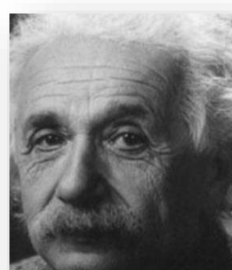
Fundamentals of Control Engineering

Al- Safwa University College

Department of Computer Engineering Techniques

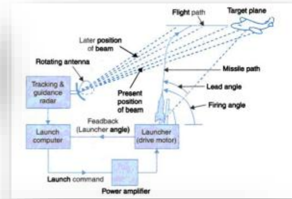
Third Year



Albert Einstein

Any one can know. The point is to understand.

INTRODUCTION



A **system** is an arrangement of physical components connected or related in such a manner as to form and/or act as an entire unit.

The word **control** is usually taken to mean *regulate*, *direct*, or *command*.

A control system : is an arrangement of physical componets connected or relate in such a manner as to command , direct, or regulate itself or another system.

HISTORICAL REVIEW

During the 1900-1940

Automatic Airplane Pilot (AAP)

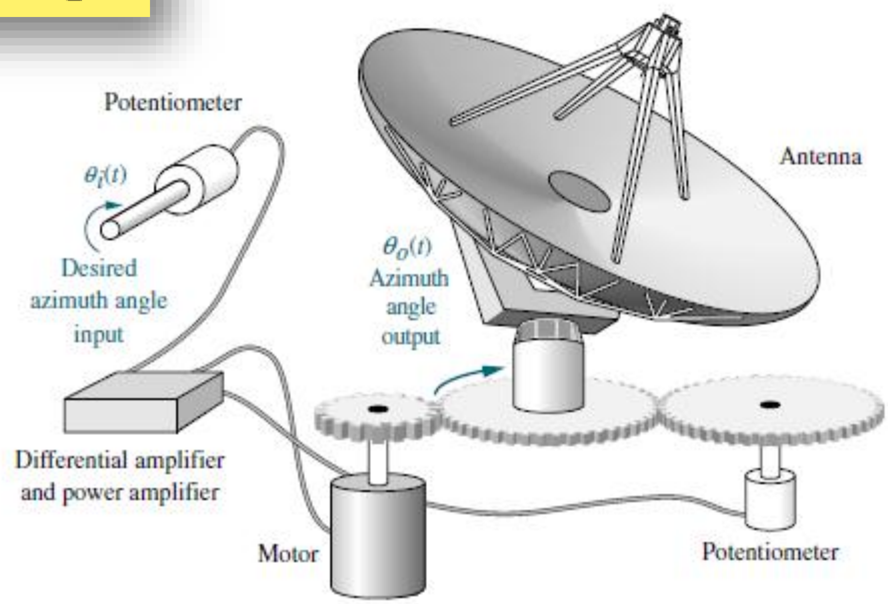
Gun Position System (GPS)

Radar Antenna Control (RAC)

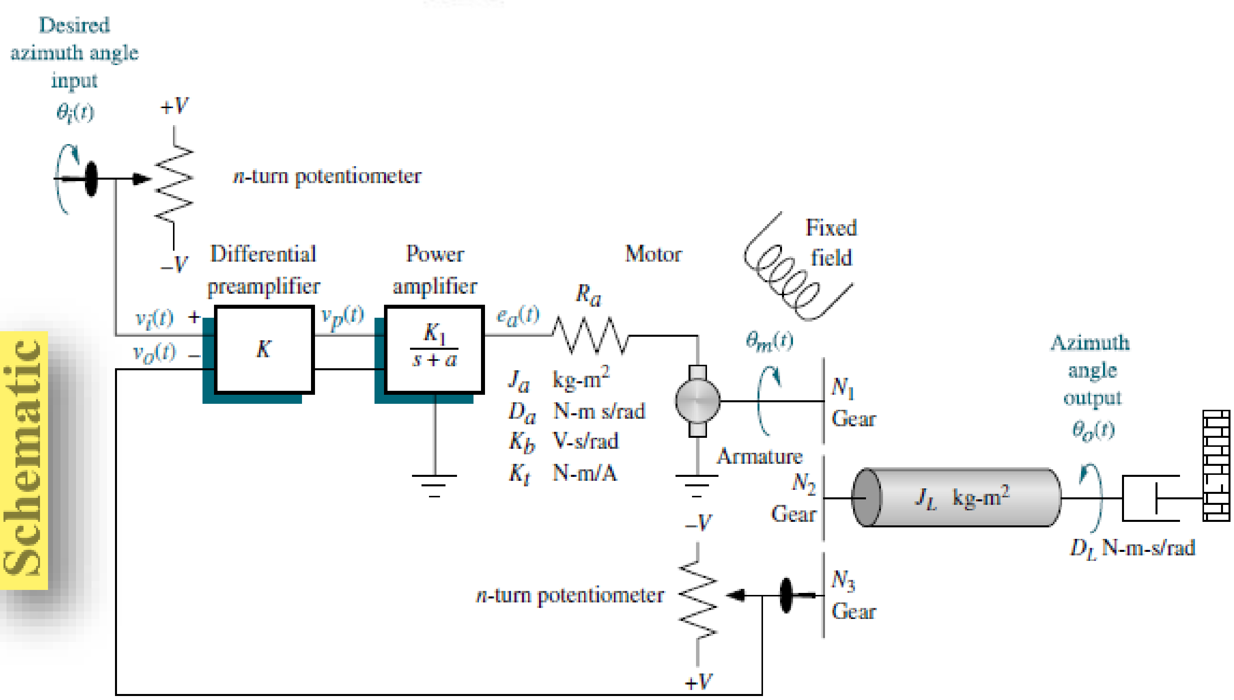
Leads to develop control systems especially servomechanism system which is a system slave to command.

Theory of cybernetics , theory of servomechanism and process control all are converging now and unified feedback control theory has emerged

Example



Schematic



Disturbance

It is uncontrolled and may be originated from outside or from within the process.

Disturbance, may be random and beyond our control

INPUT

Control Element (or) Process (or) Plant

Output

Input is the variable that affects the process.

