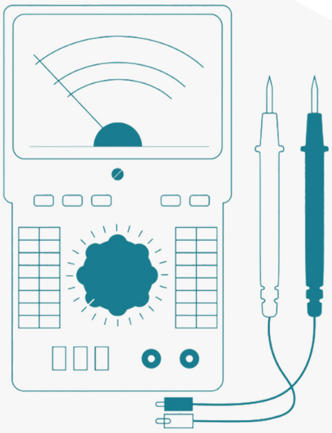
Al-Safwa University College
Department of
Computer Techniques Engineering

Measurement and Instruments

2nd Stage



Lecturer
Ehab Monther

System of Units

The principle aspects of the scientific method are accurate measurement, selective analysis, and mathematical formulation. Note that the first and most important is accurate measurements.

Measurement**:** *is the process by which one can convert physical parameters to meaningful number.*

Instrument: *may be defined as a device for determining the value or magnitude of a quantity or variable.*

The standard measure of each kind of physical quantity is the unit; the number of times the unit occurs in any given amount of the same quantity is the number of measure. With out the unit, the number of measure has no physical meaning.

Fundamental and Derived Units

To measure an unknown we must have acceptable unit standard for the property that is to be assessed. Since there are virtually hundreds of different quantities that man is called upon to measure, it would seem that hundreds of different standard units would be required. Fortunately, this is not the case. By choosing a small number of basic quantities as standards, we can define all the other in terms of these few.

The basic units are called ***fundamentals***, while all the others which can be expressed in terms of fundamental units are called ***derived*** units, and formed by multiplying or dividing fundamental units. The ***primary fundamental*** units which most commonly used are ***length, mass, and time,*** while measurement of certain physical quantities in ***thermal, electrical, and illumination*** disciplines are also represented by fundamental units. These units are used only when these particular classes are involved, and they may therefore be defined as ***auxiliary fundamental*** units. Every derived unit originates from some physical law defining that unit. For example, the voltage [volt]:

$$volt = \frac{workdone}{charge} = \frac{Joule}{coulomb} = \frac{J}{C} = \frac{Force \times distance}{current \times time} = \frac{Newton \times meter}{Amper \times second} \Rightarrow$$

$$volt = \frac{mass \times acceleration \times meter}{current \times time} = \frac{mass \times \frac{velocity}{time} \times meter}{current \times time} = \frac{mass \times \frac{distance}{time^2} \times meter}{current \times time}$$

$$volt = \frac{mass \times \frac{meter^2}{time^2}}{current \times time} = \frac{mass \times meter^2}{current \times time^3} = \frac{Kg.m^2}{A.\sec^3} = \left[Kg.m^2.A^{-1}.\sec^{-3}\right] \text{ basic S.I units}$$

A derived unit is recognized by its ***dimensions***, which can be defined as the complete algebraic formula for the derived unit. The dimensional symbols for the fundamental units of length, mass, and time are **L, M**, and **T**, respectively. So the dimensional symbol for the derived unit of voltage is $V = \frac{M.L^2}{I.T^3} = \left[M.L^2.I^{-1}.T^{-3}\right]$

Table (1) shows the six basic ***S.I*** quantity and units of measurement, with their unit symbol:

Table (1):

Quantity	*Unit*	*Symbol*
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electrical current	Ampere	A
Thermodynamic temperature	Kelvin	K
Luminous intensity	Candela	cd

Table(2) : shows the development of system of units since 1790 to our days

Quantity	*Dimensional symbol*	*British*	*CGS*	*CGSe*		*CGSm*		*MKS*	*MKSA S.I units*
length	L	ft	cm	cm	+ϵ_o=1 for free space	cm	+μ_o=1 for free space	m	m
mass	M	lb	g	g		g		kg	kg
time	T	sec	sec	sec		sec		sec	sec
current	I								Amp

