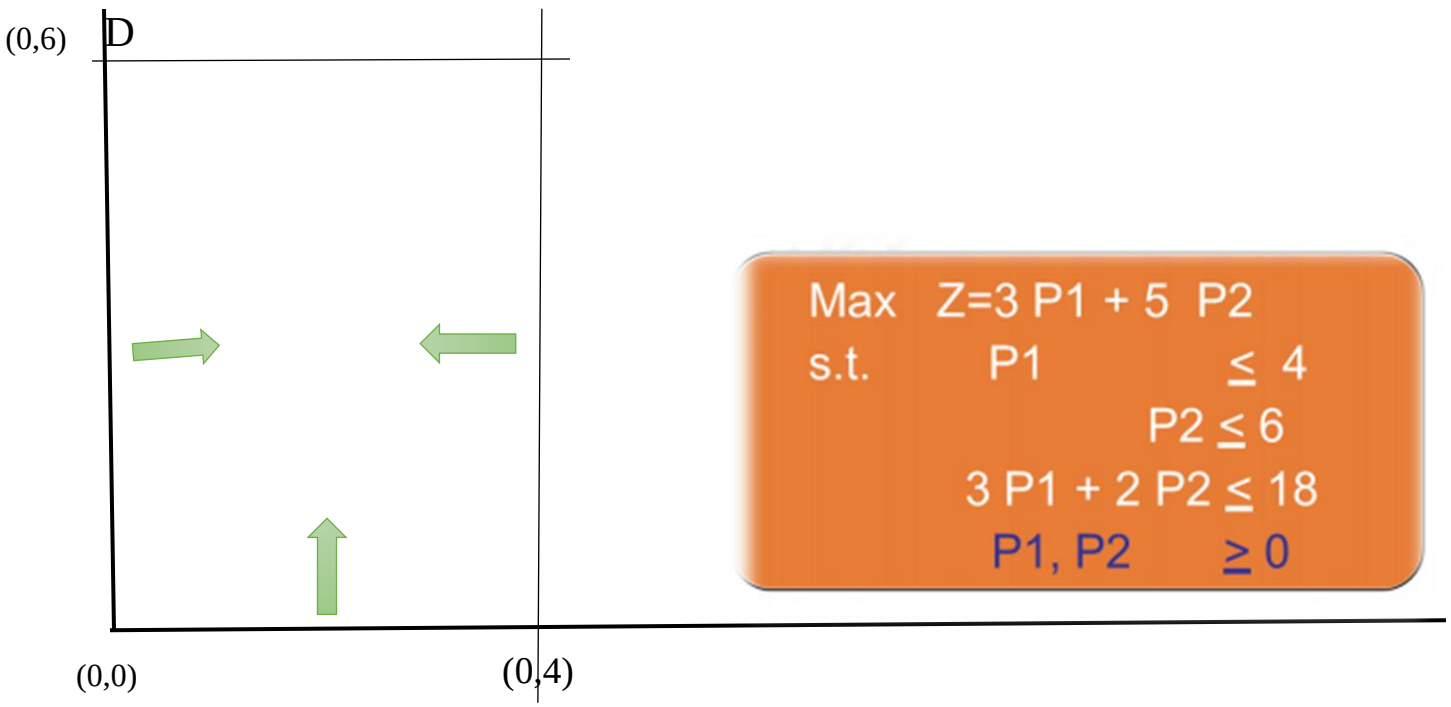
| **Objective Function: Max Corner Points** | **Z= 5x 1 +7x 2 Value of Z** |
| --- | --- |
| A (0,0) | 0 |
| B (6,0) | 30 |
| C (6,2) | 44 |
| D (5,3) | 46 |
| E (0,6.333) | 44.33 |
| Optimal Point: (5,3) Optimal Value: 46 | |