It is controlled by controller

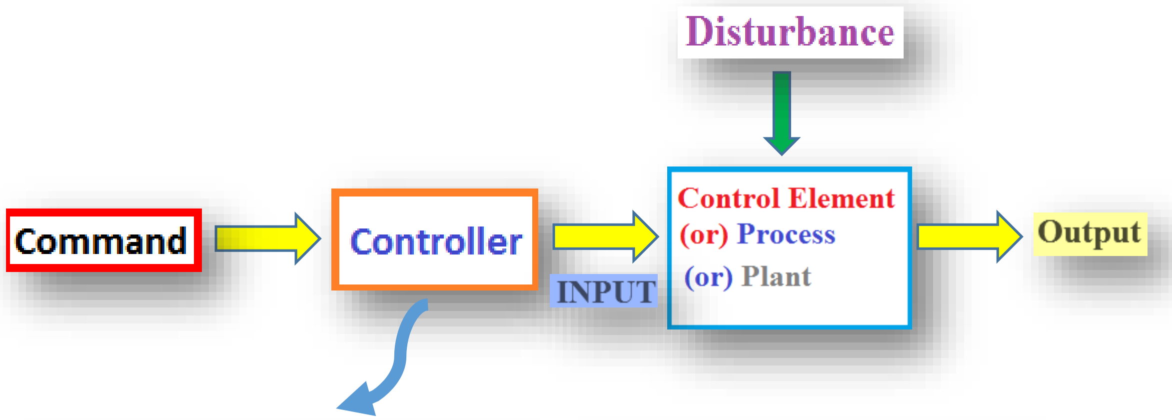
Output is the response to the input variable

Controlled element is calledsometimes as a process , or , plant .

Examples : Temperatur Gas, Liquid

Disturbance may affect the process or control system and make it non efecteive.

Also, some system parameters may change with time, therefore, these parameters will be considered some kind of disturbance sources.



The function of controller is to make the output (or response) follow the system commnds .

Controller doesnot know about "disturbance" effect and it will ignore it .

Controller will get command or information from the user and acts upon the signal to generate an input whose function is to force the controlled output to follow the command.

Information available to controller is from command "only"

Controller which may be designed for a specific information about disturbance will be no more effective in making the output to follow the command .

= Error !

Between the output and command beca-use of DISTURBANCE

This structure is called as an open – loop control .

Open – loop means that the loop has not been closed to give you information about the controlled output .

The controller **needs additional information** about the **disturbance** which is random and this is a difficult .

To solve this difficult problem ,
let the disturbance signal affect
the process (or plant) so that
this effect will appear on the output.

HOW ?? = SENSOR !!

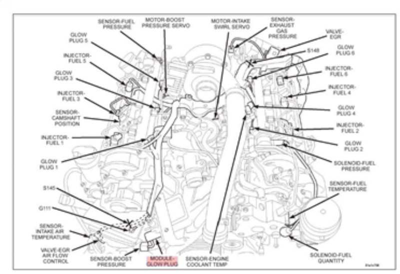
SENSOR

is a device which coverts the physical quantity into electrical quantity (volt.)

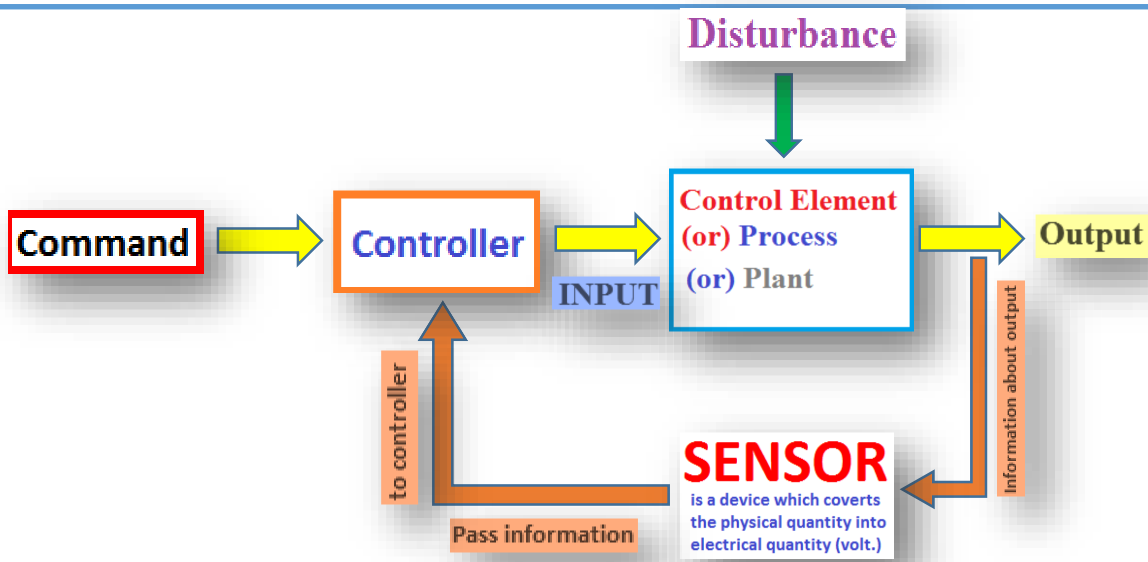
Temperature
Pressure
Hieght
Motion
Concentration
→ **SENSOR** → + Voltage -

Chrysler

oil pressure sensor



So, we will put a **sensor that gets information** about the **output** (controlled) and passes on this information to the controller .



So , you note that the controller is not intelligent , it gets information via this particular feedback structure .