The CGS electrostatic system ***(CGSe)*** used coulomb's law for the force between two electric charges. $\boldsymbol{F = k\frac{Q_1Q_2}{r^2}}$, and assume $k= 1/\epsilon_o$ to find the basic S.I units for electric charge Q which equal ($cm^{3/2}g^{1/2}s^{-1}$), then from electric charge all electrical units (I, V, E, C…) are determined by their defining equations. The same things is depended in CGS electromagnetic system ***(CGSm)***, but at this time coulomb's law determine the force between two magnetic poles with proportionality factor $k=1/\ \mu_o$ to derived electromagnetic unit of polestrength (m) which equal ($cm^{3/2}g^{1/2}s^{-1}$) then determine all other magnetic units (B, H, Φ, …). ***Rationalised system*** of units used **(MKS)** system and assignicant the value of $\boldsymbol{\mu_0=4\pi x10^{-7}}$ ***H/m***, and $\boldsymbol{\epsilon_o=8.85x10^{-14}}$ ***F/m***.

Multiples and Submultiples of units

The units in actual use are divided into submultiples for the purpose of measuring quantities smaller than the unit itself. Furthermore, multiples of units are designated and named so that measurement of quantities much larger than the unit is facilitated. Table(3) lists the decimal multiples and submultiples of units.

Table(3):

Name	*Symbol*	*Equivalent*
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	K	10^{3}
milli	m	10^{-3}
micro	µ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

Basic Definitions:

1. ***Speed, Velocity***: the rate of change of distance with respect to time

$$\upsilon = \frac{\partial x}{\partial t} \quad , \qquad \chi = \int_0^t \upsilon \partial t = \upsilon . t \quad , \qquad \upsilon = \frac{\chi}{t}$$

$\upsilon = \left[LT^{-1} \right]$ basic dimensions, $\upsilon = \left[m \sec^{-1} \right]$ basic S.I units

2. ***Acceleration:*** the rate of change of velocity during the time

$$a = \frac{\partial \upsilon}{\partial t} , \qquad \upsilon = \int_0^t a \partial t = a.t \quad , \qquad a = \frac{\upsilon}{t}$$

$a = \left[LT^{-2} \right]$ basic dimensions, $a = \left[m \sec^{-2} \right]$ basic S.I units

3. ***Momentum:***

$$p = mass \times velocity = m \times \upsilon$$

$p = \left[MLT^{-1} \right]$ basic dimensions , $p = \left[kgm \sec^{-1} \right]$ basic S.I units

4. ***Force:*** (Newton), the rate of change of momentum during the time

$F = \frac{\partial p}{\partial t} = \frac{\partial (m \upsilon)}{\partial t}$, $F = \left[MLT^{-2} \right]$ basic dimensions, $F = \left[kgm \sec^{-2} \right]$ basic S.I units

5. ***Energy:*** (Joule), the distance integral of force

$$E = \int_0^{\chi} F \partial \chi = F . \chi$$

$E = \left[ML^2 T^{-2} \right]$ basic dimensions, $E = \left[kgm^2 \sec^{-2} \right] = Joule = J$

6. ***Power:*** (Watt) , the rate of work done

$$P = \frac{\partial E}{\partial t}$$

$P = [ML^2T^{-3}]$ basic dimensions, $P = [kgm^2 \sec^{-3}]$ S.I units, $P = J.\sec^{-1}$

7. ***Potential of a point (voltage):*** work done to bring a unit charge from infinity to same point.

$$V = \frac{workdone}{charge} = \frac{Joule}{coulomb}$$

$V = [ML^2I^{-1}T^{-3}]$ basic dimensions, $V = [kgm^2A^{-1}\sec^{-3}]$ basic S.I units

8. ***Electrical current:*** the rate of flow of charge

$$I = \frac{\partial Q}{\partial t}, \quad Q = \int_0^t I\partial t, \quad Q = I.t$$

$I = [Amp]$

9. ***Resistance (ohm):*** the resistance of a load to the current flow when there is voltage difference between its terminals.