Example :

$$\begin{aligned} \text{Max} \quad & Z = 3P_1 + 5P_2 \\ \text{s.t.} \quad & P_1 \leq 4 \\ & P_2 \leq 6 \\ & 3P_1 + 2P_2 \leq 18 \\ & P_1, P_2 \geq 0 \end{aligned}$$




DD
Max Z=3 P1 + 5 P2
s.t. P1 ≤ 4
P2 ≤ 6
3 P1 + 2 P2 ≤ 18
P1, P2 ≥ 0
(0,0)
(0,4)



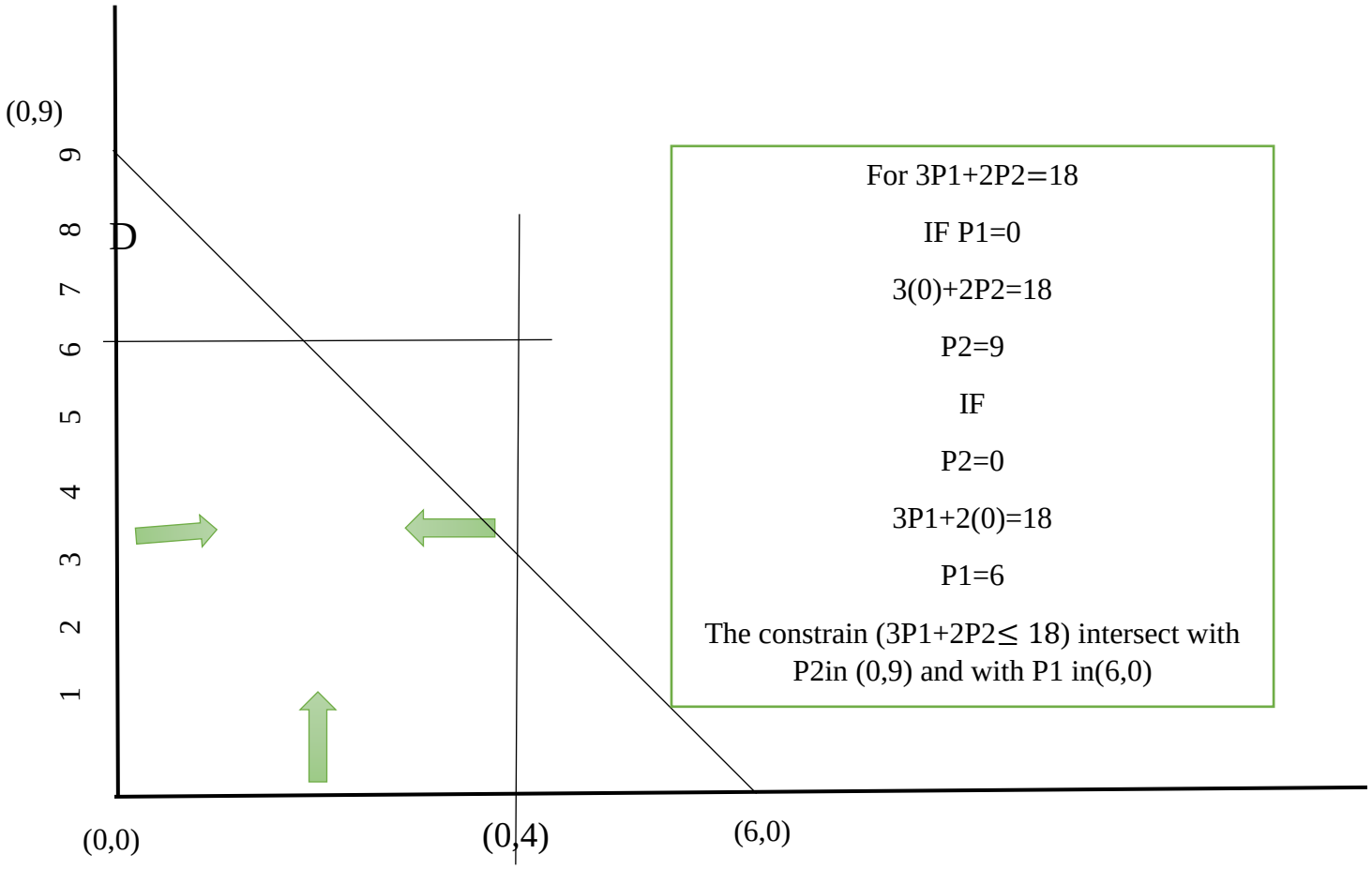

Max Z=3 P1 + 5 P2

s.t. P1 ≤ 4

P2 ≤ 6

3 P1 + 2 P2 ≤ 18

P1, P2 ≥ 0




For 3P1+2P2=18

IF P1=0

3(0)+2P2=18

P2=9

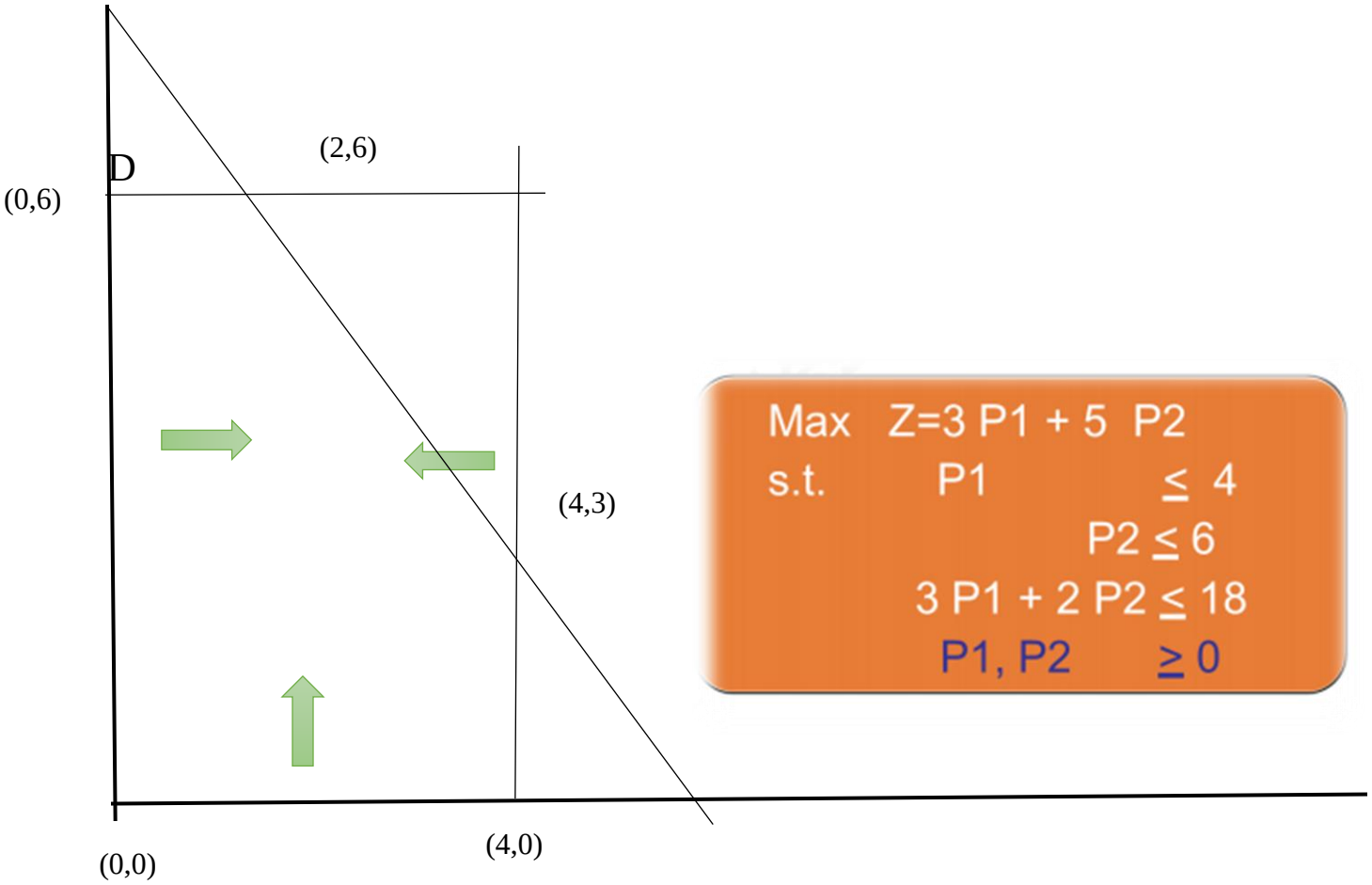
IF

P2=0

3P1+2(0)=18

P1=6

The constrain (3P1+2P2≤ 18) intersect with P2in (0,9) and with P1 in(6,0)