CONTROLLER

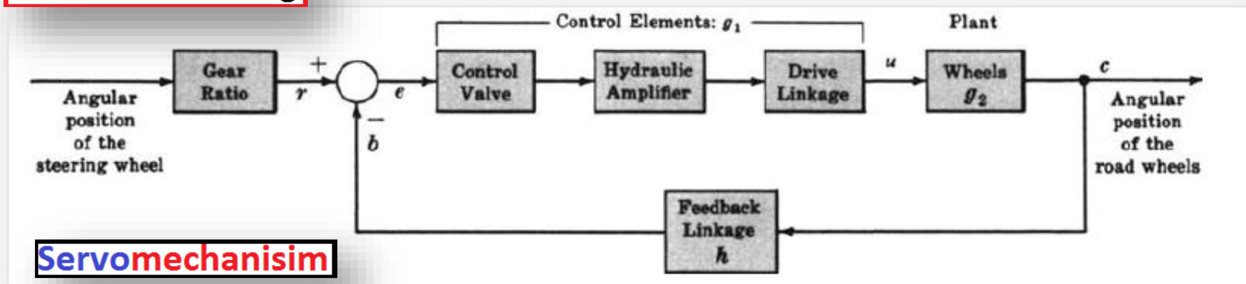
Compare, the actual output (information from sensor) and command signal, and generate an ERROR signal

Error signal will be utilized to generate suitable control signal and that control signal will modify the signal to plant to reduce the error to zero.

This system is called **CLOSED-LOOP SYSTEM** OR **FEEDBACK CONTROL SYSTEM**

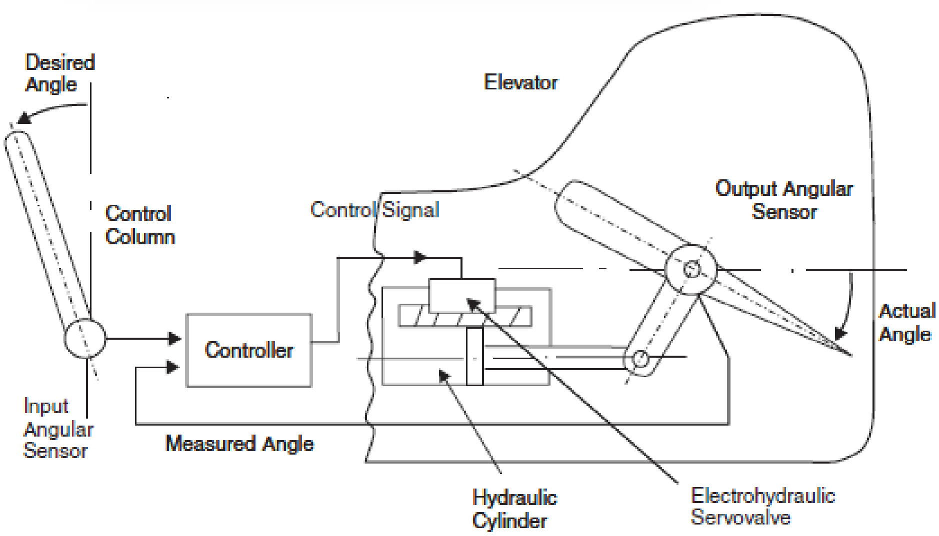
Examples

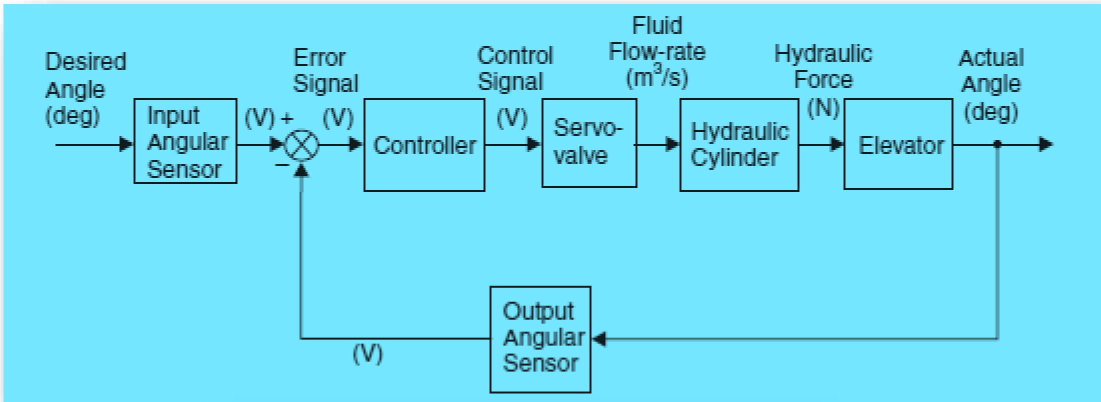
Car Power Steering



The command input is the angular position of the steering wheel. A small rotational torque applied to the steering wheel is amplified hydraulically, resulting in a force adequate to modify the output, the angular position of the front wheels.