$R = \frac{\partial V}{\partial I}$, $R = [ML^2I^{-2}T^{-3}]$ dimensions, $R = [kgm^2A^{-2}\sec^{-3}]$ basic S.I units

10. ***Capacitance (farad):***

$C = \varepsilon\frac{A}{d}$, or $C = \frac{Q}{V}$, $C = [M^{-1}L^{-2}I^2T^4]$, $C = [kg^{-1}m^{-2}A^2\sec^4]$

11. ***Electrical field:***

$$E = \frac{\partial V}{\partial x}, \quad E = [MLI^{-1}T^{-3}], \quad E = [kgmA^{-1}\sec^{-3}]$$

12. ***Permittivity Є***: how much electrical field lines can pass through some medium

$\epsilon = \frac{farad}{m}$, $\epsilon = [M^{-1}L^{-3}I^2T^4]$, $\epsilon = [kg^{-1}m^{-3}A^2\sec^4]$

13. ***Inductance(henry):***

Induce emf = inductance x rate of change of current

$$e = -L\frac{\partial i}{\partial t}, \quad \int_0^t e\partial t = L\int_0^i \partial i, \quad L = \frac{et}{I}$$

$Henry = [ML^2I^{-2}T^{-2}]$, $Henry = [kgm^2A^{-2}\sec^{-2}]$

14. ***Reluctance (S):*** the magnetic resistance to magnetic field lines in same material

$S = \frac{l}{\mu.A}$, $S = [M^{-1}L^{-2}I^2T^2]$, $S = [kg^{-1}m^{-2}A^2\sec^2]$

15. ***Magnetic flux(Φ) weber:***

$$\phi = \frac{mmf}{S} = \frac{N.I}{S}, \ \phi = \left[ML^2 I^{-1} T^{-2}\right], \ \phi = \left[kgm^2 A^{-1} \sec^{-2}\right]$$

16. ***Frequency(hertz):*** number of cycles in one second

$$f = \frac{cycles}{\sec ond} = \frac{1}{\sec}, \quad f = \left[T^{-1}\right], \quad f = \left[\sec^{-1}\right]$$

17. ***Light speed (c):***

a) Speed of light in free spaces $c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$

b) Speed of light in same medium $\upsilon = \frac{1}{\sqrt{\mu\varepsilon}}$

c) Diffraction factor $N = \frac{c}{\upsilon}$

Notes that constant and numbers have no units (unit less)

Review on measuring Instrument

1- ***Electrical and Electronic Instruments***

The measuring instrument that use mechanical movement of electromagnetic meter to measure voltage, current, power, etc. is called ***electrical*** measuring instrument, so the heart of these instruments was the d'Arsonval meter, while any measurement system use d'Arsonval meter with amplifiers to increase the sensitivity of measurements is called ***electronic*** instrument.

2- ***Analogue and Digital Instruments***

An ***analogue*** instrument are the instrument that use analogue signal (signal varying in continuous fashion and take on an infinity number of values in any given range) to display the magnitude of quantity under measurement.

The ***digital*** instrument use digital signal (signal which vary in discrete steps and take up only finite different values in a given range, like binary signal which take only two levels zero and one) to indicate the results of measurement in digital form.

3- ***Absolute and Secondary Instruments***

In ***absolute*** instrument the measured value is given in term of instrument constants and the deflection of one part of the instrument e.g. tangent galvanometer, and Rayliegh current balance. In these instruments no calibrated scale is necessary. While in ***secondary*** instruments, the quantity of the measured values is obtain by observing the out put indicate by these instruments.

Classification of Secondary Instruments

a) ***Indicating Instruments***

The magnitude of quantity being measured is obtain by deflection the pointer on scale, and the output is indicate either in analogue or digital form like *ammeter, voltmeter, and wattmeter.*

Three forces was acting on the pointer to deflect it in proportional to the quantity being measured, these forces are:

i) *Deflecting Force*

This force gives the pointer the initial force to move it from zero position, its also called operating force.

ii) *Controlling Force*

This force control and limits the deflection of the pointer on scale which must be proportional to the measured value, and also ensure that the deflection is always the same for the same values.

iii) *Damping Force*

This force is necessary in order to bring the movement system (pointer) to rise quickly to the measured value, and then stop without any oscillation.

b) ***Recording Instruments***

An instrument which makes a written record in any recorded medium to the quantity being measured in order to save information and use it in anther time or anther place. Recording instrument may record transient signal, or phenomena which can not obtain readily. This instruments like *recording devices, X-Y plotter, and oscilloscope.*

c) ***Controlling Instruments***

These instruments give an information or instruction (orders) to control on original measured quantity or control on other devices, like *a computer*.