Object function :max Z=3P1+5P1

| Corner point | value of Z |
|---|---|
| A(0,0) | 0 |
| B(4,0) | 12 |
| C(4,3) | 27 |
| D(2,6) | 36 |
| E(0,6) | 30 |

Optimum point (2,6)

Optimum value =36

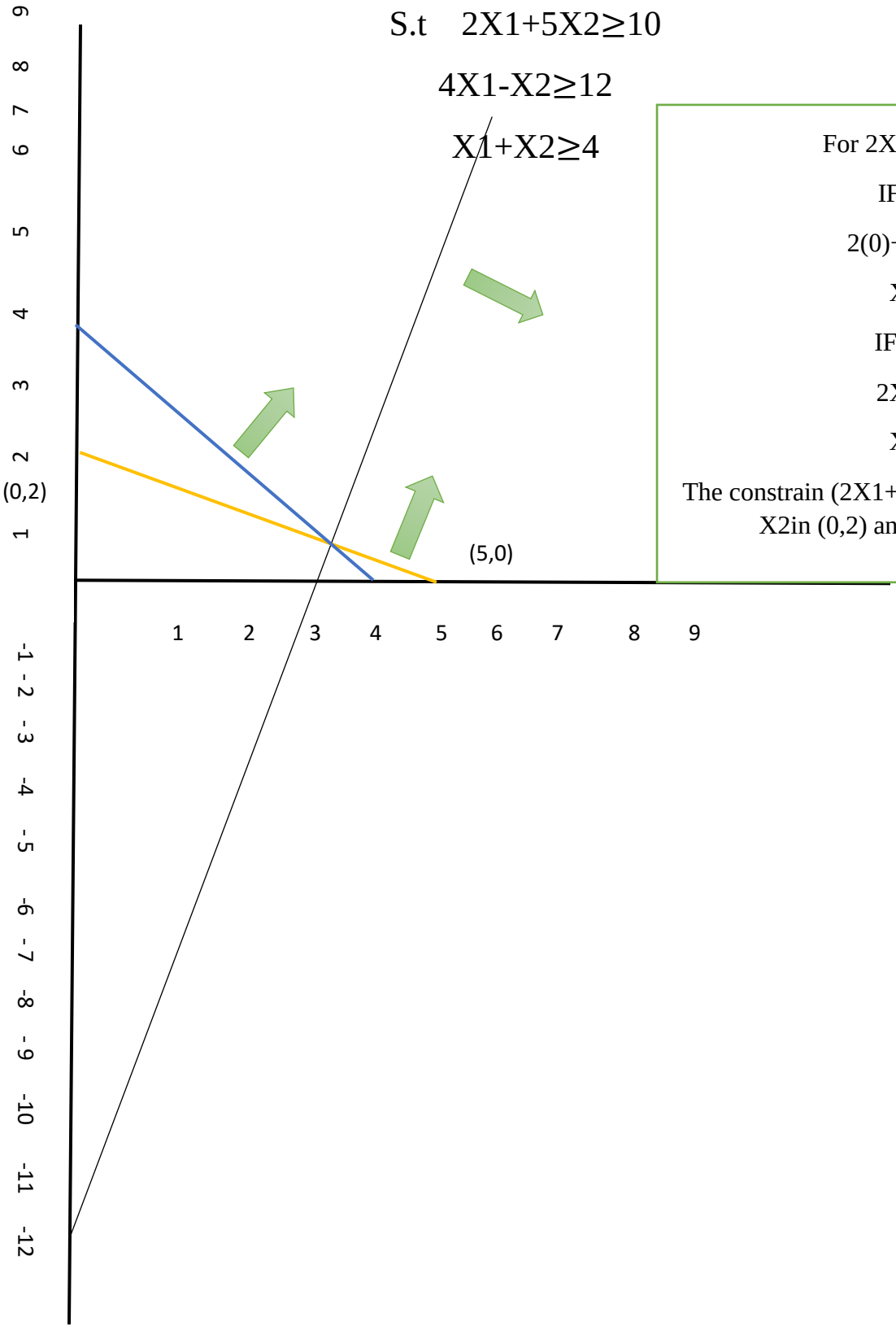
Example :