Elavator control system of high speed jet

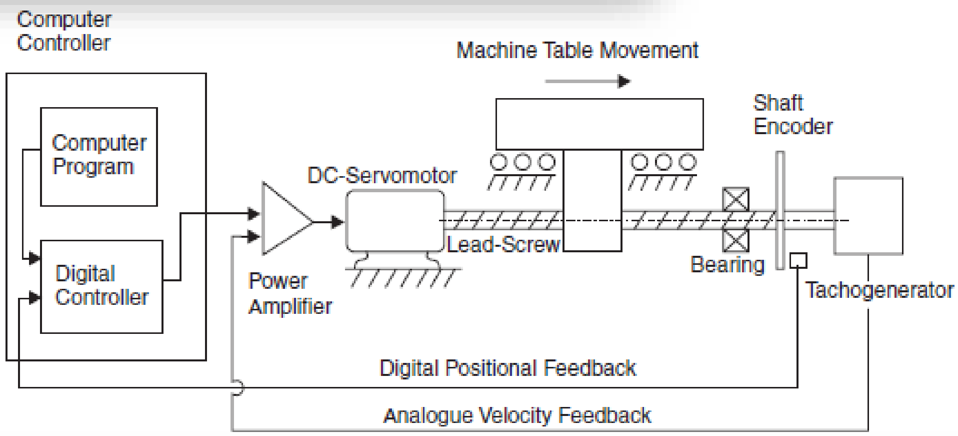


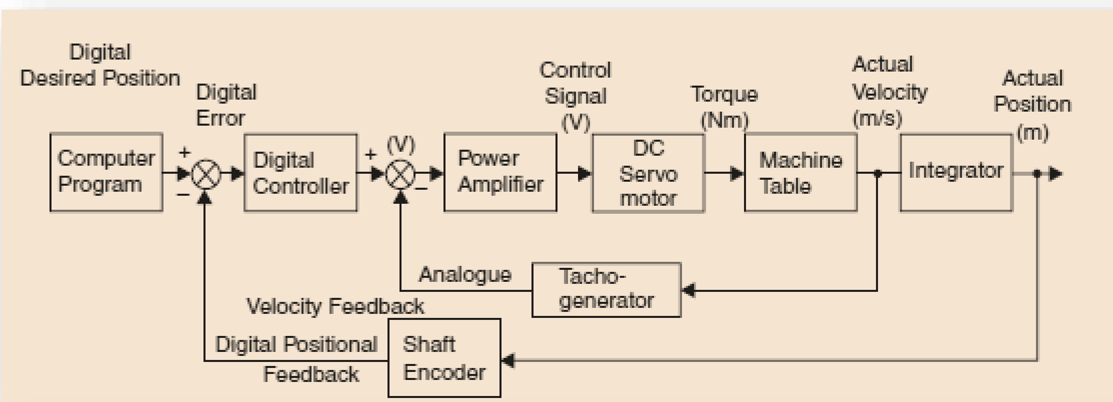


Elavator control system of high speed jet

Movement of the control column produces a signal from the input angular sensor which is compared with the measured elevator angle by the controller which generates a control signal proportional to the error. This is fed to an electrohydraulic servovalve which generates a spool-valve movement that is proportional to the control signal, thus allowing high-pressure fluid to enter the hydraulic cylinder. The pressure difference across the piston provides the actuating force to operate the elevator.

Computer Numerically Controlled (CNC) machine tool





Information relating to the shape of the work-piece and hence the motion of the machine table is stored in a computer program.

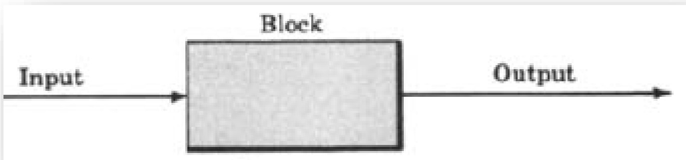
This is converted to an analogue control signal which, when amplified, drives a d.c. servomotor. Connected to the output shaft of the servomotor (in some cases through a gearbox) is a lead-screw to which is attached the machine table, the shaft encoder and a tachogenerator.

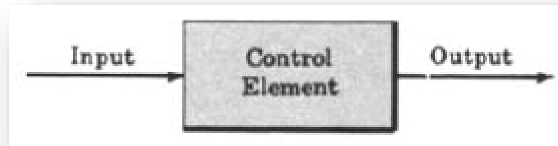
The purpose of this latter device, which produces an analogue signal proportional to velocity, is to form an inner, or minor control loop in order to dampen, or stabilize the response of the system.

Control Systems Terminology

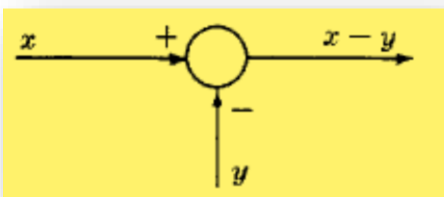
BLOCK DIAGRAMS:

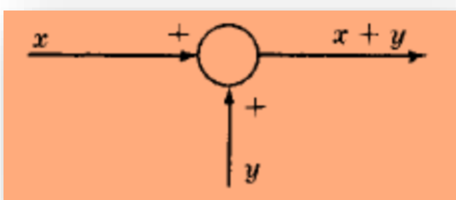
We use block diagrams to represent control system