Factors Effecting Instrument selection

1- ***Accuracy***

Its represent how ***closeness*** with which an ***instrument reading*** approaches the ***true value*** of the variable being measured.

The deviation of the measured value from the true value is the indication of how accurately reading has been made.

2- ***Precision***

It's specified the ***repeatability*** of a set of reading each made independently with the same instrument.

An estimate of precision is determined by the deviation of different reading from the mean (average) value.

Example:

To detect the deference between accuracy and precision of measurement for some voltage, we see the following cases:

i) V=6Volt (true or theoretical value) V=5.8Volt (measured or practical value)
This instrument is accurate

ii) V=6Volt (true or theoretical value) V=4.8Volt (measured or practical value)
This instrument is not accurate

iii) V=6Volt (true or theoretical value) V=5.8Volt (measured or practical value)
When we try to check the reading, we measured it again and again, and get the following results: second measure for the same reading equal V=5.8Volt, third measured V=5.8Volt, forth measured V=5.8Volt and so on.
This instrument is accurate and precise

iv) V=6Volt (true or theoretical value) V=4.8Volt (measured or practical value)
We try to check the reading, we measured it again and again, and get the following results: second measure for the same reading equal V=5Volt, third measured V=4.6Volt, forth measured V=5.2Volt and so on. This instrument is not accurate and not precise.

3- ***Range***

It is defined as that region enclosed by the limits within which a particular quantity is measured.

4- ***Span***

It is algebraic difference of the upper and lower limits of the range.

Example:

The span of (0 to10) voltmeter is $\boxed{\text{Span= 10-0=10 state}}$

But the span for (-10 to +10) voltmeter is $\boxed{\text{Span= 10-(-10) =20 state}}$

5- ***Loading effect***

It's the change of circuit parameter, characteristic, or behaves due to instrument operation with out maintains.

6- ***Sensitivity***

It's represent the ratio of output signal to a change in input, or its represent the response output of the instrument to a change of its input.

7- ***Resolution***

The smallest change in input that the instrument can response to it, or the ratio of output to smallest change in input.

8- ***Error***

The deviation of the measured value from the true value.

Types of Errors

No measurement can be made with perfect accuracy, but its important to find out what the accuracy actually is , and who different errors have enter to the measurement, so study of errors is a first steps in finding ways to reduce them.

Errors may come from different sources and are usually classified under two main heading:

1- ***Systematic Errors***

These types of errors have known reasons, and we can avoided, reduce or eliminated, and estimated it. These errors are subdivided into:

a) ***Gross (Human) Errors***

i) Misreading of instruments and observation errors.
ii) Improper choice of instrument, or the range of instrument.
iii) Incorrect adjustment or forgetting to zero.
iv) Erroneous calculations, computation mistakes, and estimation errors.
v) Neglect of loading effects.
vi) Proper position for measuring human.

b) ***Instrumentation (Equipment) Errors***

i) Damaged equipment such as defective due to loading effect or worn parts.
ii) Calibration errors.
iii) Bearing fraction.
iv) Component nonlinearities.
v) Loss during transmission.
vi) Proper position of equipment (vertical or horizontal).
vii) Static charge error.

c) ***Environmental Errors***

i) Change in temperature, pressure.
ii) Humidity.
iii) Stray electric and magnetic fields.
iv) Mechanical vibration.
v) Weather variations (day, night, and four seasons).

d) ***Measuring Errors***

Measuring human does not have enough efficiency and experience to expect the true measurement values and the reasons of errors.

2- ***Random Errors***

Those due to causes that can not be directly established because of unknown events that causes small variation in measurement, quite random and unexplained. We can reduce this type of errors after treatment the systematic errors by taking many reading for the measuring value and apply statistical analysis to determine the best true estimate of measurement readings.