Min Z=5X1+2X2

S.t 2X1+5X2≥10

4X1-X2≥12

X1+X2≥4




For 2X1+5X2≥10

IF X1=0

2(0)+5X2=10

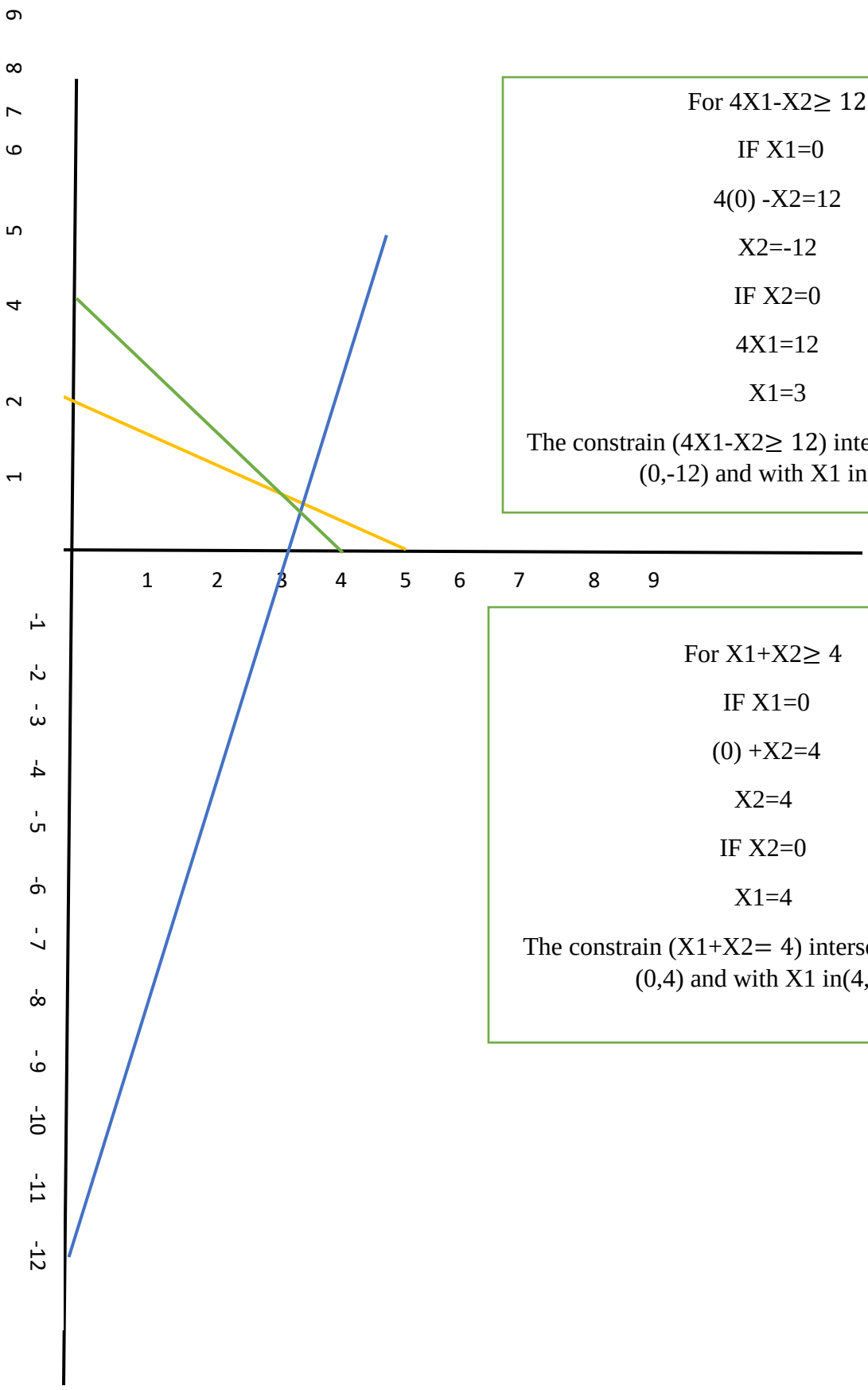
X2=2

IF X2=0

2X1=10

X1=5

The constrain (2X1+5X2≥ 10) intersect with X2in (0,2) and with X1 in(5,0)




For $4X1-X2 \geq 12$

IF X1=0

4(0) -X2=12

X2=-12

IF X2=0

4X1=12

X1=3

The constrain ($4X1-X2 \geq 12$) intersect with X2in (0,-12) and with X1 in(3,0)