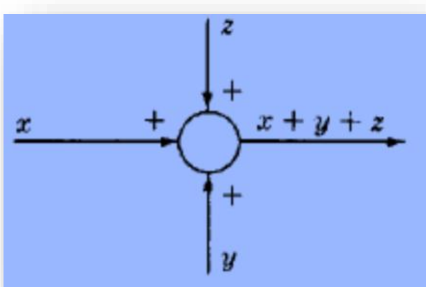


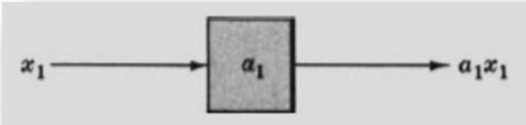


Other symbols

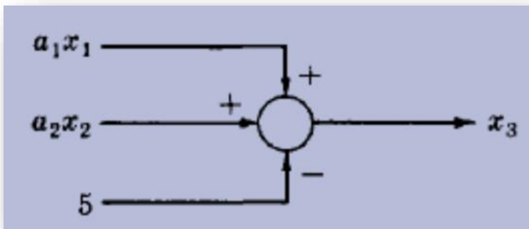




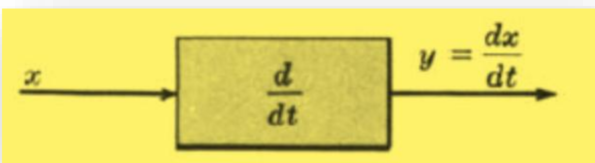




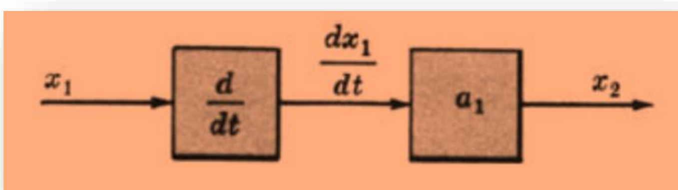
x_1
a_1
a_1x_1



a_1x_1
a_2x_2
5
+
+
−
x_3

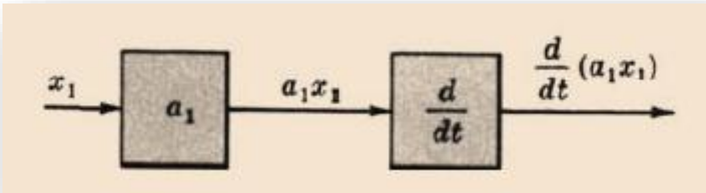


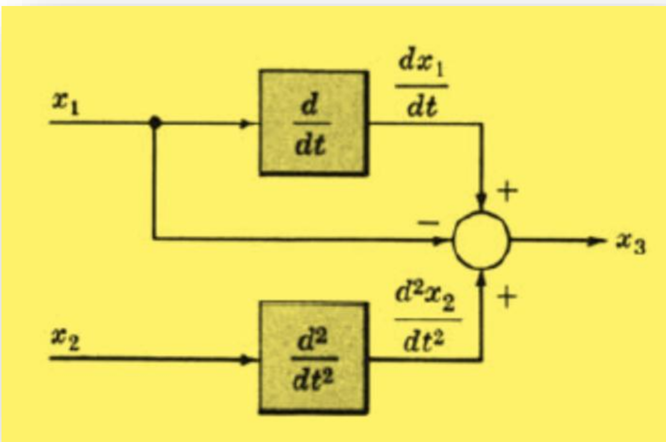
x
$\frac{d}{dt}$
$y = \frac{dx}{dt}$

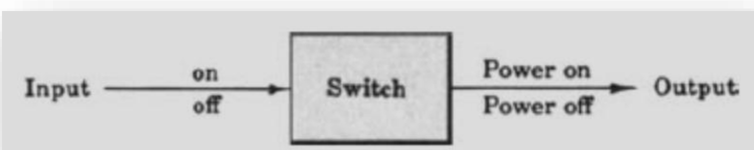


x_1
$\frac{d}{dt}$
$\frac{dx_1}{dt}$
a_1
x_2

Note:


x_1
a_1
$a_1 x_1$
$\frac{d}{dt}$
$\frac{d}{dt}(a_1 x_1)$

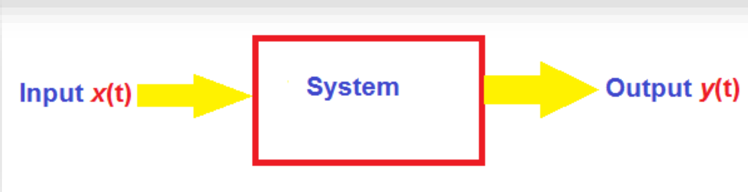

x_1
$\frac{d}{dt}$
$\frac{dx_1}{dt}$
+
−
x_3
$\frac{d^2x_2}{dt^2}$
+
x_2
$\frac{d^2}{dt^2}$


Input
on
off
Switch
Power on
Power off
Output

CHAPTER TWO

Differential Equations and Laplace Transforms for Control Systems

In general, input signal and output signal are functions of time



Accordingly, input and output signals charge other parameters.
So we have a ratio of change

Ratio of change = $\frac{change\ in\ x}{change\ in\ t}$

Change = Difference= new value – old value

Change in x = $\Delta x = x_{new} - x_{old}$

Change in t = $\Delta t = t_{new} - t_{old}$

ratio of charge is $\frac{\Delta x}{\Delta t}$

Δ: letter D in greek

Sometimes we replace Δ by d especially in equations, so $\frac{\Delta x}{\Delta t} \cong \frac{dx}{dt} = \frac{derivative\ of\ x}{derivative\ of\ t}$

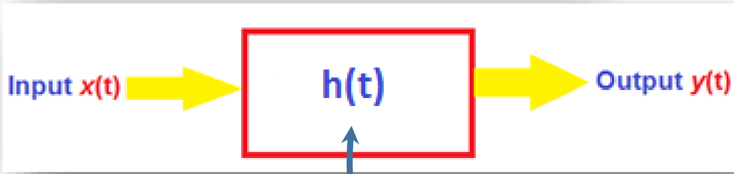
When $\frac{dx}{dt}$ is included in some equation and related to other variables, we have what is called “Differential equation”.

Differential Equations relates some functions with their derivaties

To analyze the dynamic operation of system. We need to know the relationship between the input and output in order to determine the “response” of the system.

Reaction of system to input signal.

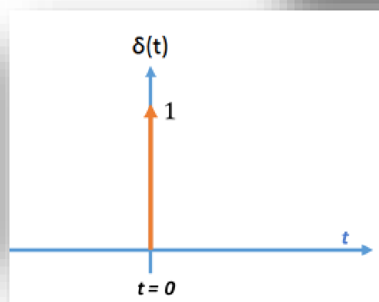
Input and output signals are in time domain.

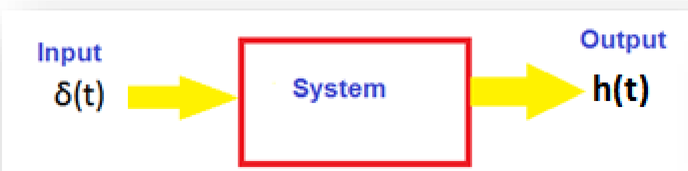


Impulse Response of System

It is the output of system when the input is impulse function

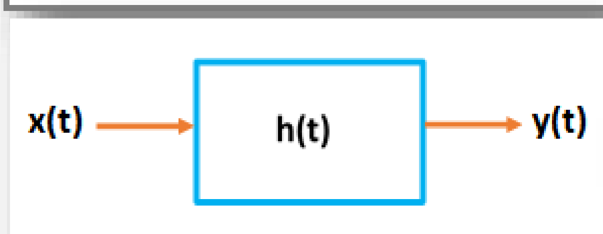
$$\delta(t) = \begin{cases} 1 \ at \ t = 0 \\ 0, other \ wise \end{cases}$$





So, δ(t) can be used to know the time response **h(t)** of any system

Then, any output can be determined according to the Input and h(t)



$y(t) = x(t) * h(t)$

Convolution is mathematical operation which gives you the result of multiplication of two signals.

Analysis (Determination) of output signal either in terms of output time response y(t) or differential equations is quite difficult.