Example (1):

(Systematic, Human errors, the proper range of measurement)

A 0 to 150V voltmeter has accuracy of 1% of full scale reading. The theoretical (true) expected value we want to measure it is 83V. Determine the practical (measured) value and the percentage of error.

Sol.:

$\boxed{\text{Tolerance= accuracy x } V_{FSD}}$

Tolerance= 1% x 150 = 0.01 x 150 = 1.5V

Measured value= true ± tolerance

Measured value= 83 ± 1.5

Measured value= 84.5V or 81.5V

The percentage error is:

$$\boxed{\textbf{errors} = \frac{\textbf{true} - \textbf{measured}}{\textbf{true}} \times 100\%}$$

$$\textbf{error} = \frac{|83 - 84.5|}{83} \times 100\% = 1.81\%\text{, or} \qquad \textbf{error} = \frac{|83 - 81.5|}{83} \times 100\% = 1.81\%$$

Or $\boxed{\textbf{error} = \frac{|\pm \textbf{Tolerance}|}{\textbf{True}} \times 100\%} = \frac{|\pm 1.5|}{83} \times 100\% = 1.81\%$

If we want to measured anther readings on the same range and determine the error, suggest we take true 60V, and 30V.

For 60V the error is:

$$\textbf{error} = \frac{|\pm \textbf{Tolerance}|}{\textbf{True}} \times 100\% = \frac{|\pm 1.5|}{60} \times 100\% = 2.5\%$$

And for 30V

$$\textbf{error} = \frac{|\pm \textbf{Tolerance}|}{\textbf{True}} \times 100\% = \frac{|\pm 1.5|}{30} \times 100\% = 5\%$$

So we can see that the error is increased as smaller voltage is measured, thus take the proper range for every measured value, the range that give big deflection on the pointer as possible.

Example (2):

(Systematic, Human errors, the difference between theoretical and practical instruments)

To measured unknown resistor by ammeter and voltmeter method. A voltmeter of sensitivity 1000Ω/V, connect in parallel with the resistor reads 100V on its 150V scale (range), while the series ammeter read 5mA. Calculate the apparent value of the resistor, actual value, and the error.

Sol.:

1- The apparent value of the resistor is:

$$\boxed{R_{ap.} = \frac{V}{I}} = \frac{100}{5mA} = 20K\Omega$$

2- The actual value of the resistor by taking the resistance of voltmeter in consider is:

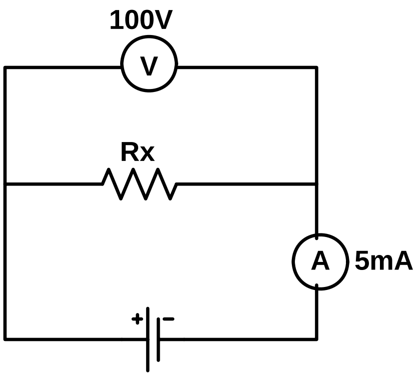
$$\boxed{R_V = 1000\frac{\Omega}{V}\times 150V = 150K\Omega}$$

$$\boxed{R_{act.} = \frac{R_{ap.}\times R_v}{R_v - R_{ap.}}} = \frac{20\times 150}{150-20} = \boxed{23.05K\Omega}$$

3- The percent error is:

$$\boxed{error = \frac{actual - apparent}{actual}\times 100\%}$$

$$error = \frac{23.05-20}{23.05}\times 100\% = 13.22\%$$



Limiting Error

In most indicating instruments the accuracy is guaranteed to a certain percentage to a full scale reading. The limits of this deviation from the specified value are known as limiting errors or guarantee errors. For example, if the resistance of a resistor is given as 500Ω±10%, the manufacture guarantees that the resistance full between the limits 450Ω and 550Ω.

Estimation of Random Errors

These errors are due to unknown causes and occur even when all systematic errors have been accounted for. In well designed experiments few random errors usually occur, but they become important in high accuracy work. The only way to offset these errors is by increasing the number of readings and using statistical means to obtain the best approximation of the true value of the quantity under measurement.