For $X1+X2 \geq 4$

IF X1=0

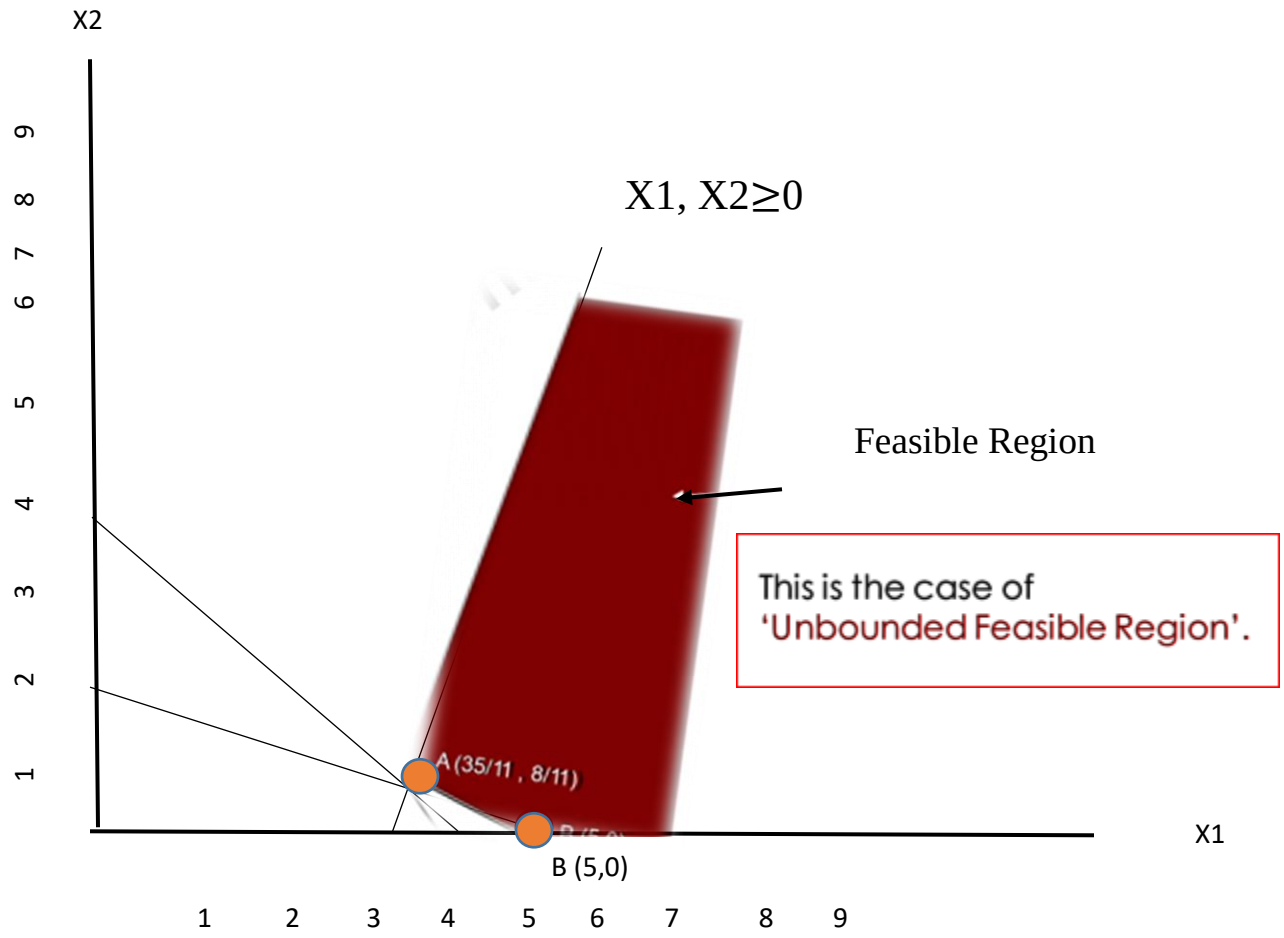
(0) +X2=4

X2=4

IF X2=0

X1=4

The constrain ($X1+X2 = 4$) intersect with X2in (0,4) and with X1 in(4,0)


X2
X1
X1, X2≥0
Feasible Region
This is the case of
'Unbounded Feasible Region'.
A (35/11 , 8/11)
B (5,0)
1
2
3
4
5
6
7
8
9

## The Interest

In this lecture, we will study the subject of "Interest" and its types.

## There are two types of Interest

1. Simple interest

2. Compound interest

Simple interest is an interest that is calculated on an amount of money that's already owned but not on previous interest.

## The following symbol and definitions

P: - principle, a sum of money invested in initial year, or The present sum of money.

R: - interest rate per of unit of time expressed as a decimal (Annual rate interest)

ex: - 5% = 0.05 , 25% = 0.25

**T:** - Time, the number of units of times over which inters Accumulates.

**I:** - simple interest

F: - compound amount, a sum of money at end of T units at interest R.

# Simple interest

Interest is the fee paid for borrowed money. The initial amount of money that put in a savings account is called (deposit) which we receive interest instead of our money that we already gave. Simple interest is calculated by applying the interest rate to the principal only,not to interest earned.

• Your deposit is called **PRINCIPAL**

• The bank takes the money and invests it.

• In return, the bank pays you **INTEREST** based on the **INTEREST RATE**.

• Simple interest is interest paid only on the PRINCIPAL.

Simple Interest Formula **(I= PRT**)

• I = interest

• P = principal

• R = the interest rate per year

• T = the time in years.

**Example1:** - an amount of 2500$ is deposit in a bank offering 5% simple interest per annum what is the interest at the end of the first year?

Solution: -

P= 2500$ R=0.05 T= 1

Simple interest (I) = P R T

= 2500 * 0.05 * 1

= 125 $

**Example2**: - Suppose you deposit $400 in a savings account. The interest rate is 5% per year. 1. Find the interest earned in 6 years. 2. Find the total of principal plus interest.