To make it easier, we use **Laplace transform (LT)**

It transforms signal from time domain to complex frequency domain or the s-domain

Pierre-Simon Laplace (1749–1827)

$$f(t) \Leftrightarrow F(s)$$

$$s = \sigma + j\omega$$

It tells how fast exponential portion of the signal is dropping (unit = Naper)

It tells you what is the oscillation frequency, (oscillation nature)

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$tu(t)$	$\frac{1}{s^2}$
$t^n u(t)$	$\frac{n!}{s^n + 1}$
$e^{-at}u(t)$	$\frac{1}{s+a}$
$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

$$L\,[x(t)] = X(s)$$

$$L[y(t)] = Y(s)$$

$$L[h(t)] = H(s)$$

$$F(s) = \int_{-\infty}^{\infty} f(t)e^{-st}\,dt$$

$$Y(s) = H(s)\,X(s)$$

$$H(s) = \frac{Y(s)}{X(s)}$$

Transfer function

Example

Determine the output transform $Y(s)$ for the differential equation

$$\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} - \frac{dy}{dt} + 6y = \frac{d^2u}{dt^2} - u$$

where y = output, u = input, and initial conditions are

$$y(0\ \ ) = \left.\frac{dy}{dt}\right|_{t=0} = 0 \qquad \left.\frac{d^2y}{dt^2}\right|_{t=0} = 0$$

Example

What is the transfer function of a system whose input and output are related by the following differential equation?

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = u + \frac{du}{dt}$$

Example

$$\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 21\frac{dy}{dt} + 26y = u$$

Example

$$\frac{d^3y(t)}{dt^3} + 13\frac{d^2y(t)}{dt^2} + 32\frac{dy(t)}{dt} + 20y(t) = 2\frac{dr(t)}{dt} + 5r(t)$$

Example

$$\frac{d^4y(t)}{dt^4} + 15\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 5y(t) = 5r(t)$$

Example

$$\frac{d^3y(t)}{dt^3} + 20\frac{d^2y(t)}{dt^2} + 10\frac{dy(t)}{dt} + 2y(t) + 2\int_0^t y(\tau)\,d\tau = \frac{dr(t)}{dt} + 5r(t)$$

Example $$\frac{dc(t)}{dt} + 2c(t) = r(t)$$

Example $$\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + 32y = 32u(t)$$

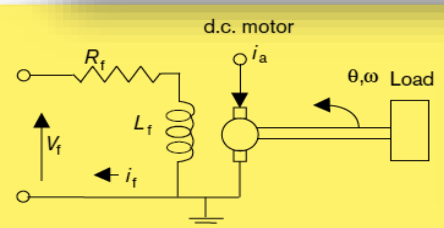
Example

$$\frac{d^3y}{dt^3}+3\frac{d^2y}{dt^2}+5\frac{dy}{dt}+y=\frac{d^3x}{dt^3}+4\frac{d^2x}{dt^2}+6\frac{dx}{dt}+8x$$

Example

$$\frac{dy}{dt}+y=u(t-T)$$

Example Find the transfer function of the DC motor whose specifications/properties shown below



D.c. motor schematic

Field circuit: $V_f(t) = R_f i_f(t) + L_f \dfrac{di_f(t)}{dt}$

Applied torque: $T_A(t) = (K_1 K_2 I_a) i_f(t)$

input signal: field voltage, $V_f(t)$

output signal: applied torque, $T_A(t)$

$V_f(t)$: Field voltage

$i_f(t)$: Field current

$i_a(t)$: Armature current

R_f: Field winding resistance = 2 Ω

L_f: Field winding inductance = 0.5 H

$K_1 K_2 I_a$, equal to 2 N m/A.

Example

In the Laser Guided Missile (LGM), control signal $u(t)$ (which depends of position of target), would control Fins angle $\beta(t)$ in order to achieve missile direction angle $\theta(t)$.

The corresponding differential equations are:

$$0.2\frac{d\beta}{dt} + \beta(t) = u(t)$$

$$\frac{d^3\theta}{dt^3} + 5\frac{d^2\theta}{dt^2} = 20u(t)$$

Determine (1) Fins transfer function
(2) "Fins to Missile direction" Transfer function

Laser Guided Missile

Example

A motor connected to load (pulley) with inertia J and friction B produces a torque proportional to the input current *i*, can be described by following differential equation

$$J\,\frac{d^2\theta}{dt} + B\,\frac{d\theta}{dt} = k\,i$$

θ is rotational postition of pulley

Considering J, B and K as constants, determine the transfer function $\theta(s)/i(s)$

Transfer Function
in terms of block diagram

$$X(s) \longrightarrow \boxed{G(s)} \longrightarrow Y(s)$$

$$G(s) = \frac{Y(s)}{X(s)}$$

G(s) is The Transfer Function

Contol System can be represented by mutiple trasfer functions or ONE TOTAL transfer function

Examples Prcatical

Unmanned Free-Swimming Submersible (UFSS) vehicle

$$\frac{\theta(s)}{\delta_e(s)} = \frac{-0.125(s+0.435)}{(s+1.23)(s^2+0.226s+0.0169)}$$

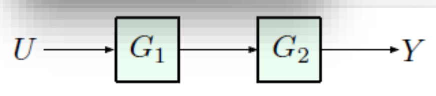
$$= \boxed{\frac{\text{Angle of Diving}}{\text{Elavator Surface}}}$$

Security Camera

$$\frac{K}{s^2+10s+K} = \boxed{\frac{\text{Camera Position}}{\text{Subject's Position}}}$$