Statistical Analysis of Data

To make statistical methods useful, the systematic errors should be small compared with random errors because statistical treatment can not improve the accuracy of measurement.

1- ***Arithmetic Mean(X`):***

It's the value lie in the medial number of measured variable and represents the most accurate measured value for the true value. Arithmetic mean is given by:

$$\overline{X} = \frac{\sum F_i \cdot X_i}{\sum F_i}$$

, where X_i is the reading values taken, and F_i is the number that each reading is occur in the measurements, or the frequency number of each reading.

2- ***Deviation From The Mean(d_i):***

Deviation is the departure of a given reading from the mean value. It's given by:

$$d_i = X_i - \overline{X}$$

The deviation from the mean may have a positive or a negative value and the algebraic sum of all the deviation must be zero in symmetrical curve.

3- ***Average Deviation(D):***

The average deviation is the sum of the ***absolute*** values of deviations divided by the number of readings

$$D = \frac{\sum \left| F_i \cdot d_i \right|}{\sum F_i}$$

where $\sum F_i = n$, and n= number of all readings

4- ***Standard Deviation(σ):***

It's the root mean square deviation, and the standard deviation represents the variation of the reading from the mean value. For a finite number of reading

$$\sigma = \sqrt{\frac{\sum F_i . (d_i)^2}{n-1}}$$

5- ***Variance(υ):***

It's defined as mean square standard deviation

$$v = \sigma^2$$

6- ***Probable Error (r):***

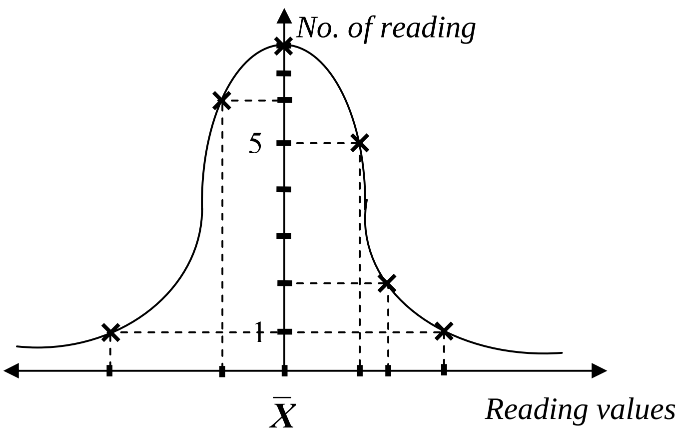
It's the maximum chance (50%) that any given measurement will have a random error no greater than ±r

$$r = \pm 0.6745\sigma$$

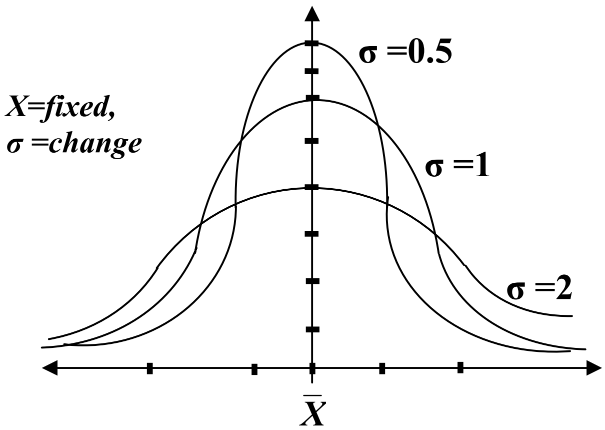
7- <u>*Gaussian Distribution Curve:*</u>