I = PRT

P = 400, R = 0.05 = 5%, T = 6 (in years)

Interest on one year = 400 x 0.05 x 1 = $20

Interest on $400 over 6 years = 400 x 0.05 x 6 = $120

Amount in account after 6 years = 400 + 120 = $520

## Maturity Value

Is the resulting value of adding the value of interest to the principal

Example3: Calculate the maturity value if the 8% $10,000 loan is for 1 year

I=PRT

P= 10,000, R=0.08, T=1

Interest on one year I= 10,000 x 0.08 x 1 = $800

M=P+I = $10,000 + $800 = $10800

## Home Work: -

1. On July 10, 2005, Sally Smith borrowed $12,000 from her Aunt Sarah. If Sally agreed to pay a 9% annual rate of interest, calculate the dollar amount of interest she must pay if the loan is for (a) 1 year, (b) 5 months, and (c) 15 months.

**Reminder**: Time is always in terms of Years. So, if you're dealing with months, you have to make your months a fraction of a year.


## Department of Computer Technology Engineering

# Compound Interest

Prepared by

Huda Majeed

**Example 7**: - if you deposit 1000$ into an account paying 12% annual interest compound annually how much will be in the account after 4 years?

**Solution: -**

P=1000$

R=12%

t=4 years

$B=p(1+R)^t$

$B=1000(1+0.12)^4$

B=1573.5$

**Example 8:** - Calculating values of balance maturity with varying compounding periods. we suppose to invest $10,000 in a 5-year with interest of 6%.

a) What is the value of the final balance if interest is compounded annually?

In this case maturity occurs at 5 years.

b) What is the value of the final balance if interest is compounded quarterly?

c) What is the value of the final balance if interest is compounded monthly?

d) What is the value of the final balance if interest is compounded daily?

e) Compare your answers from parts a, b, c, and d.

**Solution: -**

a) The annual compounding rate, has 6% of interest & as a decimal is R = 0.06. We use 1 + R = 1.06 and t = 5 years in the compound interest formula Balance after 5 years $=P\ (1+R\ )^{t}$

$=10000\ (1+0.06\ )^{5}$

= $13,382.26

b) Again, we use the compound interest formula. To find the quarterly rate we divide the R by 4. The R as a decimal is 0.06 so as a decimal the quarterly rate is

Periodic rate = quarterly rate (r) $=\frac{Annual\ Rate\ (R)}{Periodic\ per\ year\ (m)}$

Periodic rate(r) $=\frac{0.06}{4}$

=0.015

P= 10000$

r=0.015,

t= 5

m=4

compound interest formula: $B=P\ (1+r)^{mt}$

$B=10000\ (1+0.015\ )^{20}=13468.55$

c) This time we want the monthly rate, so we divide the R by 12:

Periodic rate = Monthly rate (r) $=\frac{0.06}{12}$

=0.005

Also, 5 years is 60 months, so

r=0.015,

t= 5

m=12

Balance after 60 months = principle$(1 + r)^{5*12}$

=10000*$1.005^{60}$=13488.50$

d) We assume that there are 365 days in each year, so as a decimal the daily

rate is: -

Periodic rate = Daily rate (r) =$\frac{0.06}{365}$

r=0.00016,

t= 5

m=365

m*t = 5 x 365 = 1825 days, so

Balance after 1825 days== principle$(1 + r)^{t}$

=10000*$(1 + \frac{0.06}{365})^{1826}$=13498.26$

e) We summarize the results above in the following table.

| compounding period | balance maturity |
|---|---|
| yearly | 13382.26 |
| quarterly | 13468.55 |

| | |
|---|---|
| monthly | 13488.50 |
| daily | 13498.26 |

This table shows that increasing the number of compounding periods increases the interest earned even though the R and the number of years stay the same.