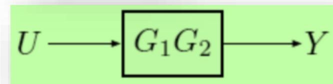
Examples

Series connection

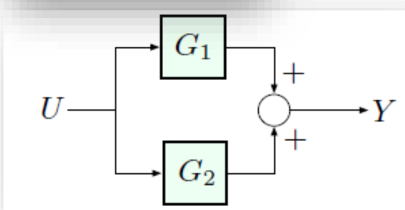


$$Y = G_1 G_2 U$$

$$\frac{Y}{U} = G_1 G_2$$

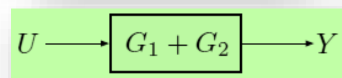


Parallel connection

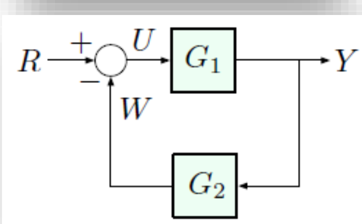


$$Y = G_1 U + G_2 U$$

$$Y = (G_1 + G_2) U$$



Negative Feedback



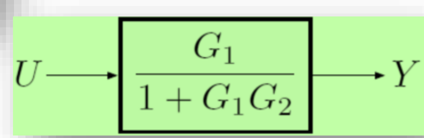
$$U = R - W$$

$$Y = G_1 U$$

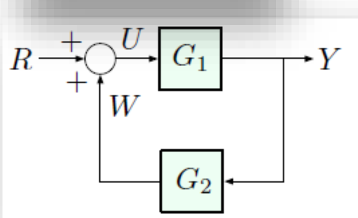
$$W = G_2 Y$$

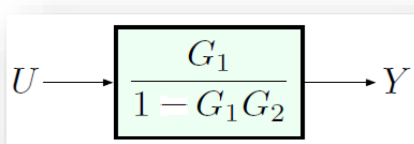
$$\therefore \quad Y = G_1 (R - W) = G_1 R - G_1 G_2 Y$$

$$Y = \frac{G_1}{1 + G_1 G_2} R$$



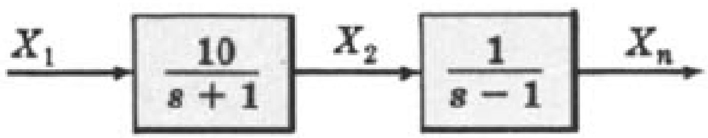
Positive Feedback

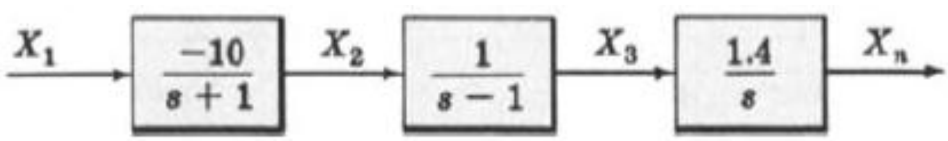


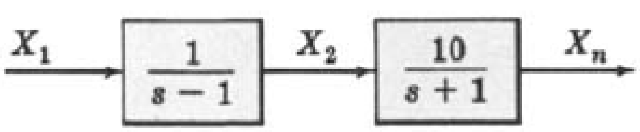


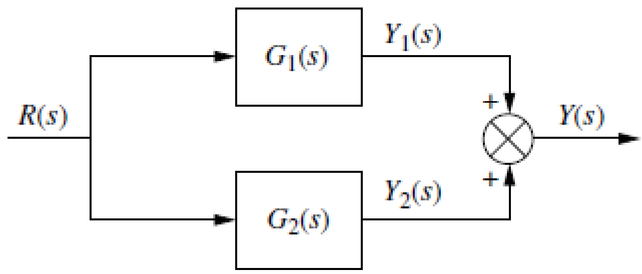
Prove !?

Find the transfer function for the following systems

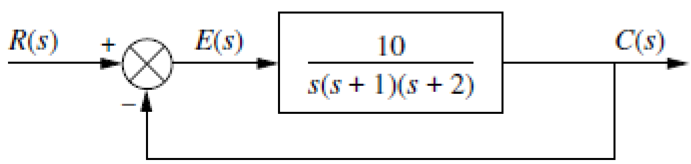




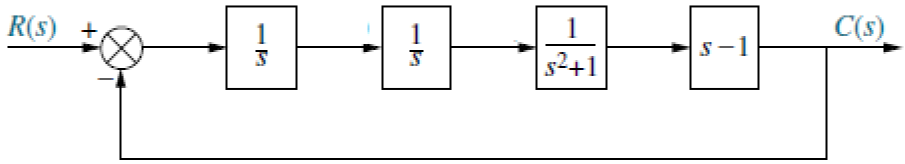


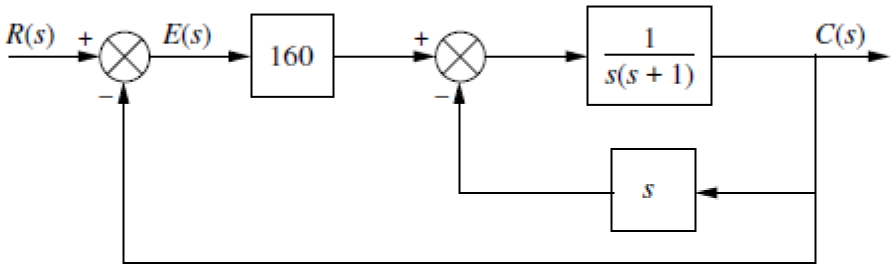


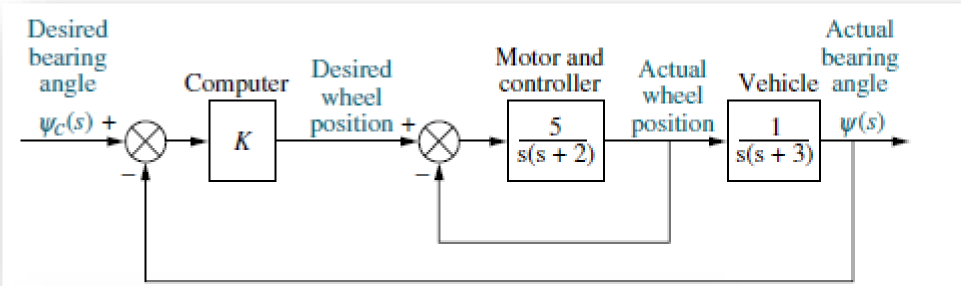
$G_1(s)$
$Y_1(s)$
$R(s)$
+
$Y(s)$
+
$Y_2(s)$
$G_2(s)$