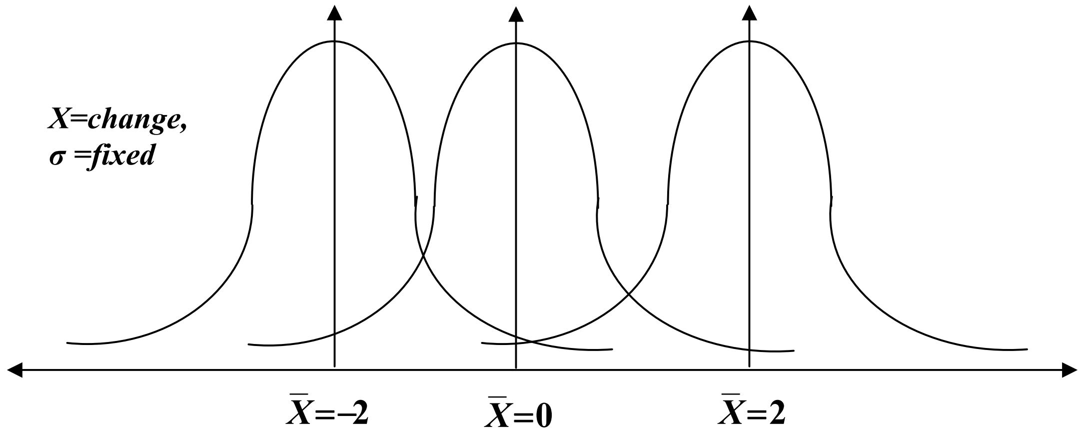
It's the normal distribution curve for random errors where $\overline{X}$ in the centre of this curve. The random errors may be positive or negative with respect to $\overline{X}$ thus lie at the two side of the curve, small errors are more probable than large errors. Gaussian curve is drawn by the following equation:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\bar{x})^2 / 2\sigma^2} \qquad -\infty < X < +\infty \qquad -\infty < \sigma < +\infty$$



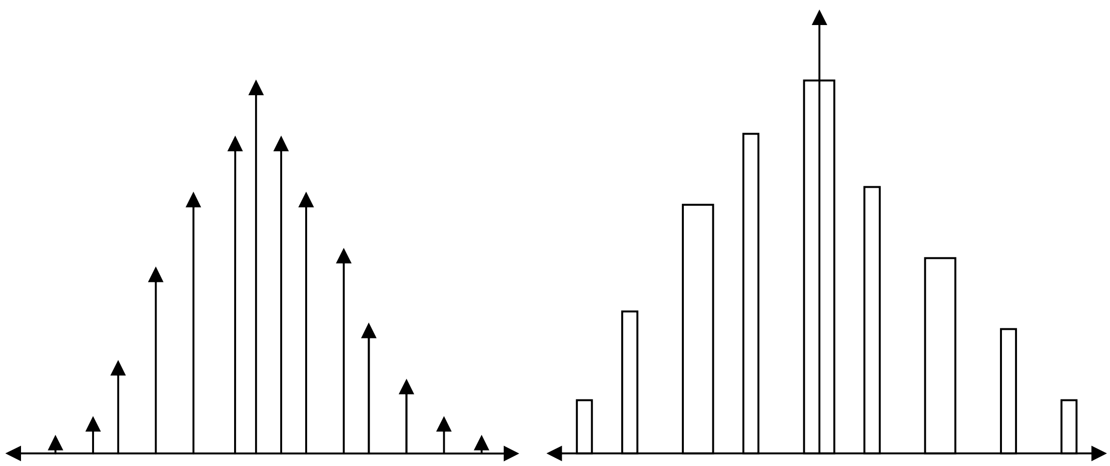
There are two factors effecting Gaussian curve shape $\overline{X}$ and σ as shown:

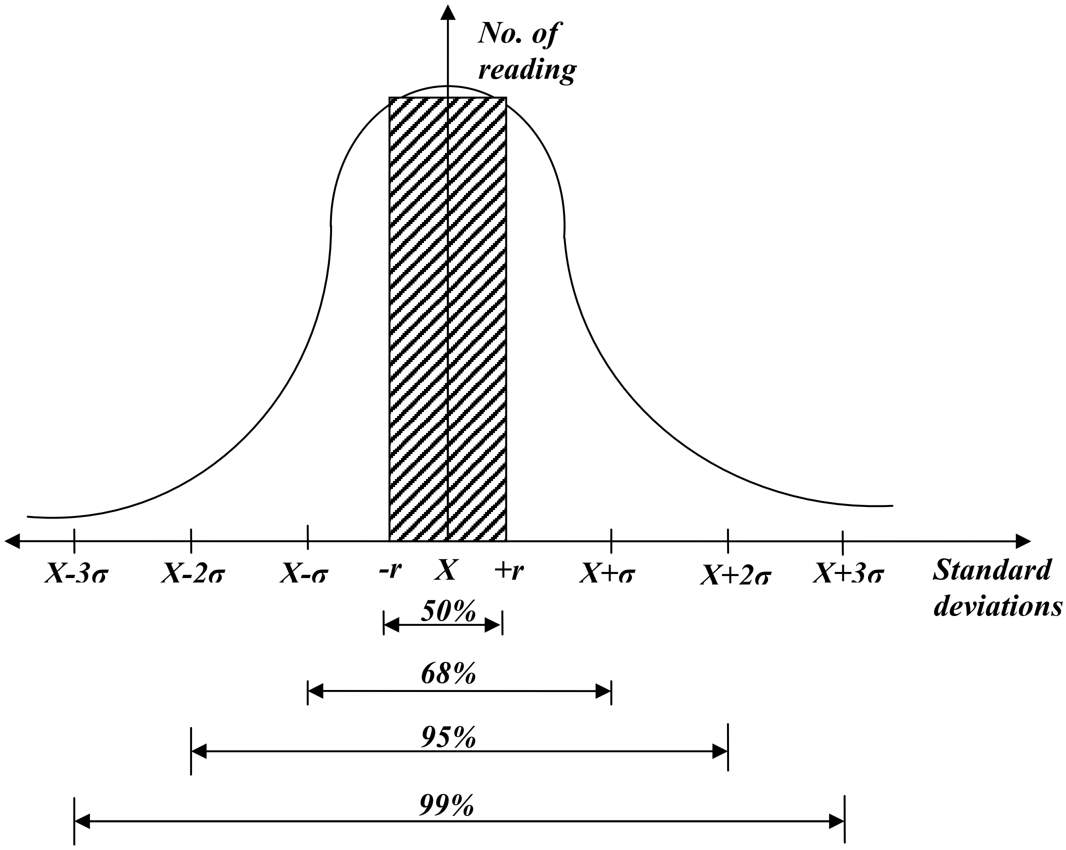




8- *Histogram:*

Graphically represent the number of observed reading against the observed value is called the histogram and the connection between the distributions of observation is called Gaussian curve





Example:

The following readings were recorded for voltage measurement:

10.1, 9.7, 10.2, 9.6, 9.7, 10.1, 9.6, 9.7, 10.1

Calculate:

1. Arithmetic mean (X)
2. Deviation from the mean (d)
3. Average deviation (D)
4. standard deviation (σ)
5. Variance (V)
6. probable error (±r)

Sol.:

Rearrangement the reading in two columns with its frequency or (number of reading), thus

Reading values	***No. of reading***	***di(xi-x)***
10.1	3	0.3
9.7	3	-0.1
9.6	2	-0.2
10.2	1	0.4

1- $\overline{X} = \dfrac{\sum F_i \cdot X_i}{\sum F_i} = \dfrac{3(10.1) + 3(9.7) + 2(9.6) + (10.1)}{9} = 9.8volt$

2- $d_i = X_i - \overline{X}$

$d_1 = 10.1 - 9.8 = 0.3volt$

$d_4 = 9.7 - 9.8 = -0.1volt$

$d_7 = 9.6 - 9.8 = -0.2volt$

$d_9 = 10.2 - 9.8 = 0.4volt$

3- $D = \dfrac{\sum |F_i \cdot d_i|}{\sum F_i} = \dfrac{3(0.3) + 3(0.1) + 2(0.2) + (0.4)}{9} = 0.22volt$

4- $\sigma = \sqrt{\dfrac{\sum F_i .(d_i)^2}{n-1}} = \sqrt{\dfrac{3(0.09) + 3(0.01) + 2(0.04) + (0.16)}{8}} = 0.26volt$

5- $\nu = \sigma^2 = (0.26)^2 = 0.067volt^2$

6- $r = \pm 0.6745\sigma = \pm 0.6745(0.26) = \pm 0.175volt$