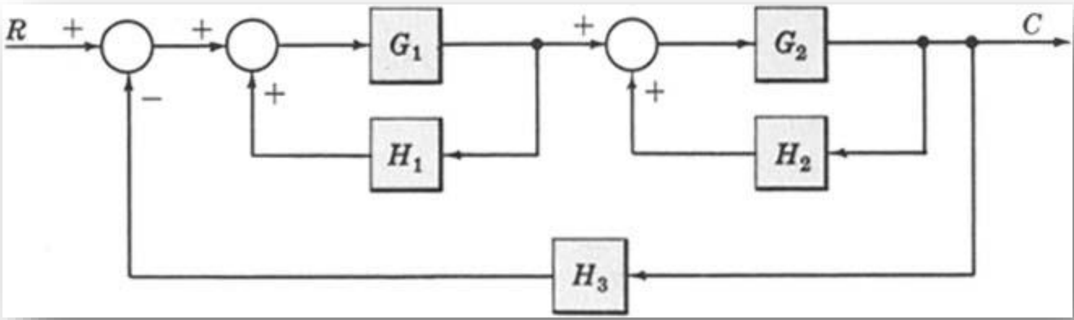
$R(s)$
+
$E(s)$
$\frac{10}{s(s+1)(s+2)}$
$C(s)$
−


R(s)
+
−
1/s
1/s
1/(s²+1)
s − 1
C(s)


R(s)
+
−
E(s)
160
+
−
1
s(s + 1)
C(s)
s



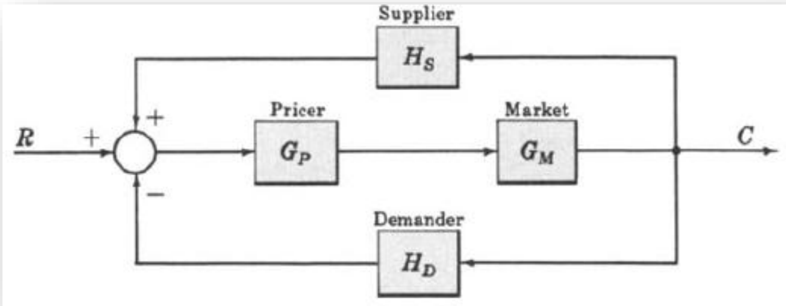
HelpMate robot used for in-hospital deliveries;

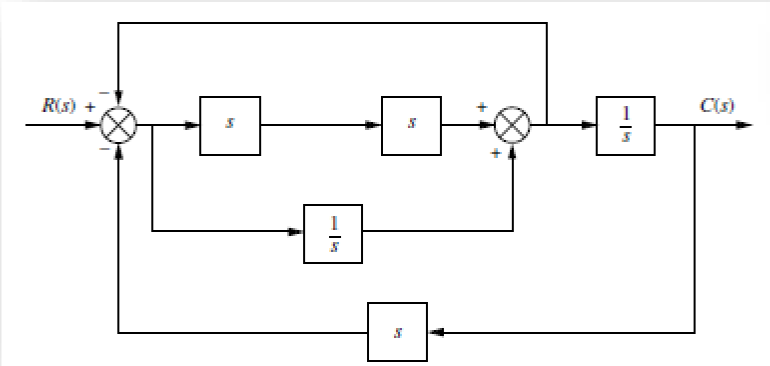


ANSWER:

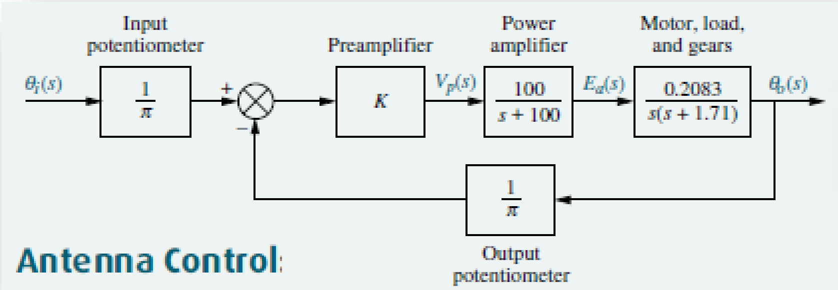
$$\frac{C}{R} = \frac{1}{H_3}$$

$H_1 = 1/G_1$ and $H_2 = 1/G_2$

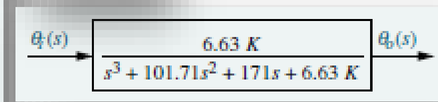

Supplier
H_S
Pricer
G_P
Market
G_M
Demander
H_D
R
+
+
−
C

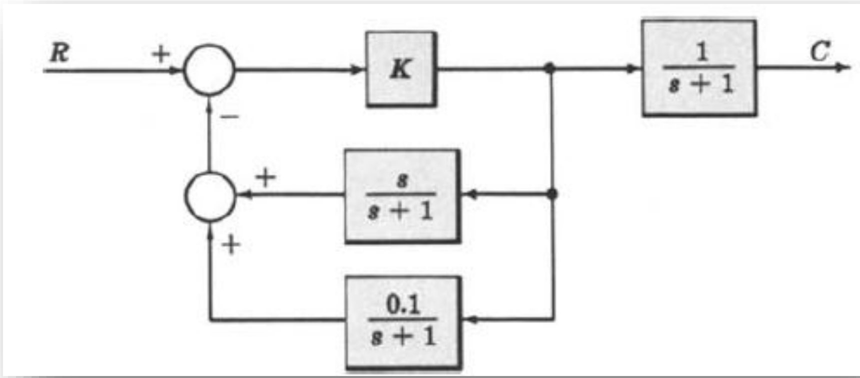


ANSWER: $T(s) = \dfrac{s^3 + 1}{2s^4 + s^2 + 2s}$


Input potentiometer
Preamplifier
Power amplifier
Motor, load, and gears
$\theta_i(s)$
$\frac{1}{\pi}$
+
−
K
$V_p(s)$
$\frac{100}{s+100}$
$E_a(s)$
$\frac{0.2083}{s(s+1.71)}$
$\theta_o(s)$
$\frac{1}{\pi}$
Antenna Control:
Output potentiometer

ANSWER:


$\theta_i(s)$
$\frac{6.63\,K}{s^3+101.71s^2+171s+6.63\,K}$
$\theta_o(s)$


R
+
K
1
s + 1
C
−
+
s
s + 1
+
0.1
s + 1

Additional Notes